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Progress in Nonlinear Dynamics and Chaos

An Application of Fuzzy Set and Approximate Reasoning for Quantitative Assessment of Group Personality

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Abstract. This paper presents an application of fuzzy set and approximate reasoning with the intention of a reflecting the fuzzy characteristic of a person's judgment in human system. Various types of approaches and techniques can be used for the purpose. When goals and constraints are stated imprecisely, decision problem grow in importance, in the investigation of social and complex systems. Using a double model based on fuzzy synthetic decision and multicriteria decision problem is demonstrated as an application. Result on test problem suggests that the model of multicriteria seem to be more accurate. It is more flexible and adaptable. This method of multicriteria decision makes it possible for all information to affect the decision and will not miss any fragmentary as well imprecise information.

Keywords: Approximate reasoning, fuzzy decision making; fuzzy synthetic and multicriteria decision; multiobjective decision making.

AMS Mathematics Subject Classification (2010): 90C29

1. Introduction

One of the most crucial problems in many decision-making methods is the precise evaluation of the pertinent data. Very often in real-life decision-making applications data are imprecise and fuzzy. A decision maker may encounter difficulty in quantifying and processing linguistic statements. Therefore it is desirable to develop decision-making methods which use fuzzy data. It is equally important to evaluate the performance of the fuzzy decision-making method. Hence, the development of useful fuzzy decision-making methods is really the need of the hour.

In first paper, on fuzzy decision making, Bellman and Zadeh [1970] suggested fuzzy model of decision making in which relevant goals and constraints are expressed in terms of fuzzy sets, and decision is determined by an appropriate aggregation of these fuzzy set. In various reputed industries, colleges, competitive examinations etc. a personality is one of the important aspect. They want to examine various aspects such as creativity, an adaptive ability to the knowledge explosion, ability of analyzing, sincerity, reasoning ability etc. as per their need.

So far, fuzzy set and approximate reasoning did not find much attention in the individual or group personality decision making problem. The evaluation of quality of individual or group in view of personality is an assessment of human system. The fuzzy set and approximate reasoning will be solicited for investigation of social and multifaceted systems as well.

2. Factors

To evaluate quality of personality, it is imperative to construct factors according to the body wishing to examine the persons or a group of persons needed by them. The factors are shown in the following table.

Factors F_i	Weights W_i	Factor Criteria	f_{in_i}
1. Social F_1	0.1	Scientific attitude	f_{11}
	0.3	Constructive outlook	f_{12}
	0.4	Cultural sensitivity to values	f_{13}
	0.2	Adjustment to environment	f_{14}
2. Mental F_2	0.1	Self-concept	f_{21}
	0.2	Self-confidence	f_{22}
	0.3	Self-learning technique	f_{23}
	0.3	Concentration	f_{24}
		Mental health	f_{25}
3.Emotional F_3	0.3	Ambitious nature	f_{31}
	0.2	Decision making	f_{32}
	0.1	Style of representation	f_{33}
	0.1	Self concept	f_{34}
	0.3	Emotional securities	f_{35}
4.operational F_4	0.2	Creativity	f_{41}
	0.1	Problem solving	f_{42}
	0.1	Enterprising responsibility	F_{43}
	0.3	Working method	f_{44}
	0.3	Working efficiency	f_{45}
5. Physique F_5	0.4	Appearance	f_{51}
	0.3	Height-weight proportion	f_{52}
	0.3	Physical health	f_{53}

3. Model

3.1. The basic level: fuzzy synthetic decision

According to Table 1, we have for the synthetic decision, the set of factors is $F_i = \{ f_{in_i} \}$, $I = 1, 2, \ldots, I = 5$, $n_i = 1, 2, \ldots, N_i$, $N_1 = 4$, $N_2 = 5$, $N_3 = 5$, $N_4 = 5$, $N_5 = 3$.

Define the evaluation set P = { P_{l_1} }, l_1 = 1, 2… L_1, L_1 = 5 if the evaluations may be divided into five grades ,

Let P = {A, B, C, D, E}. Suppose the set U is built up by the individual undergoing personality test which is to be evaluated. U = { u_m }, m = 1, 2,…, M, M = 5

The single factor evaluation for individual u_m is a fuzzy mapping f from F_i to P, f: $F_i \rightarrow$P, and the fuzzy mapping f implies a fuzzy relation which can represented by a fuzzy matrix $T_{im} \in M_{N_i L_1}$ The original data of the single factor evaluation for every individual undergoing evaluation are shown in the table2. The crisp data are normalized and we get the data of T_{im}.

