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Degree Sum Energy of Some Graphs

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Abstract. For any connected graph *G* degree sum matrix is a matrix having sum of degrees of a pair of vertices. The degree sum energy is absolute sum of degree sum eigenvalues of *G* which are simply eigen values of degree sum matrix. In this paper we deal with degree sum energy of some graphs.

Keywords: Degree sum matrix, energy of graph, Coalescence

AMS Mathematics Subject Classification (2010) : 05C50

1. Introduction

Let *G* be a simple graph with '*n*' vertices & '*m*' edges. Let the vertices of *G* be labeled as $v_1, v_2, v_3 \ldots\ldots v_n$. The degree of a vertex *v* in a graph G, denoted by *d(v)* is the number of edges incident to *v*. If all the vertices of a graph *G* have the same degree equal to *r*, then *G* is called *r*-regular graph.

The adjacency matrix *A(G)* of a graph *G* is a square matrix of order *n* whose (*i,j*) entry is equal to unity if the vertex v_i is adjacent to v_j, and is equal to zero otherwise. The eigen values of an adjacency matrix *A(G)* are denoted by $\lambda_1, \lambda_2, \lambda_3 \ldots \ldots . \lambda_n$.. The energy or adjacency energy of *G* is defined as, $E = E_\pi(G) = \sum_{i=0}^{n} |\lambda_i|$.A book which covers all its aspects is [7]. The spectra connection with connectivity of graph is discussed in [8].

This definition of energy was motivated by large number of results for the Huckel molecular orbital total π-electron energy [5]. In [9,10], authors obtained bounds for the distance eigen values and distance energy, corresponding to distance matrix of a graph. Motivated by work on Maximum Degree Energy of a Graph [2], in [1] Ramane et al. introduced the concept of degree sum matrix associated with a graph and studied some bounds for its eigenvalues as well as degree sum energy. For a brief recent survey on various types of energy defined on a graph one can refer [3].

The degree sum matrix of a simple graph *G* is denoted as *DS* (*G*) is defined as *DSM* (*G*) = [d_{ij}] where $d_{ij} = d_i + d_j$ when $i \neq j$ and 0 otherwise. The degree sum polynomial of *G* is the characteristic polynomial of DSM(*G*) denoted by

$\psi[DSM(G):\gamma]$.Since DSM(G) is a real symmetric matrix its eigen values $\gamma_1, \gamma_2, \gamma_3 \ldots\ldots \gamma_n$ can be ordered as $\gamma_1 \geq \gamma_2 \geq \gamma_3 \ldots\ldots \geq \gamma_n$.Analogues to adjacency energy, degree sum energy of a graph denoted by E_{DS} (G) $=\sum_1^n |\gamma_i|$.Degree sum polynomial of graph valued functions on regular graphs are available in [4].It's obvious that two non isomorphic graphs with same degree sum matrix will have same degree sum energy. Such graphs can be called degree sum equi energetic. Already on such family is available in [1]. Graphs with different degree sum matrix but same degree sum energy are interesting. Concept of sum distance in fuzzy graphs is available in [6].

We denote wheel graph of order n as W_n and a path of order n by P_n . We also consider a family of graphs called **Tadpole Graphs(or Lollipop Graphs)** as a graph of order $n+k$ obtained by joining a cycle C_n to a path P_{k+1} of order k+1(length k) and is denoted by $T_{n, k.}$

Example: $T_{5, 4}$

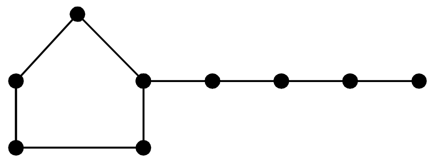

Figure 1:

For two graphs G and H, $G \cup H$ denotes the disjoint union of G and H. The Dumbbell graph $D_{a,b,c}$ consists of two vertex-disjoint cycles C_a, C_b and a path $P_{c+3} (c \geq -1)$ joining them having only its end vertices in common with the cycles. It has $a+b+c+1$ vertices and $a+b+c+2$ edges.

