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A Modified Vogel's Approximation Method for Obtaining a Good Primal Solution of Transportation Problems

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Abstract. Determining the efficient solution for large scale of transportation problems (TPs) is an important task in operations research. Vogel's Approximation Method (VAM) which is one of the well-known transportation methods in the literature was investigated to obtain an initial transportation cost (ITC). In this paper, Vogel's Approximation Method (VAM) is modified for obtaining more efficient solution of a large scale of TPs. The most attractive feature of this method is that it requires very simple arithmetical and logical calculations and avoids large number of iterations. The proposed method is easy to understand and will be very helpful for those decision makers who are dealing with logistics and supply chain related issues than the other existing methods. One can also easily implement this proposed method among the existing methods for obtaining a good primal solution of a large scale of TPs.

Keywords: Transportation problems, least cost method, VAM, Modified Vogel's approximation method

AMS Mathematics Subject Classification (2010): 90C08

1. Introduction

Transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life. Transportation models play an important role in logistics and supply chains. It helps in solving problems on distribution and transportation of resources from place to another. The problem basically deals with the determination of a cost plan for transporting a single commodity from various sources to several destinations. The aim of such TPs is to minimize the total transportation cost (TTC) of shipping goods from one location to another so that the needs of each arrival area are met and every shipping location operates within its capacity. TPs can be solved by using general simplex based integer programming methods, however it involves time-consuming computations. We are going to propose a specialized algorithm with less number of iteration for solving TPs that are much more efficient than the simplex algorithm. The basic steps for solving TPs are:

Step 1. Determination the initial feasible solution.

Step 2. Determination optimal solution using the initial solution.

The most common method used to determine efficient initial solutions for solving TPs (using a modified version of the simplex method) is Vogel's Approximation Method (VAM)**.** This method involves calculating the penalty (difference between the lowest cost and the second-lowest cost) for each row and column of the cost-matrix, and then assigning the maximum number of units possible to the least-cost cell in the row or column with the largest penalty. Vogel's approximation method (VAM) gives approximate solution while MODI and Stepping Stone (SS) methods are considered as standard techniques for obtaining optimal solution of TPs. Goyal [1] has improved Vogel's approximation method (VAM) for the unbalanced transportation problems. Ramakrishna [2] has discussed some improvement to Goyal's Modified VAM for unbalanced TPs. Moreover, Sultan [3], Sultan and Goyal [4] have studied initial basic feasible solutions and resolution of degeneracy in TPs. Few researchers have tried to give their alternate methods for overcoming major obstacles over MODI and SS methods. Adlakha and Kowalski [5,6] have suggested an alternative solution algorithm for solving certain TPs based on the theory of absolute point. Shimshak *et al.* [7] have studied on modification of Vogel's approximation method through the use of heuristics. Sharma *et al.*[8] have studied on uncapacitated TP for obtaining a good primal solution. Balakrishnan [9] has discussed Modified Vogel's approximation method for unbalanced TP. Ullah and Uddin [10] have developed an algorithmic approach to calculate the minimum time of shipment in TPs. Ullah *et al.* [11] have developed a direct analytical method for finding an optimal solution for transportation problems. Ahmed *et al.* [12] have developed an effective modification to solve transportation problems for minimizing cost. Ukil *et al.* [13] have presented time manufacturing technique used in probabilistic continuous economic order quantity review model. Kirca and Satir [14] have developed a heuristic for obtaining an initial solution for the transportation problems. Sharma and Sharma [15] have presented a new dual based procedure for the transportation problems. Hakim [16] has developed an alternative method to find initial basic feasible solution of a transportation problem. In this paper, a simple heuristic approach is proposed (MVAM) for obtaining good primal solution of wide range of TPs and the solution obtained by the proposed method is often very good in terms of minimizing TTC than the other existing methods.

2. Modified Vogel's Approximation Method

Detailed processes of proposed Modified Vogel's Approximation Method (MVAM) are given below:

Step 1: Subtract the largest entry from each of the elements of every row of the transportation table and place them on the left-top of the corresponding element.

Step 2: Subtract the largest transportation cost from each of the entries of every column of the transportation table and write them on the left-bottom of the corresponding element.

Step 3: Form a reduced matrix whose elements are the summation of left-top and left-bottom elements of step1 and step2.

Step 4: Calculate the distribution indicators by subtracting of the largest and next-to-largest element of each row and each column of the reduced matrix and write them just after and below of the supply and demand amount respectively.

Step 5: Identify the highest distribution indicator, if there are two or more highest indicators, choose the highest indicator along which the largest element is present. If there are two or more largest elements present, choose any one of them arbitrarily.

Step 6: Allocate $x_{ij} = \min(a_i, b_j)$ on the left bottom of the largest element in the $(i, j)th$ cell of the reduced matrix.