The weight of the every factor is given by $W_i \in M_{1\times N_i}$, $W_1 \in M_{1\times N_1}$ for example W_1 = (0.35 0.25 0.10) etc. W_i:Fi $\rightarrow$[0, 1] i.e. W_i is a fuzzy subset of F_i it is represented by a fuzzy vector, W_i. The data of W_i are given in Table 1.

Take T_{im} as the input. A fuzzy transform from F_i to P, F(F_i)$\rightarrow$F(P), can be determined by T_{im}, see figure1given below

$$W_i \rightarrow T_{im} \rightarrow B_{im} \rightarrow W_C^T \rightarrow T_{im}.$$

Therefore we get the synthetic decision $B_{im} = W_i \circ T_{im}$. Let $W_i(f_i)$, $T_{im}(f_i, p)$ and $W_i \circ T_{im}(p)$ denote the membership functions of W_i, T_{im} and $W_i \circ T_{im}$ respectively. These membership functions are defined by $(W_i \circ T_{im})$: (p) $\rightarrow$I such that$(W_i \circ T_{im})$ (p) $\triangleq \sum_{f_i \in F_i} W_i(f_i)\, T_{im}(f_i, p)$, B_{im}: p $\rightarrow$I .It can also be represented by a fuzzy vector .

The value r_{im} of synthetic decision can be used as input of calculation of the higher level. Making a fuzzy transform again $r_{im} = B_{im} \circ W_C^T$ where $B_{im} \circ W_C^T : F_i \rightarrow$I defined by $(B_{im} \circ W_C^T)(F_i) \triangleq \sum_{p \in P} B_{im}(p)\, W_c^T(p, F_i)$.

3.2. The higher level

We form the factor set of multicriteria decision based on basic level as follows:
The factor set of multicriteria decision: F= { F_i }

The decision criteria Set: C= { C_j } j=1, 2,…,J and J= 7 where J is decision criteria. In view of quality needed, we proposed following decision criteria.

C_1: If mental ability and physical health are good then personality is considered Satisfactory.

C_2: If emotional factor and physical health is good then personality is considered Satisfactory.

C_3: If social factor, mental ability and physical health are good then personality is considered more Satisfactory.

C_4: If social factor, emotional factor and physical health are good then personality is considered more Satisfactory.

C_5: If social and mental or emotional factor are good and operational efficiency and physical health are good then personality is considered much more Satisfactory.

C_6: If the needs of C_1, C_2, C_3, C_4 and C_5 are met at the same time then personality is considered perfect.

C_7: If the social factor is bad, operational ability is bad and bad physical health then personality is considered unsatisfactory.

Assessment Set

We define A= { A_k }, k=1,2,…, K, K=5 as a assessment set.

Assessment Function

We define assessment function A: V $\rightarrow$I by

A_1- Satisfactory: $A_1(v) = v$,

A_2- More satisfactory: $A_2(v) = v^{3/2}$,

A_3- Much more satisfactory: $A_3(v) = v^2$,

A_4- Perfect: $A_4(v) = \begin{cases} 1, v = 1, \\ 0, v \neq 0, \end{cases}$

A_5- Unsatisfactory: $A_5(v) = 1-v$,

where v$\in C_j$, V= { V_l } = {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}, l=1, 2,…, L, L=11,i.e.,V is unit assessment space. Therefore eight criteria will be

If $C_1 = F_2 \cap F_5$ then A_1

If $C_2 = F_3 \cap F_5$ then A_1

If $C_3 = F_1 \cap F_2 \cap F_5$ then A_2

If $C_4 = F_1 \cap F_3 \cap F_5$ then A_2

If $C_5 = F_1 \cap (F_2 \cup F_3) \cap F_4 \cap F_5$ then A_3

If $C_6 = F_1 \cap F_2 \cap F_3 \cap F_4 \cap F_5$ then A_4

If $C_7 = \overline{F}_1 \cap \overline{F}_2 \cap \overline{F}_3$ then A_5

The single factor assessment for every individual undergone personality assessment in the higher level is a fuzzy mapping from F to U, F: $\rightarrow$U, and it can be represented by a fuzzy matrix $R = [r_{im}] \in \mathcal{M}_{J\times M}$ where R is called as input of the higher level. Processing the input according to the decision criteria, we get a fuzzy mapping, f: C $\rightarrow$U, which can be described by a fuzzy matrix

