Annals of Pure and Applied Mathematics
Vol. 11, No. 1, 2016, 99-104
ISSN: 2279-087X (P), 2279-0888(online)
Published on 9 February 2016
www.researchmathsci.org

Characterization of Smarandache-Soft Neutrosophic Near-Ring by Soft Neutrosophic Quasi-Ideals

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Received 10 January 2016; accepted 28 January 2016

Abstract. In this paper, we introduced Samarandache-2-algebraic structure of soft neutrosophic near-ring namely Smarandache–soft neutrosophic near-ring. A Samarandache-2-algebraic structure on a set N means a weak algebraic structure S_1 on N such that there exist a proper subset M of N, which is embedded with a stronger algebraic structure S_2, stronger algebraic structure means satisfying more axioms, that is $S_1 << S_2$, by proper subset one can understand a subset different from the empty set, from the unit element if any, from the whole set. We define Smarandache-soft neutrosophic near-ring and obtain the some of its characterization through soft neutrosophic quasi-ideals.

Keywords: Soft Neutrosophic Near-ring,Soft Neutrosophic Near-field, Smarandache - Soft Neutrosophic near- ring, Soft neutrosophic Quasi-ideals

AMS Mathematics Subject Classification (2010): 46C20, 15A09

1.Introduction

In order that, new notions are introduced in algebra to better study the congruence in number theory by Florentin Smarandache [2]. By <proper subset> of a set A we consider a set P included in A, and different from A, different from empty set, and from the unit element in A-if any they rank the algebraic structures using an order relationship:
They say that the algebraic structures $S_1 << S_2$ if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1 laws, or S_2 has more laws than S_1.
For example: Semi group<< Monoid <<group<< ring<<field, or Semi group<< to commutative semi group, ring<< unitary ring etc. They define a general special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is a structure, where SM << SN. In addition, we have published [9,10,11,12].

For basic concept of near-ring we refer to Pilz, for quasi-ideals we refer Lwao Yakabe and for soft neutrosophic algebraic structure we refer to Muhammed Shabir, Mumtaz Ali, Munazza Naz, and Florentin Smarandache.

2. Preliminaries

Definition 2.1. Let $\langle N\ UI \rangle$ be a neutrosophic near-ring and (F, A) be a soft set over $\langle^{N\ UI}\rangle$.Then (F, A) is called soft neutrosophic near-ring if and only if F(a) is a neutrosophic sub near-ring of $\langle N\ UI \rangle$ for all $a \in A$.

Definition 2.2. A soft neutrosophic near-ring we mean a non-empty set (F,A) in which an addition + and multiplication $*$ are defined such that

(a) ((F,A),+) is a soft neutrosophic group
(b) ((F,A),$*$) is a soft neutrosophic semigroup
(c) $(F(n_1) + F(n_2))F(n) = F(n_1)F(n) + F(n_2)F(n)$ where $F(n), F(n_1), F(n_2)$ in (F,A).

In dealing with general soft neutrosophic near-rings the neutral element of ((F,A),+) will be denoted by F(0).

Definition 2.3. Let $K(I) = \langle KUI \rangle$ be a neutrosophic near-field and let (F, A) be a soft set over $K(I)$. Then (F, A) is said to be soft neutrosophic near-field if and only if $F(a)$ is a neutrosophic sub near-field of $K(I)$ for all $a \in A$.

Definition 2.4. Let (F,A) be a soft neutrosophic near-ring over $\langle N \cup I \rangle$.we say that (F,A) is soft neutrosophic zero-symmertic if F(n)F(0) = F(0) for every element F(n) of (F,A).

Definition 2.5. An element F(d) of soft neutrosophic near-ring (F,A) over $\langle N \cup I \rangle$ is called soft neutrosophic distributive if $F(d)(F(n_1) + F(n_2)) = F(d)F(n_1) + F(d)F(n_2)$ for all elements $F(n_1), F(n_2)$ of (F,A).

Definition 2.6. Let (H,A) and (G,B) be two non –empty soft neutrosophic subsets of (F,A).We shall define two types of products:

$(H,A)(G,B) = \{ \sum H(a_i)G(b_i) \ / \ H(a_i) \text{ in } (H,A) , G(b_i) \text{ in } (G,B) \}$ and

$(H,A)*(G,B) = \{ \sum H(a_i) (H(a_i') + G(b_i)) - H(a_i)H(a_i')) \ / \ H(a_i),H(a_i') \text{ in } (H,A) , G(b_i) \text{ in } (G,B) \}$where $\sum$ denotes all possible additions of finite terms . In the case when (G,B) consists of single element G(b) , we denote (H,A)(G,B) by (H,A)G(b) , and so on.