Step 7: If $a_i < b_j$, leave the ith row and readjust b_j as $b'_j = b_j - a_i$. If $a_i > b_j$, leave the jth column and readjust a_i as $a'_i = a_i - b_j$. If $a_i = b_j$, then leave both the ith row and jth column.

Step 8: Repeat step 4 to step7 until the rim requirement exhausted.

Step 9: Put all the allocations of the positive allocated cells of the reduced matrix to the original transportation table and calculate the TTC, $z = \sum_{i=1}^{m} \sum_{j=}^{n} c_{ij} x_{ij}$ where x_{ij} is the total allocation of the $(i, j)th$ cell and c_{ij} is the corresponding unit transportation cost.

3. Example

Let us consider the following TP to find out the minimum TTC with three sources and four destinations.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply
F_1	4	19	22	11	$a_1 = 100$
F_2	1	9	14	14	$a_2 = 30$
F_3	6	6	16	14	$a_3 = 70$
Demand	$b_1 = 40$	$b_2 = 20$	$b_3 = 60$	$b_4 = 80$	200 (Total)

At first we calculate the row differences and column differences which are shown in the next table.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply
F_1	${}^{18}_{2}4$	${}^{3}_{0}19$	${}^{0}_{0}22$	${}^{11}_{3}11$	100
F_2	${}^{13}_{5}1$	${}^{5}_{10}9$	${}^{0}_{8}14$	${}^{0}_{0}14$	30
F_3	${}^{10}_{0}6$	${}^{10}_{13}6$	${}^{0}_{6}16$	${}^{2}_{0}14$	70
Demand	40	20	60	80	200 (Total)

Now we can form the reduced matrix as follows

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply
F_1	20	3	0	14	100
F_2	18	15	8	0	30
F_3	10	23	6	2	70
Demand	40	20	60	80	200 (Total)

Now we determine the distribution indicators for each row and each column by subtracting the largest and next-to-largest element. Among the distribution indicators, 13 is the highest one and $c_{32} = 23$ is the largest element. We allocate $x_{32} = \min(a_3, b_2) = \min(70, 20) = 20$ on the left bottom of c_{32}.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators
F_1	20	3	0	14	100	6
F_2	18	15	8	0	30	3
F_3	10	${}_{20}23$	6	2	50	13
Demand	40	0	60	80	180 (Total)	
Column distribution indicators	2	8	2	12		

b_2 is exhausted and readjust a_3 as $a_3' = a_3 - b_2 = 70 - 20 = 50$. Among the distribution indicators in the second step, 12 is the highest one and $c_{14} = 14$ is the largest element. We allocate $x_{14} = \min(a_1, b_4) = \min(100, 80) = 80$ on the left bottom of c_{14}.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators	
F_1	20	3	0	${}_{80}14$	20	6	6
F_2	18	15	8	0	30	3	10
F_3	10	${}_{20}23$	6	2	50	13	4
Demand	40	0	60	0	100 (Total)		
Column distribution indicators	2	8	2	12			
	2	×	2	12			

Here, b_4 is exhausted and readjust a_1 as $a_1' = a_1 - b_4 = 100 - 80 = 20$. Among the distribution indicators in the third step, 20 is the highest one and $c_{11} = 20$ is the largest element. We allocate $x_{11} = \min(a_1', b_1) = \min(20, 40) = 20$ on the left bottom of c_{11}.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators		
F_1	${}_{20}20$	3	0	${}_{80}14$	0	6	6	20
F_2	18	15	8	0	30	3	10	10
F_3	10	${}_{20}23$	6	2	50	13	4	4
Demand	20	0	60	0	80 (Total)			
Column distribution indicators	2	8	2	12				
	2	×	2	12				
	2	×	2	×				

Here, a_1 is exhausted and readjust b_1 as $b_1' = b_1 - a_1' = 40 - 20 = 20$. Among the distribution indicators in the next step, 10 is the highest one and $c_{21} = 18$ is the largest element. We allocate $x_{21} = \min(b_1', a_2) = \min(20, 30) = 20$ on the left bottom of c_{21}.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators			
F_1	${}_{20}20$	3	0	${}_{80}14$	0	6	6	20	×
F_2	${}_{20}18$	15	8	0	10	3	10	10	10
F_3	10	${}_{20}23$	6	2	50	13	4	4	4
Demand	0	0	60	0	60 (Total)				
Column distribution indicators	2	8	2	12					
	2	×	2	12					
	2	×	2	×					
	8	×	2	×					