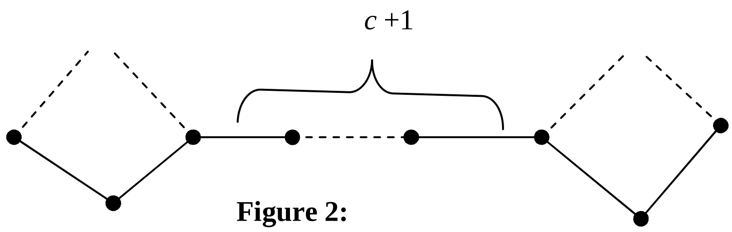

Figure 2:

For adjacency spectral characterization of the Lollipop and dumbbell graph one can refer [11,12].

Coalescence: Let H_1 & H_2 be graphs on disjoint sets of vertices respectively. Suppose $U = \{u_1, u_2 \ldots\ldots u_t\}$ is a clique in H_1 and $W = \{w_1, w_2 \ldots\ldots w_t\}$ is a clique in H_2.Let G be a graph obtained from H_1 and H_2 by identifying (coalescence into a single vertex) u_i and w_i ,

$1 \leq i \leq t$. Then G is an overlap of $H_1$and H_2 in K_t. It may be viewed as generalized coalescence denoted by H_1 o H_2.This operation is often taken to compute the chromatic polynomials of graphs [13]. In particular for t =1 we call it as coalescence on a vertex or overlap denoted by H_1 O_v H_2 .Similarly for t =2 we call it as edge coalescence denoted by H_1 O_e H_2.

2. Results

Here we obtain degree sum energy of Wheel, Path Tadpole graph and Dumbbell graph.

Theorem 2.1. The degree sum energy of wheel graph of order n W_n , is given by,

$$E_{DS}(W_n) = 6(n-2) + 2\sqrt{n^3 + 12n^2 - 36n + 32}$$

Proof: With pertinent labeling the degree sum matrix of W_n is given by,

$$DSM[W_n] = \begin{bmatrix} 0 & n+2 & n+2 & \cdots & n+2 \\ n+2 & 0 & 6 & \cdots & 6 \\ \vdots & 6 & 0 & \cdots & 6 \\ \vdots & \vdots & 6 & \ddots & \vdots \\ n+2 & 6 & \cdots & \cdots & 0 \end{bmatrix}$$

The degree sum polynomial then becomes,

$$\Psi[DSM(W_n):\gamma] = \begin{vmatrix} \gamma & -(n+2) & -(n+2) & \cdots & -(n+2) \\ -(n+2) & \gamma & -6 & \cdots & 6 \\ -(n+2) & -6 & \gamma & \cdots & 6 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -(n+2) & -6 & \cdots & \cdots & \gamma \end{vmatrix}$$

$$R_i + \frac{n+2}{\gamma}R_1 \quad i = 2, \dots\dots\dots..n$$

$$= \begin{vmatrix} \gamma & -(n+2) & -(n+2) & \cdots & -(n+2) \\ 0 & \gamma - \frac{(n+2)^2}{\gamma} & -6-\frac{(n+2)^2}{\gamma} & \cdots & -6-\frac{(n+2)^2}{\gamma} \\ \vdots & -6-\frac{(n+2)^2}{\gamma} & \gamma-\frac{(n+2)^2}{\gamma} & \cdots & -6-\frac{(n+2)^2}{\gamma} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -6-\frac{(n+2)^2}{\gamma} & -6-\frac{(n+2)^2}{\gamma} & \cdots & \gamma-\frac{(n+2)^2}{\gamma} \end{vmatrix}$$

$$= \gamma \begin{vmatrix} a & b & \cdots & b \\ b & a & \cdots & b \\ \vdots & \vdots & \ddots & \vdots \\ b & b & \cdots & a \end{vmatrix} = \gamma(a-b)^{n-2}[a+(n-2)b]$$

$$= \gamma(\gamma + 6)^{n-2}\left[\gamma - \frac{(n+2)^2}{\gamma} - (n-2)\overline{6 + \frac{(n+2)^2}{\gamma}}\right]$$

$$\Psi[DSM(W_n):\gamma] \;=\; (\gamma+6)^{(n-2)}\,[\gamma^2 - 6(n-2)\gamma - (n-1)(n+2)^2].$$

On computing eigen values and adding their absolute values theorem follows.