CR = $(\overline{C}_1\ \overline{C}_2\ \cdots\ \overline{C}_j\ \cdots\ \overline{C}_J\ )^T \in \mathcal{M}_{J\times M}$

Reasoning of likelihood

If x $=\overline{C}_1$ then y= A_1

If x $=\overline{C}_2$ then y= A_1

If x $=\overline{C}_3$ then y = A_2

IF x $=\overline{C}_4$ then y = A_2

If x $=\overline{C}_5$ then y = A_3

IF x $=\overline{C}_6$ then y = A_4

IF x $=\overline{C}_7$ then y = A_5

From this we get a fuzzy mapping U to V, f: U$\rightarrow$V, which can be represented by a fuzzy matrix

$$D_j = (d_j(m,l) \in \mathcal{M}_{M\times L} \text{ where } d_j(m,l) = 1 \wedge (1 - \bar{c}_j(u_m) + A_k(v_l)).$$

After that we get fuzzy multicriteria decision matrix

$$D_j = \bigcap_{j=1}^{J} D_j \triangleq (\prod_{j=1}^{J} d_j(m,l)) = (\bar{E}_1\ \ \bar{E}_2 \cdots\ \ \bar{E}_m)^T \in \mathcal{M}_{M\times L}$$ .Where D is also a fuzzy

map ping, from U to V, f: U$\rightarrow$V, and $\bar{E}_m$ is a fuzzy subset of the unit assessment space V, which represents the extent of the satisfaction for the individual u_m.

Assume $E_{m\alpha}$ is the α-level set of $\overline{E}_m \alpha \in [0,1] = I.$ The sets $E_{m\alpha}$ are ordinary subsets of v. For each $E_{m\alpha}$ the mean value of the elements in $E_{m\alpha}$ can be calculated as follows:

$$H_l(E_{m\alpha}) = \frac{1}{N_\alpha} \sum_{n=1}^{N_\alpha} Z_n(\alpha)$$

where, α is the level of the level set $Z_n(\alpha)$ is the element in $E_{m\alpha}, Z_n(\alpha) \in E_{m\alpha}$ and N_α is the cardinality of the finite set $E_{m\alpha}$.

We calculate the point value [4] of $\overline{E}_m$ as

$$S(m) = \frac{1}{\alpha_{max}} \sum_{l=1}^{L} H_l(E_{m\alpha}) \Delta\alpha_l$$

Where α_{max} is the maximum membership grade of $\overline{E}_m$, and $\Delta\alpha_l = \alpha_l - \alpha_{l-1,} \alpha_0 = 0$ .We find the point value for each fuzzy subset $\overline{E}_m$ which is the best satisfaction value for each individual.

4. Example

We have applied above model to assess the personality of five individuals working in same organization. Five individuals and their corresponding responses to the f_{in_i} are considered here.

The original data are normalized and we get the data of T_{im}

$T_{im} \in M_{N_i L_1}$, i=1, 2,…, 5, $L_1 = 5$, $N_1 = 4$, $N_2 = 5$, $N_3 = 5$, $N_4 = 5$, $N_5 = 3$, m=1, 2, …, M, M=5

For factor F1: setting m=1 and i=1, 2,…, 5,

$$T_{11} \in M_{N_1 \times L_1} = \begin{pmatrix} 0.1 & 0.2 & 0.5 & 0.2 & 0.0 \\ 0.2 & 0.2 & 0.2 & 0.4 & 0.0 \\ 0.2 & 0.0 & 0.5 & 0.2 & 0.1 \\ 0.1 & 0.4 & 0.2 & 0.2 & 0.1 \end{pmatrix}$$

$$T_{21} \in M_{N_2 L_1} = \begin{pmatrix} 0.1 & 0.2 & 0.5 & 0.2 & 0.0 \\ 0.0 & 0.2 & 0.2 & 0.2 & 0.4 \\ 0.1 & 0.1 & 0.5 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.2 & 0.0 \\ 0.4 & 0.2 & 0.2 & 0.1 & 0.1 \end{pmatrix}$$

