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Connectedness in soft Čech Closure Spaces

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Abstract. The aim of the present paper is to study the concept of connectedness in Čech closure spaces through the parameterization tool which is introduced by Molodtsov.

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1. Introduction

E.Čech [1] introduced the concept of closure spaces and developed some properties of connected spaces in closure spaces. According to him, a subset A of a closure space X is said to be connected in X is said to be connected in X if A is not the union of two non-empty Semi-Separated Subsets of X.

Plastria studied [2] connectedness and local connectedness of simple extensions.

Rao and Gowri [3] studied pairwise connectedness in biČech closure spaces.

Gowri and Jegadeesan [7] studied the concept of connectedness in fuzzy Čech closure spaces.

In 1999, Molodtsov [4] introduced the notion of soft set to deal with problems of incomplete information. Later, he applied this theory to several directions [5] and [6].

In this paper, through the parameterization tool of Molodtsov [4], we introduced and exhibit some results of connectedness in Čech closure spaces.

2. Preliminaries

In this section, we recall the basic definitions of soft Čech closure space.

Definition 2.1[8]. Let X be an initial universe set, A be a set of parameters. Then the function $k: P\left(X_{F_A}\right) \to P\left(X_{F_A}\right)$ defined from a soft power set $P(X_{F_A})$ to itself over X is called Čech Closure operator if it satisfies the following axioms:

(C1) $k(\emptyset_A) = \emptyset_A$.

(C2) $F_A \subseteq k(F_A)$

(C3) $k(F_A \cup G_A) = k(F_A) \cup k(G_A)$

Then (X, k, A) or (F_A, k) is called a soft Čech closure space.

Definition 2.2[8]. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft k-closed (soft closed) if $k(U_A) = U_A$.

Definition 2.3[8]. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft k-open (soft open) if $k({U_A}^C) = {U_A}^C$.

Definition 2.4[8]. A soft set $Int(U_A)$ with respect to the closure operator k is defined as $Int(U_A) = F_A - k(F_A - U_A) = [k({U_A}^C)]^C$. Here $U_A^C = F_A - U_A$.

Definition 2.5[8]. A soft subset U_A in a soft Čech closure space (F_A, k) is called Soft neighbourhood of $e_F\ if\ e_F \in Int(U_A)$.

Definition 2.6[8]. If (F_A, k) be a soft Čech closure space, then the associate soft topology on F_A is $\tau = \{{U_A}^C : k(U_A) = U_A\}$.

Definition 2.7[8]. Let (F_A, k) be a soft Čech closure space. A soft Čech closure space (G_A, k^*) is called a soft subspace of (F_A, k) if $G_A \subseteq \mathrm{F_A}$ and $k^*(U_A) = k(U_A) \cap G_A$, for each soft subset $U_A \subseteq \mathrm{G_A}$.

Definition 2.8[8]. Let (F_A, k) and (G_B, k^*) be two Soft Čech Closure spaces over X and Y respectively. For $x \in X\ and\ e: A \to B$, a map $f: (F_A, k) \to (G_B, k^*)$ is said to be soft e-continuous if $\Phi_{fe}(k(F, A)) \subseteq k^*\Phi_{fe}(F, A)$, for every soft subset $(F, A) \subseteq \mathrm{SS(X, A)}$.

On the other hand a map $f: (F_A, k) \to (G_B, k^*)$ is said to be soft e-continuous if and only if $k\Phi_{fe}^{-1}(G, B) \subseteq \Phi_{fe}^{-1}(k^*(G, B))$, for every soft subset $(G, B) \subseteq \mathrm{SS(Y, B)}$. Clearly, if $f: (F_A, k) \to (G_B, k^*)$ is said to be soft e-continuous then $\Phi_{fe}^{-1}(U_B)$ is a soft closed subset of (F_A, k) for every soft closed subset U_B of (G_B, k^*).

3. Connectedness in soft Čech closure space

In this section, we introduce soft separated sets and discuss the connectedness in soft Čech closure space.

Definition 3.1. Two non-empty soft subsets $\mathrm{U_A}$ and $\mathrm{V_A}$ of a soft Čech closure space $(\mathrm{F_A}, \mathrm{k})$ are said to be soft separated if and only if $\mathrm{U_A} \cap k[\mathrm{V_A}] = \emptyset_\mathrm{A}$ and $k[\mathrm{U_A}] \cap \mathrm{V_A} = \emptyset_\mathrm{A}$.