Here, b_1 is exhausted and readjust a_2 as $a_2' = a_2 - b_1' = 30 - 20 = 10$. Having no other alternatives, next two allocations are automatically 10 and 50 to the cell with cost c_{23} and c_{33} respectively.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators			
F_1	$_{20}20$	3	0	$_{80}14$	0	6	6	20	×
F_2	$_{20}18$	15	$_{10}8$	0	0	3	10	10	10
F_3	10	$_{20}23$	$_{50}6$	2	0	13	4	4	4
Demand	0	0	0	0	0 (Total)				
Column distribution indicators	2	8	2	12					
	2	×	2	12					
	2	×	2	×					
	8	×	2	×					

Now all the rim requirements are satisfied and the initial basic feasible solution is $x_{11} = 20,\ x_{14} = 80,\ x_{21} = 20,\ x_{23} = 10,\ x_{32} = 20,\ x_{33} = 50$ which we allocate to the original transportation table.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply
F_1	$_{20}4$	19	22	$_{80}11$	$a_1 = 100$
F_2	$_{20}1$	9	$_{10}14$	14	$a_2 = 30$
F_3	6	$_{20}6$	$_{50}16$	14	$a_3 = 70$
Demand	$b_1 = 40$	$b_2 = 20$	$b_3 = 60$	$b_4 = 80$	200 (Total)

Therefore the TTC is, $z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

$$= c_{11}\,x_{11} + c_{14}\,x_{14} + c_{21}\,x_{21} + c_{23}\,x_{23} + c_{32}\,x_{32} + c_{33}\,x_{33}$$

$$= 4\times 20 + 11\times 80 + 1\times 20 + 14\times 10 + 6\times 20 + 16\times 50$$

$$= 2040.$$

Comparison of TTC obtained in different methods is given in the following table:

Name of the Methods	Primal Solution	No. of Iterations to Get an Optimal Solution
North-West Corner Method	2820	5
Least Cost Method	2090	2
Vogel's Approximation Method	2170	3
MVAM (Proposed)	2040	1

The optimal solution obtained by adopting Modified Distribution (MODI) Method is 2040. It is seen that the value of the objective function obtained by the proposed MVAM is same as the optimal value obtained by MODI method. To apply and justify the efficiency of the proposed MVAM, we also have considered the following several supply chain (problems 1-5) TPs from different sources to several destinations.

Problem 1:

Destination → / Source ↓	E	F	G	H	Supply
A	4	5	8	4	52
B	6	2	8	1	57
C	8	7	9	10	54
Demand	60	45	8	50	

Problem 2:

Destination → / Source ↓	E	F	G	H	Supply
A	6	3	8	7	110
B	8	5	2	4	60
C	4	9	8	4	54
D	7	8	5	6	30
Demand	20	70	78	86	

Problem 3:

Destination → / Source ↓	F	G	H	I	Supply
A	5	2	4	1	30
B	5	2	1	4	20
C	6	4	8	2	12
D	4	6	5	4	30
E	2	8	4	5	46
Demand	31	50	30	27	

Problem 4:

Destination → / Source ↓	E	F	G	H	I	Supply
A	100	150	200	140	35	400
B	50	70	60	65	80	200
C	40	90	100	150	130	150
Demand	100	200	150	160	140	

Problem 5:

Destination → / Source ↓	E	F	G	Supply
A	3	15	17	580
B	45	30	30	240
C	13	25	42	330
Demand	310	540	300	

Comparisons of initial solution of the above (1-5) TPs obtained by all procedures are given in the following table with number of iterations:

Initial solutions obtained by all procedures:

No. of Problems		Methods				Optimal Solution (MODI)
		NWCM	LCM	VAM	MVAM (Proposed)	
1	Primal Solution	914	674	750	**674**	674
	No. of Iterations to Get an Optimal Solution	4	1	2	**1**	
2	Primal solution	1010	988	988	**968**	968
	No. of Iterations to Get an Optimal Solution	3	2	2	**1**	
3	Primal solution	621	423	391	**381**	381
	No. of Iterations to Get an Optimal Solution	7	4	2	**1**	
4	Primal solution	92450	63550	66300	**63300**	63300
	No. of Iterations to Get an Optimal Solution	6	2	3	**1**	
5	Primal solution	25530	21450	21030	**20550**	20550
	No. of Iterations to Get an Optimal Solution	3	2	3	**1**	

4. Conclusion

In this article, North-West Corner Method (NWCM), Least Cost Method (LCM), Vogel's Approximation Method (VAM) and proposed Modified Vogel's Approximation Method (MVAM) are used to find the initial basic feasible solution and are compared to optimal solution obtained by MODI method. It is seen that, the results obtained by the proposed MVAM is almost same as optimal solution obtained by MODI method for several TPs and better than the solution obtained by the other existing methods (viz. NWCM, LCM and VAM). More efficient initial solution is obtained by the proposed MVAM for a wide range of TPs within a few numbers of iterations.

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