Definition 2.7. A soft neutrosophic subgroup (H,A) of ((F,A),+) is called an (F,A)-subgroup of (F,A) if $(F,A)(H,A) \subset (H,A)$ .For instance, (F,A)F(a) is an (F,A)-subgroup of (F,A) for every element F(a) in (F,A).

Definition 2.8. Let (F,A) be the soft neutrosophic near-ring over $\langle N \cup I \rangle$.The set

$(F,A)_0 = \{ F(n) \text{ in } (F,A) \ / \ F(n)F(0) = F(0)\}$ is called the soft neutrosophic zero symmetric part of (F,A) ;

$(F,A)_c = \{F(n) \text{ in } (F,A) \ / \ F(n)F(0) = F(n)\}$ is called the soft neutrosophic constant part of (F,A).

Now we have introduced our basic concept, called **Smarandache–soft neutrosophic–near ring.**

Definition 2.9 .A Soft neutrosophic–near ring is said to be Smarandache–soft neutrosophic–near ring, if a proper subset of it is a soft neutrosophic–near field with respect to the same induced operations.

Definition 2.10. A soft neutrosophic subgroup (L_Q,A) of ((F,A),+) is called a soft neutrosophic quasi-ideal of (F,A), if (L_Q,A) (F,A) ∩ (F,A) (L_Q,A) ∩ (F,A) ∗ (L_Q,A) ⊂ (L_Q,A).

For instance, every (F,A)-sugroup of (F,A) and F(d)(F,A) with a distributive element F(d) of (F,A) are soft neutrophic quasi-ideals of (F,A).

Clearly {F(0)} and (F,A) are soft neutrosophic quasi-ideals of (F,A). If (F,A) has no soft neutrosophic quasi-ideals except {F(0)} and (F,A), we say that (F,A) is L_Q- simple.

We recall the following properties of soft neutrosophic quasi-ideals:

(a) The intersection of any set of soft neutrosophic quasi-ideals of (F,A) is a soft neutrosophic quasi-ideal of (F,A).

(b) Suppose that (F,A) is soft neutrosophic zero-symmetric.Then a soft neutrosophic subgroup (L_Q,A) of ((F,A),+) is a soft neutrosophic quasi-ideal of (F,A) if and only if (L_Q,A)(F,A) ∩ (F,A) (L_Q,A) ⊂ (L_Q,A).

3. Characterization of Smarandache - soft neutrosophic near-ring:

A soft neutrosophic near ring (F,A) over $\langle N \cup I \rangle$ is called a soft neutrosophic near-field , if its non-zero elements form a group with respect to the multiplication defined in (F,A).
As usual, in this section, we will exclude those soft neutrosophic near-fields which are isomorphic to this soft neutrosophic near-field.So every soft neutrosophic near-field is zero-symmetric and L_Q-simple.

In this section we are going to characterize those zero-symmetric soft neutrosophic near-rings which are soft neutrosophic near-fields. We start with the following lemma.

Lemma 3.1. Let F(n) be a right cancellable element of a smarandache – soft neutrosophic near-ring (F,A) over $\langle N \cup I \rangle$ contained in the (F,A) – subgroup (F,A)F(n), then (F,A) has a right identity element F(e) such that F(n) = F(e)F(n) = F(n)F(e). In particular, if F(n) is a cancellable element of (F,A) contained in (e)F(n), then (F,A) has a two –sided identity element.

Proof: Since F(n) is contained in (F,A)F(n),there exists an element F(e) in (F,A) such that F(e)F(n) = F(n). Then F(x)F(e)F(n) = F(x)F(n) for every element F(x) of (F,A),whence F(x)F(e) = F(x), that is F(e) is right identity element of (F,A) such that F(n) = F(e)F(n) = F(n)F(e).

If F(n) is a cancellable element contained in (F,A)F(n), then the equations F(n) = F(e)F(n) = F(n)F(e) imply that F(x)F(e) = F(x) and F(e)F(x) = F(x) for every element F(x) in (F,A).

Now we characterize those soft neutrosophic zero-symmetric near-rings which are soft neutrosophic near-fields.