Remark 3.2. In other words, two non-empty $\mathrm{U_A}$ and $\mathrm{V_A}$ of a soft Čech closure space $(\mathrm{F_A}, k)$ are said to be soft separated iff $(\mathrm{U_A} \cap k[\mathrm{V_A}]) \cup (k[\mathrm{U_A}] \cap \mathrm{V_A}) = \emptyset_\mathrm{A}$.

Theorem 3.3. In a soft Čech closure space $(\mathrm{F_A}, k)$, every soft subsets of soft separated sets are also soft separated.

Proof. Let $(\mathrm{F_A}, k)$ be a soft Čech closure space. Let $\mathrm{U_A}$ and $\mathrm{V_A}$ are soft separated sets and $G_A \subset U_A\ and\ H_A \subset V_A$ .Therefore,$\mathrm{U_A} \cap k[\mathrm{V_A}] = \emptyset_\mathrm{A}$ and $k[\mathrm{U_A}] \cap \mathrm{V_A} = \emptyset_\mathrm{A}$ ... ... (1)
Since, $G_A \subset U_A \Rightarrow k[G_A] \subset k[U_A] \Rightarrow k[G_A] \cap \mathrm{H_A} \subset k[U_A] \cap H_A$

$\Rightarrow k[G_A] \cap \mathrm{H_A} \subset k[U_A] \cap V_A$
$\Rightarrow k[G_A] \cap \mathrm{H_A} \subset \emptyset_\mathrm{A}$ …… by (1)
$\Rightarrow k[G_A] \cap \mathrm{H_A} = \emptyset_\mathrm{A}$.
Since, $H_A \subset V_A \Rightarrow k[H_A] \subset k[V_A] \Rightarrow k[H_A] \cap \mathrm{G_A} \subset k[V_A] \cap G_A$
$\Rightarrow k[H_A] \cap \mathrm{G_A} \subset k[V_A] \cap U_A$
$\Rightarrow k[H_A] \cap \mathrm{G_A} \subset \emptyset_\mathrm{A}$ …… by (1)
$\Rightarrow k[H_A] \cap \mathrm{G_A} = \emptyset_\mathrm{A}$.
Hence, $\mathrm{U_A}$ and $\mathrm{V_A}$ are also soft separated.

Theorem 3.4. Let $(\mathrm{G_A}, k^*)$ be a subspace of a soft Čech closure space $(\mathrm{F_A}, k)$ and $let\ U_A, \mathrm{V_A} \subset \mathrm{G_A}$, then $\mathrm{U_A}$ and $\mathrm{V_A}$ are soft separated in F_A if and only if $\mathrm{U_A}$ and $\mathrm{V_A}$ are soft separated in $\mathrm{G_A}$.

Proof. Let $(\mathrm{F_A}, k)$ be a soft Čech closure space and $(\mathrm{G_A}, k^*)$ be a subspace of $(\mathrm{F_A}, k)$. Let $\mathrm{U_A}, \mathrm{V_A} \subset \mathrm{G_A}$. Assume that, $\mathrm{U_A}$ and $\mathrm{V_A}$ are soft separated in F_A implies that $\mathrm{U_A} \cap k[\mathrm{V_A}] = \emptyset_\mathrm{A}$ and $k[\mathrm{U_A}] \cap \mathrm{V_A} = \emptyset_\mathrm{A}$. That is, $(\mathrm{U_A} \cap k[\mathrm{V_A}]) \cup (k[\mathrm{U_A}] \cap \mathrm{V_A}) = \emptyset_\mathrm{A}$.
Now, $(\mathrm{U_A} \cap k^*[\mathrm{V_A}]) \cup (k^*[\mathrm{U_A}] \cap \mathrm{V_A}) = \big(\mathrm{U_A} \cap (k[\mathrm{V_A}] \cap \mathrm{G_A})\big) \cup \big((k[\mathrm{U_A}] \cap \mathrm{G_A}) \cap \mathrm{V_A}\big)$
$= (\mathrm{U_A} \cap \mathrm{G_A} \cap k[\mathrm{V_A}]) \cup (k[\mathrm{U_A}] \cap \mathrm{G_A} \cap \mathrm{V_A})$
$= (\mathrm{U_A} \cap k[\mathrm{V_A}]) \cup (k[\mathrm{U_A}] \cap \mathrm{V_A})$
$= \emptyset_\mathrm{A}$.
Therefore, $\mathrm{U_A}$ and $\mathrm{V_A}$ are soft separated in $\mathrm{F_A}$ if and only if $\mathrm{U_A}$ and $\mathrm{V_A}$ are soft separated in $\mathrm{G_A}$.

