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On Pseudo regular Fuzzy Graphs

***N.R.Santhi Maheswari*[1] and *C.Sekar*[2]**

[1]Department of Mathematics, G.Venkataswamy Naidu College, Kovilpatti-628502
Tamil Nadu, India. e-mail: nrsmaths@yahoo.com
[2]Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur-628216
Tamil Nadu, India. e-mail: sekar.acas@gmail.com

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Abstract. In this paper, pseudo degree, pseudo regular fuzzy graphs and totally pseudo regular fuzzy graphs are defined. Comparative study between pseudo regular fuzzy graph and totally pseudo regular fuzzy graph is done. A necessary and sufficient condition under which they are equivalent is provided. Characterization of pseudo regular fuzzy graph in which underlying crisp graph is a cycle is investigated. Also, whether the results hold for totally pseudo regular fuzzy graphs is examined.

Keywords: 2-degree, average degree of a vertex in graph, regular fuzzy graph, totally regular fuzzy graph.

AMS Mathematics Subject Classification (2010): 05C12, 03E72, 05C72

1.Introduction

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by $V(G)$ and $E(G)$ respectively. The degree of a vertex v is the number of edges incident at v, and it is denoted by $d(v)$. A graph G is regular if all its vertices have the same degree. The 2-degree of v [4] is the sum of the degrees of the vertices adjacent to v and it is denoted by t(v). We call $\frac{t(v)}{d(v)}$, the average degree of v . A graph is called pseudo-regular if every vertex of G has equal average degree [3].

Fuzzy set theory was first introduced by Zadeh in 1965 [17]. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. In 1975, A. Rosenfeld introduced the concept of fuzzy graphs [8]. Now, fuzzy graphs have many applications in branches of engineering and technology.

2.Review of literature

Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2008 [7]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008 [6]. Mathew, Sunitha and Anjali introduced some connectivity concepts in bipolar fuzzy graphs [15]. Akram and Dudek introduced the notions of regular bipolar fuzzy graphs [1] and also introduced intuitionistic fuzzy graphs [2]. Samanta and Pal introduced the concept of irregular bipolar fuzzy graphs [13]. Maheswari and Sekar introduced (2, k)-regular fuzzy graphs and totally (2,k)-regular fuzzy graphs [9]. Maheswari and Sekar introduced m-neighbourly irregular fuzzy graphs [12]. Maheswari and Sekar introduced neighbourly edge irregular fuzzy graphs [10]. Maheswari and Sekar introduced neighbourly edge irregular bipolar fuzzy graphs [11]. Pal and Hossein introduced irregular interval-valued fuzzy graphs [16]. Sunitha and Mathew discussed about growth of fuzzy graph theory [14].

Regular fuzzy graphs play a central role in combinatorics and theoretical computer science. These motivate us to define pseudo degree in fuzzy graphs and pseudo regular fuzzy graphs and discussed some of its properties. This paper deals with significant properties of pseudo regular fuzzy graphs.

3. Preliminaries

We present some known definitions and results for ready reference to go through the work presented in this paper.

By graph, we mean a pair G*=(V,E), where V is the set and E is a relation on V. The elements of V are vertices of G* and the elements of E are edges of G*.

Definition 3.1. 2-degree of v is defined as the sum of the degrees of the vertices adjacent to v and it is denoted by $t(v)$ [4].

Definition 3.2. Average degree of v is defined as $\frac{t(v)}{d(v)}$, where $t(v)$ is the 2-degree of v and $d(v)$ is the degree of v and it is denoted by $d_a(v)$ [3].

Definition 3.3. A graph is called pseudo-regular if every vertex of G has equal average-degree [3].

Definition 3.4. A Fuzzy graph denoted by $G : (\sigma, \mu)$ on the graph $G^* : (V,E)$*:* is a pair of functions (σ, μ), where $\sigma : V \rightarrow [0; 1]$ is a fuzzy subset of a set V and $\mu : V X V \rightarrow [0; 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u, v) = \mu(uv) \leq \sigma(u) \Lambda \sigma(v)$ is satisfied, where σ and μ are called membership function [7].

Definition 3.5. Let $G : (\sigma, \mu)$ be a fuzzy graph on G^*: (V,E). The degree of a vertex u is $d_G(u) = \Sigma\mu(uv)$, for $uv \in E$ and $\mu(uv) = 0$, for uv not in E, this is equivalent to $d_G(u) = \Sigma \mu(uv)$, $u \neq v$ *and* $uv \in E$ [7].