Theorem 2.2 The degree sum energy of path $P_{n,}$ is given by

$$E_{DS}(P_n) = 4n\text{-}10 + 2\sqrt{4n^2 - 10n + 13}$$

Proof: With pertinent labeling the degree sum matrix of P_n is given by,

$$\text{DSM}\,[P_n]= \begin{bmatrix} 0 & 3 & 3 & \dots & 3 & 2 \\ 3 & 0 & 4 & \dots & 4 & 3 \\ 3 & 4 & 0 & \cdots & 4 & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 3 & 4 & 4 & \cdots & 0 & 3 \\ 2 & 3 & \cdots & \cdots & 3 & 0 \end{bmatrix}$$

The degree sum polynomial is

$$\psi[DSM(P_n):\gamma] = \begin{vmatrix} \gamma & -3 & -3 & \dots & -3 & -2 \\ -3 & \gamma & -4 & \dots & -4 & -3 \\ \vdots & -4 & \gamma & \dots & \dots & -3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -3 & -4 & -4 & \dots & \gamma & -3 \\ -2 & -3 & -3 & \dots & -3 & \gamma \end{vmatrix}$$

$R_n - R_1$, $R_i - R_2$, $\qquad i = 3, \dots \dots n-1$

$$= \begin{vmatrix} \gamma & -3 & -3 & \cdots & -3 & -2 \\ -3 & \gamma & -4 & \cdots & -4 & -3 \\ 0 & -(4+\gamma) & \gamma+4 & 0 & \cdots & 0 \\ \vdots & -(4+\gamma) & 0 & (\gamma+4) & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -(4+\gamma) & \vdots & \vdots & \gamma+4 & 0 \\ -(\gamma+2) & -(4+\gamma) & \cdots & \cdots & \cdots & \gamma+2 \end{vmatrix}$$

$C_1 + C_n \; gives$

$$= \begin{vmatrix} \gamma-2 & -3 & -3 & \cdots & -3 & -2 \\ -6 & \gamma & -4 & \cdots & -4 & -3 \\ 0 & -(4+\gamma) & \gamma+4 & \cdots & \vdots & \vdots \\ \vdots & -(4+\gamma) & 0 & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -(4+\gamma) & 0 & \cdots & \cdots & \gamma+2 \end{vmatrix}$$

$C_2 + \sum_{i=3}^{n-1} C_i \; gives,$

$$= \begin{vmatrix} \gamma-2 & -3(n-2) & \cdots & \cdots \\ -6 & \gamma-4(n-3) & \cdots & \vdots \\ 0 & 0 & \gamma+4 & \vdots \\ 0 & 0 & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \cdots & \cdots & \cdots & \gamma+2 \end{vmatrix}$$

$$\therefore \quad \psi[DSM(P_n):\gamma] = \left(\gamma+4\right)^{(n-3)}\left(\gamma+2\right)\left[\gamma^2 - 2(2n-5)\,\gamma - 2(5n-6)\right]$$

On calculating eigen values and adding their absolute values theorem follows.

Theorem 2.3. The degree sum energy of tadpole graph $T_{n,k}$ is given by

$E_{DS}[T_{n,k}] = 4(n+k-3) + |p| + |q| + |s|$ where p,q,s are the roots of the equation,

$$\{\gamma^3 - 4(n+k-3)\gamma^2 - [34n+34k-52]\gamma + [48-56n-56k]\} = 0$$

Proof: With pertinent labeling the degree sum matrix of $T_{n,k}$ is given by,

$$DSM\,[T_{n,k}] = \begin{bmatrix} 0 & 5 & 5 & \dots & 5 & 4 \\ 5 & 0 & 4 & \dots & 4 & 3 \\ 5 & 4 & 0 & \cdots & 4 & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 5 & 4 & 4 & \cdots & 0 & 3 \\ 4 & 3 & \cdots & \cdots & 3 & 0 \end{bmatrix}$$

The rest of the proof follows on similar lines as in the proof of the previous theorem.

Corollary 2.4. Two non-isomorphic Tadpole graphs having same $n + k$, have same degree sum energy. Such graphs are called degree sum equienergetic graphs.