Definition 3.5. A soft Čech closure space $(\mathrm{F_A}, k)$ is said to be disconnected if it can be written as two disjoint non-empty soft subsets $\mathrm{U_A}$and $\mathrm{V_A}$ such that $k[\mathrm{U_A}] \cap k[\mathrm{V_A}] = \emptyset_\mathrm{A}$ and $k[\mathrm{U_A}] \cup k[\mathrm{V_A}] = \mathrm{F_A}$.

Definition 3.6. A soft Čech closure space $(\mathrm{F_A}, k)$ is said to be connected if it is not disconnected.

Example 3.7. Let the initial universe set $X = \{u_1, u_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters. Let $A = \{x_1, x_2\} \subseteq E$and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$.
Then $P\big(X_{F_A}\big)$ are,
$F_{1A} = \{(x_1, \{u_1\})\}, F_{2A} = \{(x_1, \{u_2\})\}, F_{3A} = \{(x_1, \{u_1, u_2\})\}, F_{4A} = \{(x_2, \{u_1\})\}$,
$F_{5A} = \{(x_2, \{u_2\})\}, F_{6A} = \{(x_2, \{u_1, u_2\})\}, F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$,
$F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}, F_{9A} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}$,
$F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, F_{11A} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$,
$F_{12A} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}, F_{13A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$,
$F_{14A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, F_{15A} = F_A, F_{16A} = \emptyset_A$.
An operator $k: P(X_{F_A}) \to P(X_{F_A})$is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = k(F_{2A}) = k(F_{3A}) = F_{3A}, k(F_{4A}) = k(F_{6A}) = F_{6A}, k(F_{5A}) = F_{5A}$,
$k(F_{8A}) = k(F_{10A}) = k(F_{14A}) = F_{14A}, k(F_{7A}) = k(F_{9A}) = k(F_{11A}) = k(F_{12A})$
$= k(F_{13A}) = k(F_A) = F_A,\ k(\emptyset_A) = \emptyset_A$.
Taking, $U_A = F_{1A}\ and\ V_A = F_{4A}$, $k[U_A] \cap k[V_A] = \emptyset_A$ and $k[\mathrm{U_A}] \cup k[\mathrm{V_A}] = \mathrm{F_A}$.
Therefore, the soft Čech closure space(F_A, k) is disconnected.

Example 3.8. Let us consider the soft subsets of F_A that are given in *example 3.7.* An operator $k: P(X_{F_A}) \to P(X_{F_A})$is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = k(F_{7A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{4A}) = k(F_{5A}) = k(F_{6A}) = F_{6A}$,
$k(F_{2A}) = F_{10A}, k(F_{9A}) = k(F_{10A}) = k(F_{12A}) = F_{12A}$,
$k(F_{3A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) = F_A$, $k(\emptyset_A) = \emptyset_A$. Here, the soft Čech closure space (F_A, k) is connected.

Remark 3.9. The following example shows that connectedness in soft Čech closure space does not preserves hereditary property.

Example 3.10. In *example 3.8.*, the soft Čech closure space (F_A, k) is connected. Consider (G_A, k^*) be the subspace of (F_A, k) such that $G_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$. Taking, $U_A = \{(x_1, \{u_1\})\}$ *and* $V_A = \{(x_1, \{u_2\})\}$, $k^*[U_A] \cap k^*[V_A] = \emptyset_A$ and $k^*[U_A] \cup k^*[V_A] = G_A$. Therefore, the Soft Čech closure subspace (G_A, k^*) is disconnected.

Theorem 3.11. Connectedness in soft topological space (F_A, τ) need not imply that the soft Čech closure space (F_A, k) is connected.
Proof. Let us consider the soft subsets of F_A that are given in *example 3.7.* An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{1A}, k(F_{2A}) = F_{12A}, k(F_{4A}) = F_{4A}, k(F_{5A}) = F_{14A}, k(F_{7A}) = F_{7A}$,
$k(F_{3A}) = k(F_{6A}) = k(F_{10A}) = k(F_{11A}) = k(F_{12A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) =$
$= F_A, k(F_{8A}) = F_{14A}, k(F_{9A}) = F_{12A}$, $k(\emptyset_A) = \emptyset_A$.
Here, the two disjoint non empty soft subsets $U_A = F_{1A}$ *and* $V_A = F_{2A}$ satisfies $k[U_A] \cap k[V_A] = \emptyset_A$ and $k[U_A] \cup k[V_A] = F_A$. Therefore, the soft Čech closure space (F_A, k) is disconnected. But, it's associated soft topological space (F_A, τ), the only non empty soft open and soft closed subset is F_A. Hence, (F_A, τ) is connected.