$$T_{31} \in M_{N_3 L_1} = \begin{pmatrix} 0.2 & 0.2 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.5 & 0.1 & 0.0 \\ 0.1 & 0.2 & 0.2 & 0.3 & 0.5 \\ 0.2 & 0.2 & 0.4 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.2 & 0.5 & 0.0 \end{pmatrix}$$

$$T_{41} \in M_{N_4 \times L_1} = \begin{pmatrix} 0.2 & 0.1 & 0.5 & 0.1 & 0.1 \\ 0.0 & 0.2 & 0.1 & 0.2 & 0.4 \\ 0.1 & 0.2 & 0.2 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.5 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.4 & 0.2 & 0.0 \end{pmatrix}$$

$$T_{51} \in M_{N_5 \times L_1} = \begin{pmatrix} 0.2 & 0.2 & 0.4 & 0.2 & 0.0 \\ 0.2 & 0.5 & 0.2 & 0.1 & 0.0 \\ 0.1 & 0.2 & 0.5 & 0.1 & 0.1 \end{pmatrix}$$

Using

W_c = (1.0 0.8 0.6 0.4 0.2) and $B_{im} = W_i \circ T_{im}$

B_{11} =(0.17 0.16 0.35 0.26 0.06), B_{21} =(0.28 0.19 0.68 0.17 0.12), B_{31} =(0.16 0.2 0.27 0.17 0.12), B_{41} =(0.14 0.16 0.4 0.17 0.1), B_{51} =(0.14 0.26 0.42 0.08 0.06)

We find

r_{11} =0.624, r_{21} =0.500, r_{31} =0.932, r_{41} =0.596, r_{11} =0.644. The basic calculations are completed, and we get R as the input of the higher level.

$$R = \begin{bmatrix} 0.264 & 0.87 & 0.63 & 0.286 & 0.606 \\ 0.500 & 0.614 & 0.918 & 0.58 & 0.732 \\ 0.932 & 0.422 & 0.728 & 0.554 & 0.652 \\ 0.596 & 0.552 & 0.64 & 0.530 & 0.616 \\ 0.644 & 0.62 & 0.772 & 0.680 & 0.656 \end{bmatrix}$$

$$CR = \begin{bmatrix} 0.50 & 0.614 & 0.772 & 0.58 & 0.656 \\ 0.644 & 0.422 & 0.728 & 0.554 & 0.652 \\ 0.264 & 0.614 & 0.63 & 0.286 & 0.606 \\ 0.264 & 0.422 & 0.63 & 0.286 & 0.606 \\ 0.264 & 0.552 & 0.63 & 0.286 & 0.606 \\ 0.264 & 0.422 & 0.63 & 0.286 & 0.606 \\ 0.356 & 0.13 & 0.228 & 0.32 & 0.344 \end{bmatrix}$$

Fuzzy mapping f: U$\longrightarrow$V, represented by a fuzzy matrix

$$D_j = (d_j(m,l) \in \mathcal{M}_{M\times L} \text{ where } d_j(m,l) = 1 \wedge (1 - \bar{C}_j(u_m) + A_k(v_l)).$$

where j = 1,2,…,M, K = 7, m = 1, 2, 3, …,M, M = 5, l = 1, 2,…,L, L = 11, k = 1, 2,…,K, K = 5

For j=1, m=1, l=1, 2,…,L, L = 11

$$D_1 = d_1(1,l) = 1 \wedge (1 - \bar{C}_1(u_m) + A_1(v_l)).$$

$D_1 =$

$$\begin{bmatrix} 0.500 & 0.600 & 0.700 & 0.800 & 0.900 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0.386 & 0.486 & 0.586 & 0.686 & 0.786 & 0.886 & 0.986 & 1 & 1 & 1 & 1 \\ 0.228 & 0.328 & 0.428 & 0.528 & 0.628 & 0.728 & 0.828 & 0.928 & 1 & 1 & 1 \\ 0.420 & 0.520 & 0.620 & 0.720 & 0.820 & 0.920 & 1 & 1 & 1 & 1 & 1 \\ 0.344 & 0.444 & 0.544 & 0.644 & 0.744 & 0.844 & 0.944 & 1 & 1 & 1 & 1 \end{bmatrix}$$