Definition 3.6. Let $G : (\sigma, \mu)$ be a fuzzy graph on G^* : (V,E). If $d(v) = k$ for all $v \in V$, then G is said to be a regular fuzzy graph of degree k [7].

Definition 3.7. The total degree of a vertex u is defined as $td(u) = \Sigma\mu(uv) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$ [9]. If each vertex of G has the same total degree k, then G is said to be a totally regular fuzzy graph of degree k or k-totally regular fuzzy graph [7].

4. Pseudo regular fuzzy graphs

In this section, we define pseudo degree, pseudo regular fuzzy graph and totally pseudo regular fuzzy graph and discussed about its properties.

Definition 4.1. Let $G : (\sigma, \mu)$ be a fuzzy graph on G^* : (V, E). The 2-degree of a vertex v in G is defined as the sum of degrees of the vertices adjacent to v and is denoted by $t_G(v)$. That is, $t_G(v) = \Sigma d_G(u)$, where $d_G(u)$ is the degree of the vertex u which is adjacent with the vertex v.

Definition 4.2. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. A pseudo (average) degree of a vertex v in fuzzy graph G is denoted by $d_a(v)$ and is defined by $d_a(v) = \frac{t_G(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v.

Definition 4.3. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V,E)$. If $d_a(v) = k$, for all u *in* V; then G is called k- pseudo regular fuzzy graph.

Example 4.4. Consider a fuzzy graph on G*(V, E).

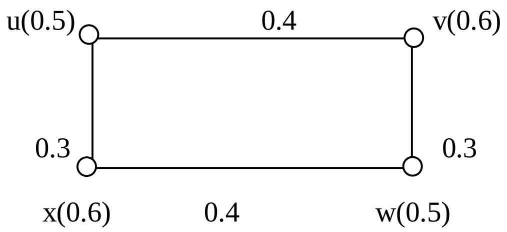

Figure 1:

Here, d(u) = 0.7; d(v) = o.7; d(w) = 0.7; d(x) = 0.7 and $d_{G^*}(u) = 2$, for all u ϵ V. Now, u is adjacent to v and x. So, $d_a(u) = \frac{d(v)+d(x)}{2} = 0.7$. So, $d_a(v) = 0.7$, for all v ϵ V. Hence G is o.7- pseudo regular fuzzy graph.

5 . Totally pseudo regular fuzzy graphs

Definition 5.1. Let $G : (\sigma, \mu)$ be a fuzzy graph on G*(V,E). The total pseudo degree of a vertex v is G is denoted by $td_a(v)$ and is defined as $td_a(v) = d_a(v) + \sigma(v)$, for all v ϵ V.

Definition 5.2. Let G be a fuzzy graph on G*(V,E). If all the vertices of G have the same total pseudo degree k, then G is said to be a totally k- pseudo regular fuzzy graph.

Example 5.3. Consider a fuzzy graph on G*(V,E).

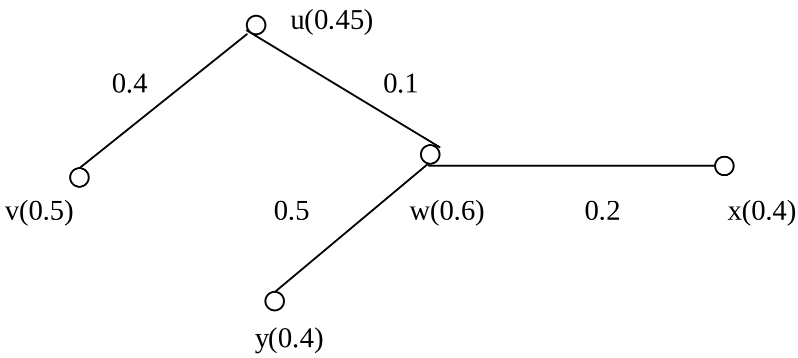

Figure 2:

Here, $d_a(u)=0.45$, $d_a(v)=0.4$, $d_a(w)=0.3$, $d_a(x)=0.5$, $d_a(y)=0.5$ and t $d_a(u) = 0.9$, for all u ϵ V. So, G is totally 0.9 –pseudo regular fuzzy graphs.

Remark 5.4. ***A pseudo regular fuzzy graph need not be a totally pseudo regular fuzzy graph.***

Example 5.5. Consider a fuzzy graph on G*(V, E).