Theorem 3.12. If soft Čech closure space is disconnected such that $F_A = k[U_A]/k[V_A]$ and let G_A be a connected soft subset of F_A then G_A need not to be holds the following conditions $(i) G_A \subseteq k[U_A]$ $(ii) G_A \subseteq k[V_A]$
Proof. Let us consider the soft subsets of F_A that are given in *example 3.7.* An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = k(F_{2A}) = k(F_{3A}) = F_{3A}, k(F_{4A}) = k(F_{6A}) = F_{6A}, k(F_{5A}) = F_{5A}$, $k(\emptyset_A) = \emptyset_A$,
$k(F_{8A}) = k(F_{10A}) = k(F_{14A}) = F_{14A}, k(F_{7A}) = k(F_{9A}) = k(F_{11A}) = k(F_{12A}) =$
$k(F_{13A}) = k(F_A) = F_A$. Taking, $U_A = F_{3A}$ *and* $V_A = F_{4A}$ then we get, $F_A = k[U_A]/k[V_A]$. Here, the soft Čech closure space (F_A, k) is disconnected. Let $G_A = F_{7A}$ be the connected soft subset of F_A. Clearly, G_A does not lie entirely within either $k[U_A]$ *or* $k[V_A]$.

Theorem 3.13. If the soft topological space (F_A, τ) is disconnected then the soft Čech closure space (F_A, k) is also disconnected.

Proof. Let the soft topological space (F_A, τ) is disconnected, implies that it is the union of two disjoint non empty soft subsets $\mathrm{U_A}$ and $\mathrm{V_A}$ such that $[\mathrm{U_A} \cap \tau - \mathrm{cl}(\mathrm{V_A})] \cup [\tau - \mathrm{cl}(\mathrm{U_A}) \cap \mathrm{V_A}] = \emptyset_\mathrm{A}$. Since, $k[\mathrm{U_A}] \subset \tau - cl(\mathrm{U_A})$ for every $\mathrm{U_A} \subset \mathrm{F_A}$ and $\tau - \mathrm{cl}(\mathrm{U_A}) \cap \tau - \mathrm{cl}(\mathrm{V_A}) = \emptyset_\mathrm{A}$ then $k[\mathrm{U_A}] \cap k[\mathrm{V_A}] = \emptyset_\mathrm{A}$. Since, $\mathrm{U_A} \cup \mathrm{V_A} = \mathrm{F_A}$, $\mathrm{U_A} \subseteq k[\mathrm{U_A}]$ and $\mathrm{V_A} \subseteq k[\mathrm{V_A}]$ implies that $\mathrm{U_A} \cup \mathrm{V_A} \subseteq k[\mathrm{U_A}] \cup k[\mathrm{V_A}]$, $\mathrm{F_A} \subseteq k[\mathrm{U_A}] \cup k[\mathrm{V_A}]$. But, $k[\mathrm{U_A}] \cup k[\mathrm{V_A}] \subseteq F_A$. Therefore, $k[\mathrm{U_A}] \cup k[\mathrm{V_A}] = \mathrm{F_A}$. Hence, $(\mathrm{F_A}, k)$ is also disconnected.

Definition 3.14. A soft Čech closure space $(\mathrm{F_A}, k)$ is said to be feebly disconnected if it can be written as two disjoint non-empty soft subsets $\mathrm{U_A}$and $\mathrm{V_A}$ such that $U_A \cap k[V_A] = \emptyset_A$ and $U_A \cup k[V_A] = F_A$.

Result 3.15. Every disconnected soft Čech closure space (F_A, k) is feebly disconnected but the following example shows that the converse is not true.

Example 3.16. In *example 3.8* Consider, $F_{2A} = \{(x_1, \{u_2\})\}$ and $F_{1A} = \{(x_1, \{u_1\})\}$. Which satisfies the condition $F_{2A} \cap k[F_{1A}] = \emptyset_A$ and $F_{2A} \cup k[F_{1A}] = F_A$.Therefore, the soft Čech closure space (F_A, k) is feebly disconnected . But, the soft Čech closure space (F_A, k) is connected.