Theorem 2.5. The degree sum energy of dumbbell graph $D_{a,b,c}$ is given by ,

$$E_{DS}[D_{a,b,c}] = [4(a+b+c) - 2 + 2\sqrt{4(a+b+c)^2 + 22(a+b+c) - 1}]$$

Proof: With pertinent labeling the degree sum matrix of $D_{a,b,c}$ is given by,

$$DSM\ [D_{a,b,c}] = \begin{bmatrix} 0 & 6 & 5 & \dots & 5 & 5 \\ 6 & 0 & 5 & \dots & 5 & 5 \\ 5 & 5 & 0 & \cdots & 4 & 4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 5 & 5 & 4 & \cdots & 0 & 4 \\ 5 & 5 & \cdots & \cdots & 4 & 0 \end{bmatrix}$$

The rest of the proof follows on similar lines as in proof the previous theorem.

Corollary 2.6. From the expression for energy $E_{DS}[D_{a,b,c}]$ given above it's clear that two non-isomorphic Dumbbell graphs having same $a+b+c$ have same degree sum energy.

3. Degree sum energy for Coalescence of graphs

In this section we discuss the degree sum energy of coalescence regular graphs, complete graphs and cycles.

Theorem 3.1. The degree sum energy of the vertex coalescence of two r-regular graphs of $G_1 \& G_2$ of order n_1, n_2 respectively is given by $E_{DS}(G_1 \circ_v G_2) = 2r[n_1 + n_2 - 3 + \sqrt{n_1^2 + n_2^2 + 3n_2 + 2n_1 n_2 - 6n_1 - 9}\ ]$

Proof: The degree sum of the coalescence $G_1 \circ G_2$ has the form ,

$$DSM[G_1 \circ_v G_2] = \left[\begin{array}{cccc|cccc} 0 & 3r & \cdots & 3r & 3r & \cdots & \cdots & 3r \\ 3r & 0 & \cdots & 3r & 2r & \cdots & \cdots & 2r \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ 3r & 3r & \cdots & 0 & 2r & \cdots & \cdots & 2r \\ \hline 3r & 3r & \cdots & 3r & 0 & 2r & \cdots & 2r \\ \vdots & \vdots & \cdots & \vdots & 2r & 0 & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ 3r & 3r & \cdots & 3r & 2r & \cdots & \cdots & 0 \end{array}\right]$$

Degree sum polynomial is then given by

$$\Psi[DSM(G_1 \circ_v G_2):\gamma] = \left|\begin{array}{cccc|cccc} \gamma & -3r & \cdots & -3r & -3r & \cdots & \cdots & -3r \\ -3r & \gamma & \cdots & -3r & -2r & \cdots & \cdots & -2r \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ -3 & -3r & \cdots & \gamma & -2r & \cdots & \cdots & -2r \\ \hline -3r & -3r & \cdots & -3r & \gamma & -2r & \cdots & -2r \\ \vdots & \vdots & \cdots & \vdots & -2r & \cdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ -3r & \cdots & \cdots & -3r & -2r & \cdots & \cdots & \gamma \end{array}\right|$$

Performing $R_i - R_2,\ i = 3,4,\ldots .n_1 + n_2 - 1$

$$\Psi[DSM(G_1 \circ_v G_2):\gamma] = \left|\begin{array}{ccccc|cccc} \gamma & -3r & -3r & \cdots & -3r & -2r & -2r & \cdots & -2r \\ -3r & \gamma & -2r & \cdots & -2r & \vdots & \vdots & \cdots & \vdots \\ \vdots & -(\lambda+2r) & \gamma+2r & 0 & \cdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & 0 & \gamma+2r & \cdots & \vdots & \vdots & \cdots & \vdots \\ -3r & -(\lambda+2r) & \cdots & \cdots & \gamma+2r & -2r & \cdots & \cdots & -2r \\ \hline 0 & -(\gamma+2r) & 0 & \cdots & 0 & \gamma+2r & 0 & \cdots & \vdots \\ 0 & -(\gamma+2r) & 0 & \cdots & 0 & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -(\gamma+2r) & 0 & \cdots & 0 & 0 & \cdots & \cdots & \gamma+2r \end{array}\right|$$

Performing $C_2 + \sum_3^{n_1+n_2-1} C_i$

$$= \begin{vmatrix} \gamma & -3r(n_1+n_2-2) & -3r & -3r & \cdots & -3r \\ -3r & \gamma-2r(n_1+n_2-3) & -2r & -2r & \cdots & -2r \\ 0 & 0 & \gamma+2r & 0 & \cdots & 0 \\ 0 & 0 & 0 & \gamma+2r & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \gamma+2r \end{vmatrix} \text{ X } \left|(\gamma+2r)\mathrm{I}_{n_2-1}\right|$$