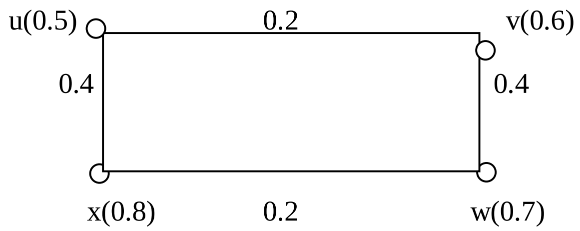

Figure 3:

The graph is 0.6-pseudo regular fuzzy graph. But $td_a(u) \neq td_a(v)$. Hence G is not a totally pseudo regular fuzzy graph.

Remark 5.6. *A totally pseudo regular fuzzy need not be a pseudo regular fuzzy graph.*

Example 5.7. Consider a fuzzy graph on G*(V, E).

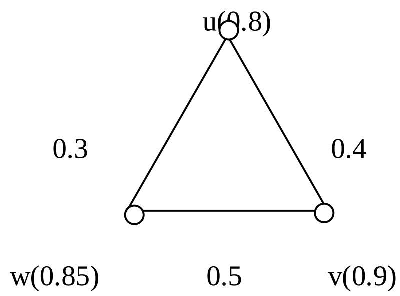

Figure 4:

The graph is totally 1.65 – pseudo regular fuzzy graph. But $d_a(u) \neq d_a(v)$. Hence G is not a pseudo regular fuzzy graph.

Remark 5.8. A pseudo regular fuzzy graph which is totally pseudo regular fuzzy graph.

Example 5.9. Consider a fuzzy graph on G*(V, E).

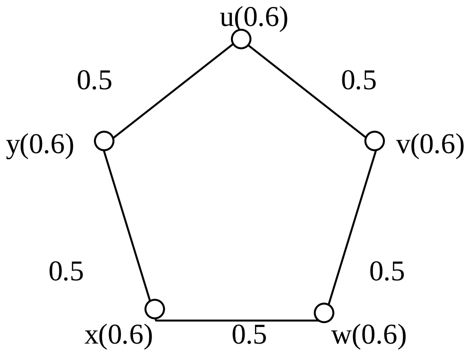

Figure 5:

The graph is 1- pseudo regular fuzzy graph and totally 1.6- pseudo regular fuzzy graph.

Theorem 5.10. Let G:(σ, μ) be a fuzzy graph on G*(V, E). Then σ is a constant function if and only if the following are equivalent.

(i) G is a pseudo regular fuzzy graph.

(ii) G is a totally pseudo regular fuzzy graph.

Proof. Assume that σ is a constant function. Let σ(u) = c, for all u∊V. Suppose G is a pseudo regular fuzzy graph. Then $d_a(u) = k$, for all u∊V. Now, $td_a(u) = d_a(u)+\sigma(u) = k+c$. Hence G is a totally pseudo regular fuzzy graph. Thus (i) ⟹ (ii) is proved. Suppose G is a totally pseudo regular fuzzy graph. Then $td_a(u) = k$, for all u∊V ⟹ $d_a(u)+\sigma(u) = k$, for all u∊V ⟹ $d_a(u)+ c = k$, for all u∊V ⟹ $d_a(u) = k-c$, for all u∊V. Hence G is a pseudo regular fuzzy graph. Thus (ii) ⟹ (i) is proved. Hence (i) and (ii) are equivalent.

Conversely, suppose (i) and (ii) are equivalent. Let G be a pseudo regular fuzzy graph and a totally pseudo regular fuzzy graph. Then $d_a(u) = k_1$ and $td_a(u) = k_2$, for all u∊V. Now $td_a(u) = k_2$ for all u∊V ⟹ $d_a(u)+\sigma(u) = k_2$, for all u∊V ⟹ $k_1+\sigma(u) = k_2$, for all u∊V ⟹ $\sigma(u) = k_2-k_1$, for all u∊V. Hence σ is a constant function.

Theorem 5.11. Let G:(σ, μ) be a fuzzy graph on G*(V, E). If G is both pseudo regular and totally pseudo regular fuzzy graph then σ is a constant function.

Proof. Assume that G is both pseudo regular and totally pseudo regular fuzzy graph. Then $d_a(u) = c$ and $td_a(u) = k$, for all u∊V. Now, $td_a(u) = k \Longrightarrow d_a(u)+\sigma(u) = k \Longrightarrow c+\sigma(u) = k \Longrightarrow \sigma(u) = k-c$ = constant. Hence σ is a constant function.