Theorem 3.17. Let $\mathrm{G_A}$ be a connected subset of the connected soft Čech closure space (F_A, k), then $(k[G_A], k^*)$ need not be connected.
Proof. The above theorem is proved by the following counter example.

Example 3.18. Let us consider the soft subsets of F_A that are given in *example 3.7.* An operator $k: P(X_{F_A}) \rightarrow P(X_{F_A})$is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{1A}, k(F_{2A}) = k(F_{3A}) = F_{14A}, k(F_{4A}) = k(F_{7A}) = F_{7A}, k(F_{5A}) = F_{6A},$
$k(F_{6A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(\emptyset_A) = \emptyset_A,$
$k(F_{9A}) = k(F_{10A}) = k(F_{12A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) = F_A.$
The soft Čech closure space $(\mathrm{F_A}, k)$ is connected. Consider the connected soft subset $\mathrm{G_A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$. Then $k[\mathrm{G_A}] = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$ and $P\big(X_{k[\mathrm{G_A}]}\big)$ are $k[\mathrm{G_{1A}}] = \{(x_1, \{u_1\})\}$, $k[\mathrm{G_{2A}}] = \{(x_2, \{u_1\})\}$, $k[\mathrm{G_{3A}}] = \{(x_2, \{u_2\})\}$,
$k[\mathrm{G_{4A}}] = \{(x_2, \{u_1, u_2\})\}, k[\mathrm{G_{5A}}] = \{(x_1, \{u_1\}), (x_2, \{u_1\})\},$
$k[\mathrm{G_{6A}}] = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}, k[\mathrm{G_{7A}}] = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\} = k[\mathrm{G_A}],$
$k[\mathrm{G_{8A}}] = \emptyset_A.$

An operator $k^*: P\big(X_{k[\mathrm{G_A}]}\big) \rightarrow P\big(X_{k[\mathrm{G_A}]}\big)$is defined from soft power set $P\big(X_{k[\mathrm{G_A}]}\big)$ to itself over X as follows.
$k^*(k[\mathrm{G_{1A}}]) = k[\mathrm{G_{1A}}],, k^*(k[\mathrm{G_{2A}}]) = k^*(k[\mathrm{G_{5A}}]) = k[\mathrm{G_{5A}}], k^*(k[\mathrm{G_{3A}}]) = k[\mathrm{G_{4A}}],$
$k^*(k[\mathrm{G_{4A}}]) = k^*(k[\mathrm{G_{6A}}]) = k^*(k[\mathrm{G_{7A}}]) = k[\mathrm{G_A}], k^*(k[\mathrm{G_{8A}}]) = \emptyset_A$. Taking,
$k[\mathrm{G_{1A}}]$ *and* $k[\mathrm{G_{3A}}]$, $k^*\big[k[\mathrm{G_{1A}}]\big] \cap k^*\big[k[\mathrm{G_{3A}}]\big] = \emptyset_\mathrm{A}$ and $k^*[k[\mathrm{G_{1A}}]] \cup k^*\big[k[\mathrm{G_{3A}}]\big] = k[\mathrm{G_A}]$. Therefore, $(k[\mathrm{G_A}], k^*)$ is disconnected.

Definition 3.19. Let (F_{A}, k) and $(\mathrm{G}_{\mathrm{A}}, k^*)$ are two Soft Čech closure spaces over X and Y respectively. For $x \in X$ and $e: A \to B$, a map $f: (F_{\mathrm{A}}, k) \to (\mathrm{G}_{\mathrm{A}}, k^*)$ is said to be morphism if $\varphi_{fe}(k(U_A)) \subset k^*\big(\varphi_{fe}(U_A)\big)$, for every soft subset $U_A \in SS(X, A)$.

Theorem 3.20. The image of a connected soft Čech closure space under morphism need not be connected.