As the lower triangular block is 0, expanding directly gives,

$$= (\gamma + 2r)^{n_2-1}(\gamma + 2r)^{n_1-2}[\gamma^2 - 2r(n_1 + n_2 - 3)\gamma - 9r^2(n_1 + n_2 - 2)]$$

$$= (\gamma + 2r)^{n_2+n_2-3}[\gamma^2 - 2r(n_1 + n_2 - 3)\gamma - 9r^2(n_1 + n_2 - 2)]$$

So that calculating eigen values we get degree sum energy of coalescence as,

$$E_{DS}(G_1 \circ_v G_2) = 2r[(n_1 + n_2 - 3) + \sqrt{n_1^2 + n_2^2 + 3n_2 + 2n_1n_2 - 6n_1 - 9}]$$

Example: Let $r = 2$, $G_1 \simeq C_3$, $G_2 \simeq C_4$ so that $n_1 = 3$ $n_2 = 4$ gives

$E_{DS}(G_1 \circ G_2) = 47.241$

Note: Irrespective of the structure, the degree sum energy of vertex coalescence of two regular graphs of same regularity remains same. Hence we have another family of degree sum equienergetic graphs

Theorem 3.2 If G_1 and G_2 are r- regular graphs of order n_1 and n_2 respectively then degree sum energy of edge coalescence $G_1 \mathrm{o_e} G_2$ is $E_{DS}(G_1 \circ_e G_2) = 2(3r-1)(n_1 + n2-5+4r-2+a$ where $a = \{(4r-2)+(3r-1)(n_1 + n_2 - 5)\}^2 + 8(3r-1)^2(n_1 + n_2 - 4) - 4(4r-2)(3r-1)(n_1 + n_2 - 5)$

Proof: The degree sum matrix of the coalescence $G_1 \mathrm{o_e} G_2$ has the form

$$DSM[G_1 \mathrm{o_e} G_2] = \begin{bmatrix} 0 & 4r-2 & 3r-1 & \cdots & 3r-1 & 3r-1 & \cdots & \cdots & 3r-1 \\ 4r-2 & 0 & 3r-1 & \cdots & 3r-1 & 3r-1 & \cdots & \cdots & 3r-1 \\ 3r-1 & 3r-1 & 0 & \cdots & 3r-1 & 3r-1 & \cdots & \cdots & 3r-1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 3r-1 & 3r-1 & 3r-1 & \cdots & 0 & 3r-1 & \cdots & \cdots & 3r-1 \\ 3r-1 & 3r-1 & 3r-1 & \cdots & 3r-1 & 0 & \cdots & \cdots & 3r-1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 3r-1 & 3r-1 & 3r-1 & \cdots & 3r-1 & 3r-1 & \cdots & \cdots & 0 \end{bmatrix}$$

Degree sum polynomial is then given by

$$\Psi[DSM(G_1 \mathrm{o_e} G_2): \lambda]$$
$$= \begin{vmatrix} \gamma & -(4r-2) & -(3r-1) & \cdots & -(3r-1) & -(3r-1) & \cdots & \cdots & -(3r-1) \\ -(4r-2) & \gamma & -(3r-1) & \cdots & -(3r-1) & -(3r-1) & \cdots & \cdots & -(3r-1) \\ -(3r-1) & -(3r-1) & \gamma & \cdots & -(3r-1) & -(3r-1) & \cdots & \cdots & -(3r-1) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -(3r-1) & -(3r-1) & -(3r-1) & \cdots & \gamma & -(3r-1) & \cdots & \cdots & -(3r-1) \\ -(3r-1) & -(3r-1) & -(3r-1) & \cdots & -(3r-1) & \gamma & \cdots & \cdots & -(3r-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -(3r-1) & -(3r-1) & -(3r-1) & \cdots & -(3r-1) & -(3r-1) & \cdots & \cdots & \gamma \end{vmatrix}$$