For j=2, $D_2 = d_2(2,l) = 1 \wedge (1 - \bar{C}_2(u_2) + A_1(v_l)).$

$D_2 =$

$$\begin{bmatrix} 0.356 & 0.456 & 0.556 & 0.656 & 0.756 & 0.856 & 0.956 & 1 & 1 & 1 & 1 \\ 0.578 & 0.609 & 0.778 & 0.878 & 0.978 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0.272 & 0.372 & 0.472 & 0.572 & 0.672 & 0.772 & 0.872 & 0.972 & 1 & 1 & 1 \\ 0.446 & 0.546 & 0.646 & 0.746 & 0.846 & 0.946 & 1 & 1 & 1 & 1 & 1 \\ 0.348 & 0.448 & 0.458 & 0.648 & 0.748 & 0.848 & 0.948 & 1 & 1 & 1 & 1 \end{bmatrix}$$

For j=3, $D_3 = d_3(3,l) = 1 \wedge (1 - \bar{C}_3(u_m) + A_2(v_l)).$

$D_3 =$

$$\begin{bmatrix} 0.736 & 0.768 & 0.825 & 0.900 & 0.989 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0.386 & 0.418 & 0.475 & 0.550 & 0.639 & 0.79 & 0.851 & 0.972 & 1 & 1 & 1 \\ 0.37 & 0.402 & 0.459 & 0.534 & 0.623 & 0.724 & 0.835 & 0.956 & 1 & 1 & 1 \\ 0.714 & 0.746 & 0.803 & 0.854 & 0.878 & 0.967 & 1 & 1 & 1 & 1 & 1 \\ 0.394 & 0.426 & 0.483 & 0.558 & 0.647 & 0.748 & 0.859 & 0.98 & 1 & 1 & 1 \end{bmatrix}$$

For j=4, $D_4 = d_4(4,l) = 1 \wedge (1 - \bar{C}_4(u_m) + A_2(v_l)).$

$D_4 =$

$$\begin{bmatrix} 0.736 & 0.768 & 0.825 & 0.9 & 0.989 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0.578 & 0.61 & 0.667 & 0.742 & 0.831 & 0.932 & 1 & 1 & 1 & 1 & 1 \\ 0.37 & 0.402 & 0.459 & 0.534 & 0.623 & 0.729 & 0.835 & 0.956 & 1 & 1 & 1 \\ 0.714 & 0.746 & 0.803 & 0.878 & 0.967 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0.394 & 0.426 & 0.483 & 0.558 & 0.647 & 0.748 & 0.859 & 0.98 & 1 & 1 & 1 \end{bmatrix}$$

For j=5, $D_5 = d_5(4,l) = 1 \wedge (1 - \bar{C}_5(u_m) + A_3(v_l)).$

$D_5 =$

$$\begin{bmatrix} 0.736 & 0.746 & 0.776 & 0.826 & 0.896 & 0.986 & 1 & 1 & 1 & 1 & 1 \\ 0.448 & 0.458 & 0.488 & 0.538 & 0.608 & 0.698 & 0.808 & 0.938 & 1 & 1 & 1 \\ 0.37 & 0.380 & 0.410 & 0.46 & 0.53 & 0.620 & 0.73 & 0.86 & 1 & 1 & 1 \\ 0.714 & 0.724 & 0.754 & 0.804 & 0.874 & 0.964 & 1 & 1 & 1 & 1 & 1 \\ 0.394 & 0.404 & 0.434 & 0.484 & 0.554 & 0.644 & 0.754 & 0.884 & 1 & 1 & 1 \end{bmatrix}$$

For j=6, $D_6 = d_6(m,l) = 1 \wedge (1 - \bar{C}_6(u_m) + A_4(v_l))$.

$D_6 =$

$$\begin{bmatrix} 0.644 & 0.644 & 0.644 & 0.644 & 0.644 & 0.644 & 0.644 & 0.644 & 0.644 & 0.644 & 1 \\ 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 1 \\ 0.772 & 0.772 & 0.772 & 0.772 & 0.772 & 0.772 & 0.772 & 0.772 & 0.772 & 0.772 & 1 \\ 0.68 & 0.68 & 0.68 & 0.68 & 0.68 & 0.68 & 0.68 & 0.68 & 0.68 & 0.68 & 1 \\ 0.656 & 0.656 & 0.656 & 0.656 & 0.656 & 0.656 & 0.656 & 0.656 & 0.656 & 0.656 & 1 \end{bmatrix}$$