Proof. Let us consider the soft subsets of F_A that are given in *example 3.7*. An operator $k: P(X_{F_A}) \to P(X_{F_A})$is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{1A}, k(F_{2A}) = k(F_{9A}) = F_{12A}, k(F_{4A}) = F_{4A}, k(F_{5A}) = k(F_{8A}) = F_{14A},$
$k(F_{3A}) = k(F_{6A}) = k(F_{10A}) = k(F_{11A}) = k(F_{12A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) =$
$F_A, k(F_{7A}) = F_{7A},\ k(\emptyset_A) = \emptyset_A$. Then, (F_{A}, k) is soft Čech closure space.
Let the initial universe set $Y = \{v_1, v_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters. Let $B = \{x_1, x_2\} \subseteq E$and $G_B = \{(x_1, \{v_1, v_2\}), (x_2, \{v_1, v_2\})\}$.Then $P\big(X_{G_B}\big)$ are
$G_{1B} = \{(x_1, \{v_1\})\}, G_{2B} = \{(x_1, \{v_2\})\}, G_{3B} = \{(x_1, \{v_1, v_2\})\}, G_{4B} = \{(x_2, \{v_1\})\},$
$G_{5B} = \{(x_2, \{v_2\})\}, G_{6B} = \{(x_2, \{v_1, v_2\})\}, G_{7B} = \{(x_1, \{v_1\}), (x_2, \{v_1\})\},$
$G_{8B} = \{(x_1, \{v_1\}), (x_2, \{v_2\})\}, G_{9B} = \{(x_1, \{v_2\}), (x_2, \{v_1\})\},$
$G_{10B} = \{(x_1, \{v_2\}), (x_2, \{v_2\})\}, G_{11B} = \{(x_1, \{v_1\}), (x_2, \{v_1, v_2\})\},$
$G_{12B} = \{(x_1, \{v_2\}), (x_2, \{v_1, v_2\})\}, G_{13B} = \{(x_1, \{v_1, v_2\}), (x_2, \{v_1\})\},$
$G_{14B} = \{(x_1, \{v_1, v_2\}), (x_2, \{v_2\})\}, G_{15B} = G_A, G_{16B} = \emptyset_A.$

An operator $k^*: P\big(X_{G_B}\big) \to P\big(X_{G_B}\big)$ is defined from soft power set $P(X_{G_B})$ to itself over Y as follows.
$k^*(G_{1B}) = k^*(G_{5B}) = G_{8B}, k^*(G_{2B}) = G_{3B}, k^*(G_{3B}) = k^*(G_{9B}) = k^*(G_{13B}) = G_{13B},$
$k^*(G_{4B}) = G_{4B}, k^*(G_{6B}) = k^*(G_{8B}) = k^*(G_{11B}) = G_{11B}, k^*(G_{7B}) = G_{7B},$
$k^*(G_{10B}) = G_{14B}, k^*(G_{12B}) = k^*(G_{14B}) = k^*(G_B) = G_B, k^*(\emptyset_B) = \emptyset_B.$
Here, (G_{B}, k^*) is the soft Čech closure space.
Let $f : (F_{\mathrm{A}}, k) \to (G_{\mathrm{B}}, k^*)$ and $e: A \to B$ are the map defined in such a way that $f(u_1) = v_2$; $f(u_2) = v_1$ and $e(x_1) = x_2$; $e(x_2) = x_1$. Therefore, $f : (F_{\mathrm{A}}, k) \to (G_{\mathrm{B}}, k^*)$ is morphism. Taking, the connected soft subset $F_{12A} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$, we get the image $G_{13B} = \{(x_1, \{v_1, v_2\}), (x_2, \{v_1\})\}$. But $f(F_{12A}) = G_{13B}$ is disconnected.
Therefore, image of a connected soft Čech closure space under morphism need not be connected.

Definition 3.20. Each soft point $(x, u) \in F_A$ belongs to a unique soft component of F_A, called the soft component of (x, u). A soft component E_A of a soft Čech closure space (F_{A}, k) is a maximal soft connected subset of F_A ; that is, E_A is soft connected and E_A is not a proper soft subset of any soft connected subset of F_A. Clearly, E_A is non-empty. The central facts about the soft components of a space are contained in the following theorem.

Theorem 3.21. Let the soft components of a soft Čech closure space (F_{A}, k) form a partition of F_A, then every soft connected subset of F_A is contained in some soft component.

Proof. The proof of the theorem shows in the following example.