Remark 5.12. The converse of the theorem 5.11 need not be true.

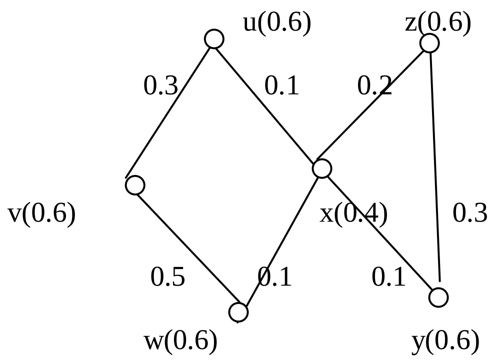

Figure 6:

Here, σ is a constant function. But G is neither pseudo regular fuzzy graph nor a totally pseudo regular fuzzy graph.

Theorem 5.13. Let G:(σ, μ) be a fuzzy graph on G*(V, E), *a cycle of length n. If* μ *is a constant function, then G is a pseudo regular fuzzy graph.*
Proof. If μ is a constant function say μ $(uv) = c$; for all $uv \in E$. Then $d_a(u) = 2c$, for all $u \in V$: Hence G is c- pseudo regular fuzzy graph.

Remark 5.14. *Converse of the above theorem 5.13 need not be true.*

Example 5.15. Fuzzy graph given in example 4.4 is a pseudo regular fuzzy graph. But μ is not constant.

Theorem 5.16. Let G:(σ, μ) be a fuzzy graph on G*(V, E), an even cycle of length n. If the alternate edges have same membership values, then G is a pseudo regular fuzzy graph.
Proof. If the alternate edges have the same membership values, then

$$\mu(ei) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2, & \text{if } i \text{ is even.} \end{cases}$$

If $c_1 = c_2$; then μ is a constant function. So, by above theorem G is a pseudo regular fuzzy graph. If $c_1 \neq c_2$; then $d_G(v) = c_1 + c_2$ for all $v \in V$: So, $t_G(v) = 2c_1+2c_2$ and $d_G{}^*(v) = 2$. Hence G is a $(c_1 + c_2)$-pseudo regular fuzzy graph.

Remark 5.17. The above theorem 5.16 does not hold for a totally pseudo regular fuzzy graph.

Example 5.18. Consider a fuzzy graph on $G^*(V, E)$.

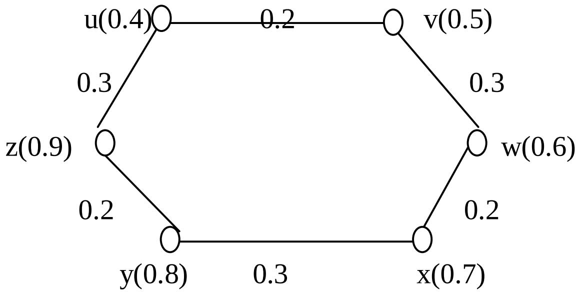

Figure 7:

Hence alternate edges have the same membership values, but G is not a totally pseudo regular fuzzy graph.

Result 5.19. If v is a pendant vertex, then pseudo degree of v is the degree of the vertex which is adjacent with v. (or) If v is a pendant vertex, then $d_a(v)=d_G(u)$, where u is the vertex adjacent with v.

Theorem 5.20. If G is a regular fuzzy graph on $G^*(V,E)$, an r-regular graph, then $d_a(v)=d_G(v)$, for all v∈G.

Proof. Let G is a k-regular fuzzy graph on $G^*(V,E)$, an r-regular graph. Then $d_G(v)=k$, for all v∈G and $d_G^*(v)=r$, for all v∈G. So, $t_G(v)=\sum d_G(v_i)$, where each v_i (for i=1,2,. . . r) is adjacent with vertex v$\Longrightarrow t_G(v)=\sum d_G(v_i)=r\,k$. Also,

$$d_a(v)=\frac{t_G(v)}{d_{G^*}(v)} \Longrightarrow d_a(v)=\frac{t_G(v)}{r} \Longrightarrow d_a(v)=\frac{kr}{r} \Longrightarrow d_a(v)=k \Longrightarrow d_a(v)= d_G(v).$$

Theorem 5.21. Let G be a fuzzy graph on $G^*(V,E)$, an r-regular graph. Then G is pseudo regular fuzzy graph if G is a regular fuzzy graph.