Theorem 3.1. Let (F,A) be a smarandache - soft neutrosophic near-ring over $\langle N \cup I \rangle$ which is soft neutrosophoic zero-symmetric with more than one element. Then (H,A) is a soft neutrosophic near-field if and only if (H,A) has cancellable and distributive element contained in a minimal soft neutrosophic quasi-ideal of (F,A),where (H,A) is a proper subset of (F,A),which is soft neutrosophic near-field.

Proof: Assume that (H,A) is soft neutrosophic near-field .Then (H,A) is a minimal soft neutrosophic quasi-ideal of (F,A) and (H,A) has a two-sided identity element which is cancellable and distributive.

Conversely, assume that the soft neutrosophic zero-symmetric near-ring (H,A) has a cancellable and distributive element H(n) contained in a minimal soft neutrosophic quasi-ideal (L_Q,A) of (F,A).Then H(n) (H,A) ∩ (H,A) H(n) is a soft neutrosophic quasi-ideal of (H,A) and it contains the non-zero element $H(n)^2$.

Moreover H(n) (H,A) ∩ (H,A) H(n) ⊂ (L_Q,A)(H,A) ∩ (H,A) (L_Q,A) ⊂ (L_Q,A).

Hence we have (L_Q,A) = H(n)(H,A) ∩ (H,A)H(n). Therefore (L_Q,A) ⊂ (H,A)H(n).

So, by Lemma, (H,A) has a two-sided identity element H(e).

On the other hand, $H(n)^2$(H,A) ∩ (H,A)$H(n)^2$ is also soft neutrosophic quasi-ideal of (H,A), since $H(n)^2$ is distributive. Moreover, it contains the non-zero element $H(n)^3$ and is contained in the minimal soft neutrosophic quasi-ideal (L_Q,A). Hence we have (L_Q,A) = $H(n)^2$(H,A) ∩ (H,A)$H(n)^2$. Thus H(n) in (L_Q,A) ⊂ (H,A)$H(n)^2$ and H(n) = H(e)H(n) = H(x)$H(n)^2$ for some H(x) of (H,A). Therefore H(e) = H(x)H(n) in (H,A)H(n). Dually we obtain that H(e) in H(n)(H,A). So H(e) in H(n)(H,A) ∩ (H,A)H(n) = (L_Q,A), whence (H,A) = H(e)(H,A) ∩ (H,A)H(e) ⊂ (L_Q,A) ,that is , (H,A) = (L_Q,A) . This relation and the minimality of (L_Q,A) imply that (H,A) is L_Q – simple. So (H,A) is a soft neutrosophic near-field.

Theorem 3.2. Let (F,A) be a smarandache - soft neutrosophic near-ring over $\langle N \cup I \rangle$ which is soft neutrosophoic zero-symmetric with more than one element.Then the followings are equivalent :

(i) (H,A) is a soft neutrosophic near-field;

(ii) (H,A) has a cancellable element contained in a minimal soft neutrosophic (H,A) – subgroup of $_{(H,A)}$(H,A);

(iii) (H,A) has a cacellable element contained in a minimal soft neutrosophic quasi-ideal of (H,A), Where (H,A) is a soft neutrosophic near-field.

Proof: The implications (i) ⇒ (ii) and (i) ⇒ (iii) are equivalent.

(ii) ⇒ (i)

Assume H(n) to be a cancellable element contained in a minimal soft neutrosophic (H,A) – subgroup (H_1,A) of $_{(H,A)}$(H,A). Then (H,A)H(n) is an (H,A) – subgroup of $_{(H,A)}$(H,A) containing the non-zero element $H(n)^2$ and (H,A)H(n) ⊂ (H,A)(H_1,A) ⊂ (H_1,A). So (H,A)H(n) = (H_1,A) by the minimality of (H_1,A) and (H_1,A) has a two-sided identity element H(e) by the lemma.

On the other hand, (H,A)$H(n)^2$ is an soft neutrosophic (H,A) – subgroup of $_{(H,A)}$(H,A) containing the non-zero element $H(n)^2$ and (H,A)$H(n)^2$ ⊂ (H,A)H(n) = (H_1,A). So (H,A)$H(n)^2$ = (H_1,A) by the minimality of (H_1,A). This implies H(n) in (H_1,A) = (H,A)$H(n)^2$. Thus H(e)H(n) = H(n) = H(x)$H(n)^2$ for some H(x) of (H,A).

Therefore H(e) = H(x)H(n) in (H,A)H(n) = (H_1,A) , that is (H_1,A) = (H,A). This relation and the minimality of (H_1,A) imply that $_{(H,A)}$(H,A) is (H,A) – simple. So (H,A) is a soft neutrosophic near-field.