For j=7, $D_7 = d_7(m,l) = 1 \wedge (1 - \bar{C}_7(u_m) + A_5(v_l))$.

$$D_7 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.944 & 0.844 & 0.744 & 0.644 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.97 & 0.87 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.97 & 0.87 & 0.772 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.98 & 0.88 & 0.78 & 0.68 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.956 & 0.856 & 0.756 & 0.656 \end{bmatrix}$$

The fuzzy multicriteria decision matrix is

$$D_j = \bigcap_{j=1}^{7} D_j \triangleq \left(\prod_{j=1}^{7} d_j(m,l)\right)$$

For j = 1, i.e., for individual u_1

$$D_1 = \bigcap_{j=1}^{7} D_1 \triangleq \left(\prod_{j=1}^{7} d_1(1,l)\right)$$

We get from the first row in D, the fuzzy subset of V,

$$\bar{E}_1 = \frac{0.0457}{0} + \frac{0.0775}{0.1} + \frac{0.1324}{0.2} + \frac{0.2261}{0.3} + \frac{0.4286}{0.4} + \frac{0.5435}{0.5} + \frac{0.6157}{0.6} + \frac{0.607}{0.7} + \frac{0.5435}{0.8} + \frac{0.4791}{0.9} + \frac{0.644}{1}$$

$$D = \begin{bmatrix} 0.0457 & 0.0775 & 0.1324 & 0.2261 & 0.4286 & 0.5435 & 0.6157 & 0.6079 & 0.5435 & 0.4791 & 0.644 \\ 0.0194 & 0.0285 & 0.0613 & 0.1150 & 0.2159 & 0.3711 & 0.5898 & 0.7932 & 0.87 & 0.8439 & 0.87 \\ 0.0024 & 0.0058 & 0.0135 & 0.0306 & 0.067 & 0.1410 & 0.2837 & 0.431 & 0.7488 & 0.6716 & 0.772 \\ 0.0464 & 0.0778 & 0.1324 & 0.2264 & 0.3865 & 0.2746 & 0.68 & 0.6664 & 0.5984 & 0.5305 & 0.68 \\ 0.0048 & 0.0096 & 0.0198 & 0.0413 & 0.0847 & 0.1692 & 0.4332 & 0.5433 & 0.5615 & 0.4959 & 0.656 \end{bmatrix}$$

along with equation

$$H_l(E_{1\alpha}) = \frac{1}{N_\alpha}\sum_{n=1}^{N_\alpha} Z_n(\alpha) = \frac{1}{N_{0.111}}\left(\frac{0+0.1+0.2+0.3+0.4+.5+.6+.7+0.8+0.9+1}{11}\right) = 0.5$$

The Series of $E_{1\alpha}, H_l(E_{1\alpha})$ and $\Delta\alpha_l$ is as shown below

l	Range of α	$E_{1\alpha}$	$H_l(E_{1\alpha})$	$\Delta\alpha_l$
1	$0<\alpha\le 0.0457$	{0. 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.5	0.0457
2	$0.0457<\alpha\le 0.0775$	{ 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.55	0.0318
3	$0.0775<\alpha\le 0.1324$	{ 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.6	0.0549
4	$0.1324<\alpha\le 0.2261$	{ 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.65	0.0937
5	$0.2261<\alpha\le 0.4286$	{ 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.74	0.2025
6	$0.4286<\alpha\le 0.4791$	{ 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.85	0.05025
7	$0.4791<\alpha\le 0.5435$	{ 0.6, 0.7, 0.8, 0.9, 1}	0.72	0.0644
8	$0.5435<\alpha\le 0.6079$	{ 0.7, 0.8, 0.9, 1}	0.133	0.0644
9	$0.6079<\alpha\le 0.6157$	{ 0.6, 1}	0.8	0.0078
10	$0.6157<\alpha\le 0.644$	{1}	1	0.0283
11	-	-	-	-

along with equation

$$S(1) = \frac{1}{\alpha_{max}}\sum_{l=1}^{11} H_1(E_{1\alpha})\Delta\alpha_l$$

$$= \frac{1}{0.644}\left[\begin{array}{l} 0.5(0.457)+0.55(0.0318)+0.6(0.0549)+0.65(0.0937)+ \\ 0.74(0.2025)+0.85(0.0505)+0.72(0.0644)+0.133(0.0644)+ \\ 0.8(0.0078)+1(0.0283) \end{array}\right]$$

$$= 0.9674$$

and the rest.