Performing $R_i - R_3$, $i = 4, \ldots . n_1 + n_2 - 2$

$$\Psi\left[DSM(G_1 \mathrm{o_e} G_2):\gamma\right]$$

$$= \left|\begin{array}{ccccc|cccc}
\gamma & -(4r-2) & -(3r-1) & -(3r-1) & -(3r-1) & -(3r-1) & -(3r-1) & \cdots & -(3r-1) \\
-(4r-2) & \gamma & -(3r-1) & -(3r-1) & -(3r-1) & -(3r-1) & -(3r-1) & \cdots & -(3r-1) \\
-(3r-1) & -(3r-1) & \gamma & -(3r-1) & -(3r-1) & -(3r-1) & -(3r-1) & \cdots & -(3r-1) \\
0 & 0 & -(\gamma+3r-1) & (\gamma+3r-1) & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & -(\gamma+3r-1) & 0 & (\gamma+3r-1) & 0 & 0 & \cdots & 0 \\
\hline
0 & 0 & -(\gamma+3r-1) & \cdots & 0 & (\gamma+3r-1) & 0 & \cdots & 0 \\
0 & 0 & -(\gamma+3r-1) & \cdots & 0 & \vdots & \vdots & \cdots & \vdots \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & -(\gamma+3r-1) & \cdots & 0 & 0 & \cdots & \cdots & (\gamma+3r-1)
\end{array}\right|$$

Performing $C_3 + \sum_4^{n_1+n_2-2} C_i$

$$= \begin{vmatrix} \gamma & -(4r-2) & -(n_1+n_2-4)(3r-1) \\ -(4r-2) & \gamma & -(n_1+n_2-4)(3r-1) \\ -(3r-1) & -(3r-1) & \gamma-(3r-1)(n_1+n_2-5) \end{vmatrix} \ \mathrm{X}\ \left|(\gamma+3r-1)I_{n_1+n_2-5}\right|$$

$\Psi[DS(G_1 \mathrm{o_e} G_2):\lambda] =$

$(\gamma+3r-1)^{n_1+n_2-5}(\gamma+4r-2)[\gamma^2 - \{(4r-2)+(3r-1)(n_1+n_2-5)\}\gamma +$

$(4r-2)(3r-1)(n_1+n_2-5) - 2(3r-1)^2(n_1+n_2-4)]$

Hence the theorem.

Example: Let $r = 2$, $G_1 \simeq C_3$, $G_2 \simeq C_4$ then $G_1 O e G_2$ is $E_{DS}(G_1 \mathrm{o_e} G_2) = 38.5764$

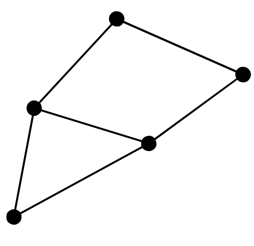

Figure 3:

Theorem 3.3. Degree sum energy of vertex coalescence of two cycles $C_{m,}$ and C_n is given by, $E_{DS}[C_m \mathrm{o_v} C_n] = 4\sqrt{(m+n-3)^2 + 9(m+n-2)} + 4(m+n-2)$

Proof: Let C_m and C_n be the two cycles of order m ,n respectively. The degree sum matrix of vertex coalescence is given by $DSM\ [C_m \mathrm{o_v} C_n]$

$$DSM\,[C_m \text{ o}_v\, C_n]=\begin{bmatrix} 0 & 6 & 6 & 6 & \cdots & 6 \\ 6 & 0 & 4 & 4 & \cdots & 4 \\ 6 & 4 & 0 & 4 & \cdots & 4 \\ 6 & 4 & 4 & 0 & \cdots & 4 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 6 & 4 & 4 & 4 & \cdots & 0 \end{bmatrix}$$

The rest of the proof follows on similar lines as in proof the previous theorem.

Theorem 3.4 Degree sum energy of edge coalescence of two cycles $C_{m,}$and C_n is given by, $E_{DS}[C_m \text{ o}_e\, C_n] = 2(2m+2n-7)+2\sqrt{4m^2+4n^2-20m-20n+8mn+41}$

Proof: The degree sum matrix of edge coalescence of cycles C_m & C_n is given by,

$$DSM\,[C_m \text{ o}_e\, C_n] = \begin{bmatrix} 0 & 6 & 4 & 4 & \cdots & 4 \\ 6 & 0 & 4 & 4 & \cdots & 4 \\ 4 & 4 & 0 & 4 & \cdots & 4 \\ 4 & 4 & 4 & 0 & \cdots & 4 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 4 & 4 & 4 & 4 & \cdots & 0 \end{bmatrix}$$

The rest of the proof follows on similar lines as in proof the previous theorem.

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