(iii)⇒ (i)

Assume H(n) to be cancellable element contained in a minimal soft neutrosophic quasi-ideal (L_Q,A) of (H,A). Then (H,A)H(n) is a soft neutrophic quasi-ideal of (H,A) containing a non-zero element $H(n)^2$, and $H(n)^2$ in (L_Q,A).

So $(L_Q,A) \cap$ (H,A)H(n) is a non-zero soft neutrosophic quasi-ideal of (H,A) contained in the minimal soft neutrosophic quasi-ideal (L_Q,A), whence $(L_Q,A) = (L_Q,A) \cap$ (H,A)H(n). Thus H(n) in $(L_Q,A) \subset$ (H,A)H(n) and (H,A) has a two-sided identity element H(e) by lemma.

On the other hand, $(H,A)H(n)^2$ is also a soft neutrosophic quasi-ideal of (H,A) containing a non-zero element $H(n)^2$. Similarly to the above consideration we obtain H(n) in $(L_Q,A) \subset (H,A)H(n)^2$ and $H(e)H(n) = H(n) = H(x)H(n)^2$ for some H(x) of (H,A).

Therefore H(e) = H(x)H(n) and H(e) = H(n)H(x) because H(e) is a two-sided identity element and H(n) is cancellable. Thus H(e) = H(n)H(x) = H(x)H(n) in $(L_Q,A)(H,A) \cap (H,A)(L_Q,A) \subset (L_Q,A)$, that is $(L_Q,A) = (H,A)$. This relation and the minimality of (L_Q,A) imply that (H,A) is L_Q – simple. So (H,A) is a soft neutrosophic near-field.

Proposition 3.1. Let (F,A) be a smarandache - soft neutrosophic near-ring $\langle N \cup I \rangle$ is L_Q - simple, then either (F,A) is soft neutrosophic zero-symmetric or (F,A) is constant.

Proof: Since the soft neutrosophic zero-symmetric part $(F,A)_0$ of (F,A) is a soft neutrosophic quasi-ideal of (F,A), either $(F,A)_0 = (F,A)$ or $(F,A)_0 = \{F(0)\}$, that is, either (F,A) is soft neutrosophic zero-symmetric or (F,A) is constant.

Theorem 3.3. Let (F,A) be a smarandache-soft neutrosophic near-ring over $\langle N \cup I \rangle$ with more than one element .Then the following conditions are equivalent:

(i) (H,A) is a soft neutrosophic near-field;

(ii) (H,A) is L_Q – simple and (H,A) has a left identity;

(iii) (H,A) is L_Q – simple , $H(d) \neq \{H(0)\}$ and for each non-zero element H(n) of (H,A) there exists an element $H(n_1)$ of (H,A) such that $H(n_1)H(n) \neq H(0)$, where (H,A) is a proper subset of (H,A).

Proof:

(i)⇒(ii)

Clearly (H,A) has a left identity and (H,A) is soft neutrosophic zero-symmetric. Let (L_Q,A) be a soft neutrosophic quasi-ideal of (H,A) and $L_Q(a)$ a non-zero element of (L_Q,A), then $(H,A) = L_Q(a)(H,A) = (H,A)\,L_Q(a)$. Hence $(H,A) = L_Q(a)(H,A) \cap (H,A)\,L_Q(a) \subseteq (L_Q,A)(H,A) \cap (H,A)\,(L_Q,A) \subseteq (L_Q,A)$, whence $(L_Q,A) = (H,A)$

(ii) ⇒ (iii)

If (H,A) has a left identity H(e), then H(e) is non-zero and distributive . Hence $H(d) \neq \{H(0)\}$ and $H(e)H(n) = H(n) \neq H(0)$ for every non-zero element H(n) of (H,A).

(iii)⇒(i)

$H(d) \neq \{H(0)\}$ implies that (H,A) is not constant. Hence (H,A) is soft neutrosophic zero-symmetric by proposition 3.1. Moreover, let H(n) be a non-zero element of (H,A), then (H,A)H(n) is a soft neutrosophic quasi-ideal of (H,A) and $H(n_1)H(n)$ in (H,A)H(n), where $H(n_1)$ is an element of (H,A) such that $H(n_1)H(n) \neq H(0)$. Hence (H,A)H(n) = (H,A). Therefore, (H,A) is a soft neutrosophic near-field.

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