Thus we have the following satisfaction values

1. S(1) = 0.9674
2. S(2) = 0.7696
3. S(3) = 0.7797
4. S(4) = 0.7153
5. S(5) = 0.8134

From these satisfaction values the best is u_1 and next from good to bad: u_5, u_3, u_2, u_4.

Actions	Evaluation	Factors																					
		f_{11}	f_{12}	f_{13}	f_{14}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}	f_{51}	f_{52}	f_{53}
u_1	A	1	2	2	1	1	0	1	5	4	2	2	1	2	1	2	0	1	1	2	2	2	1
	B	2	2	0	4	2	2	1	2	2	2	2	2	2	2	1	2	2	2	2	2	5	2
	C	5	2	5	2	5	2	5	1	2	3	5	2	4	2	5	1	2	5	4	4	2	5
	D	2	4	2	2	2	2	2	2	1	2	1	2	1	5	1	2	4	1	2	2	1	1
	E	0	0	1	1	0	4	1	0	1	1	0	3	1	0	1	4	1	1	0	0	0	1
u_2	A	1	1	1	1	0	1	1	2	1	1	1	0	1	1	0	1	1	1	1	2	2	1
	B	1	0	1	1	2	1	1	2	1	0	1	1	1	1	2	1	2	2	2	2	4	1
	C	2	2	4	1	2	2	2	5	2	1	4	1	2	2	5	2	2	2	4	2	2	5
	D	4	5	2	5	2	2	2	1	4	3	2	3	4	4	1	2	4	5	2	4	2	2
	E	2	2	2	2	4	4	4	0	2	5	2	5	2	2	2	4	1	0	1	0	0	1
u_3	A	2	2	1	5	5	1	1	2	5	4	1	1	1	5	1	1	2	5	2	5	2	5
	B	1	2	1	2	1	4	1	5	2	2	1	1	5	4	2	1	1	2	4	2	2	2
	C	4	2	4	1	2	2	1	1	1	2	4	2	2	1	1	5	4	2	2	2	4	1
	D	2	4	2	2	1	2	2	1	2	2	2	5	2	0	4	2	2	1	1	1	2	2
	E	1	0	2	0	1	1	5	1	0	0	2	1	0	0	2	1	1	0	1	0	0	0
u_4	A	1	2	1	1	2	1	1	1	2	2	2	1	0	0	1	1	0	1	1	2	2	2
	B	1	2	2	1	4	1	1	2	1	5	1	1	2	1	2	2	2	1	1	2	4	2
	C	3	1	5	5	2	2	1	4	4	1	2	2	2	1	5	4	2	1	5	4	3	4
	D	5	4	1	2	1	4	3	2	2	1	4	5	4	6	1	2	5	5	2	2	1	1
	E	0	1	1	1	1	2	4	1	1	1	0	1	2	2	1	1	1	2	1	0	0	1
u_5	A	0	1	2	2	5	2	1	2	6	2	1	1	2	2	2	2	1	1	1	2	2	1
	B	1	2	2	4	0	4	1	5	2	5	2	1	4	2	2	5	2	2	1	5	3	1
	C	2	2	2	2	2	1	1	2	1	1	5	1	2	4	4	1	2	4	5	2	4	6
	D	4	4	4	1	2	2	2	1	1	1	1	6	1	1	1	1	5	2	2	1	1	1
	E	3	1	0	1	1	1	5	0	0	1	1	1	1	1	1	1	0	1	1	0	0	1

Table 1: Characteristics of crisp values

5. Conclusion

The double model presented in paper is more capable of capturing of humans appraisal of ambiguity when complex decision-making problems are considered. This is because it provides a flexible and realistic way to accommodate real life data. The experimental results reveal that if we use models together it is more in accordance with the thinking process of a human being to make a decision on a multifaceted subject. The experiment also shows that by using two models together decisions results are more likely to represent the actual facts and the results are more acceptable.

It needs to be emphasized here that these decisions making processes are best used as decision tools. This study provides a general view of using two methods together under certain situations. A broader understanding of the characteristics of the methods and evaluation criteria is required for successful solutions of real-life fuzzy mulicriteria decision making problems. Besides using two models together, one can build multiple models, so as to meet the needs of the decision problem and identification of multifaceted systems.

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