Proof. Let G be a k-regular fuzzy graph on $G^*(V,E)$, an r-regular graph. $\Longrightarrow d_a(v)= d_G(v)$, for all v∈G. $\Longrightarrow d_a(v)=k$, for all v∈G$\Longrightarrow$ all the vertices have same pseudo degree k. Hence G is k-pseudo regular fuzzy graph.

Theorem 5.22. *Let G* : (σ, µ)*be a fuzzy graph such that* $G^*(V, E)$ *is any cycle of length n. If G is a pseudo regular fuzzy graph, then G is a fuzzy cycle.*

Proof.

Case 1: Let *G* be a fuzzy graph on $G^*(V,E)$, an odd cycle of length n. Then by theorem 5.13, *G* is pseudo regular fuzzy graph only if µ is constant. Hence all edges have same membership values. So there does not exist unique edge *uv* such that $\mu(uv) = \Lambda\{\mu(xy) : xy \in E\}$ Hence *G* is a fuzzy cycle.

Case 2: Let *G* be a fuzzy graph on $G^*(V,E)$, an even cycle of length n.

Then by theorems 5.13 and 5.16, *G* is pseudo regular fuzzy graph only if µ is constant (or) alternate edges have same membership values. So there does not exist unique edge *uv* such that $\mu(uv) = \Lambda\{\mu(xy) : xy \in E\}$. Hence *G* is a fuzzy cycle.

Remark 5.23. The above theorem 5.23 does not hold for a pseudo totally regular fuzzy graph.

Example 5.24. Consider a fuzzy graph on G*(V;E), a cycle on 6 vertices.

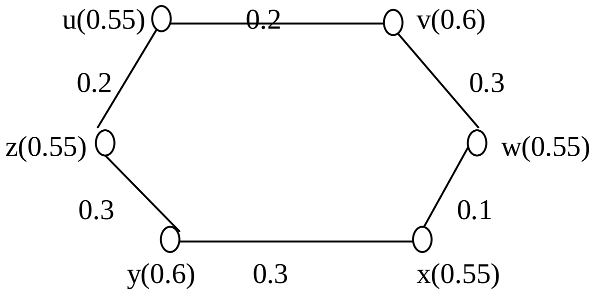

Figure 8:

Here, G is totally 1- pseudo regular fuzzy graph. But there exist unique edge wx such that $\mu(wx) = \Lambda\{ \mu(xy) : xy \in E\} = 0.1$. Hence G is not a fuzzy cycle.

Theorem 5.25. A pseudo regular fuzzy graph on any cycle does not have fuzzy bridge.

Proof.

Case 1: Let G be a fuzzy graph on $G^*(V,E)$, an odd cycle of length n. Then by theorem 5.13, G is pseudo regular fuzzy graph only if μ is constant. So, the removal of any edge does not reduce the strength of connectedness between any pair of vertices. Hence G has no fuzzy bridge.

Case 2: Let G be a fuzzy graph on $G^*(V,E)$, an even cycle of length n. Then by theorems 5.13 and 5.16, G is pseudo regular fuzzy graph only if μ is constant (or) alternate edges have same membership values. So, the removal of any edge does not reduce the strength of connectedness between any pair of vertices. Hence G has no fuzzy bridge.

Remark 5.26. The above theorem 5.26 does not hold for pseudo totally regular fuzzy graph.

Example 5.27. Consider a fuzzy graph on $G^*(V,E)$

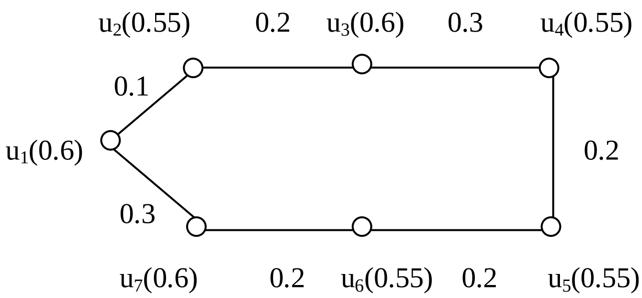

Figure .10

Hence G is totally 1-pseudo regular fuzzy graph. But the edge (u_2u_3) is a fuzzy bridge, since $\mu(u_2u_3) = 0.2$ and if we remove the edge, then we have $\mu(23) = \Lambda\{\mu(xy) : xy \in E\} = 0.1$.

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