Example 3.22. Let us consider the soft subsets of F_A that are given in *example 3.7.* An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = k(F_{5A}) = F_{8A}, k(F_{2A}) = F_{3A}, k(F_{4A}) = F_{4A}, k(F_{7A}) = F_{7A}$,
$k(F_{3A}) = k(F_{9A}) = k(F_{13A}) = F_{13A}, k(F_{10A}) = F_{14A}, k(\emptyset_A) = \emptyset_A$,
$k(F_{6A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{12A}) = k(F_{14A}) = k(F_A) = F_A$.
The soft components of F_A are $k(F_{4A})$ and $k(F_{10A})$. Any other soft connected subset of F_A , such as $\{(x_1, \{u_1, u_2\})\}$ is contained in $k(F_{10A})$.

4. Local soft connectedness

In this section, we devoted to study and characterize local soft connectedness in soft Čech closure space.

Definition 4.1. A soft Čech closure space (F_A, k) is said to be locally soft connected at (x, u) if for every soft neighbourhood U_A of (x, u), there is a soft connected neighbourhood V_A of (x, u) contained in U_A. If F_A is locally soft connected at each of its soft points, then F_A is said to be locally soft connected.

Example 4.2. Let us consider the soft subsets of F_A that are given in *example 3.7.* An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{1A}, k(F_{2A}) = F_{2A}, k(F_{3A}) = F_{3A}, k(F_{4A}) = F_{4A}, k(F_{5A}) = F_{5A}, k(F_{6A}) = F_{6A}$,
$k(F_{7A}) = F_{7A}, k(F_{8A}) = F_{8A}, k(F_{9A}) = F_{9A}, k(F_{10A}) = F_{10A}, k(F_{11A}) = F_{11A}$,
$k(F_{12A}) = F_{12A}, k(F_{13A}) = F_{13A}, k(F_{14A}) = F_{14A}, k(F_A) = F_A, k(\emptyset_A) = \emptyset_A$.
For if $(x, u) \in F_A$, then $\{(x, u)\}$ is a soft open connected set containing (x, u) which is contained in every soft open set containing (x, u). Therefore, (F_A, k) is locally soft connected.

Result 4.3. The following example shows that a locally soft connected Čech closure space (F_A, k) is need not imply soft connectedness.

Example 4.4. In *example 4.2*, the discrete soft Čech closure space (F_A, k) is locally soft connected but not soft connected.

Theorem 4.5. Every soft open subspace of a locally soft connected Čech closure space (F_A, k) is locally soft connected.
Proof. Let (G_A, k^*) be the soft open subspace of a locally soft connected Čech closure space (F_A, k). Let $(y, u) \in G_A$ be arbitrary, then $(y, u) \in F_A$. Let U_A be a soft k^*-open neighbourhood of (y, u), then U_A be the soft k-open neighbourhood of (y, u) in F_A. Also F_A is locally soft connected. Hence, there exist soft connected soft k-open neighbourhood V_A of (y, u) such that $(y, u) \in V_A \subset U_A$. Consequently, (G_A, k^*) is locally soft connected at (y, u).

Theorem 4.6. A soft Čech closure space (F_A, k) is locally soft connected if and only if the soft components of every soft open subspace of F_A are soft open in F_A.

Proof. Let (G_A, k^*) be a soft open subspace of (F_{A}, k). Let $E_A \subset G_A$ be any component of G_A, then E_A is maximal soft connected set in (G_A, k^*). Since, $G_A \subset F_A$. Then, E_A is also maximal soft connected in (F_{A}, k). Let (F_{A}, k) be locally soft connected. It is enough to show that, E_A is soft open in (F_{A}, k). Let $(y, u) \in E_A$ be arbitrary, then $(y, u) \in G_A \subset F_A$ or $(y, u) \in F_A$. Since, (F_A, k) is locally soft connected this implies given any soft k-open neighbourhood U_A of (y, u), there exist soft connected soft k-open neighbourhood V_A such that $(y, u) \in V_A \subset U_A$. $(y, u) \in E_A$ is maximal soft connected and $(y, u) \in V_A$ is soft connected. This implies, $V_A \subset E_A$. Thus, given any $(y, u) \in E_A$, there exist a soft k-open neighbourhood V_A such that $(y, u) \in V_A \subset E_A$. This implies, E_A is soft open in (F_{A}, k). Conversely, Let E_A be soft open in (G_A, k^*), then E_A is soft open in (F_{A}, k). Let $(y, u) \in E_A$. Now, $(y, u) \in E_A \subset G_A$, where E_A, G_A are soft k-open in (F_{A}, k). Also, E_A is soft connected. This implies that (F_{A}, k) is locally soft connected at (y, u). From this the required result follows.

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