Annals of Pure and Applied Mathematics
Vol. 11, No. 1, 2016, 73-77
ISSN: 2279-087X (P), 2279-0888(online)
Published on 6 January 2016
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

On the Acharya Index of Graph

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Received 11 December 2015; accepted 31 December 2015

Abstract. Motivated by the research on Terminal Wiener index, Terminal distance matrix of connected graph and its chemical applications and its mathematical properties, we define Acharya Index $AI_\lambda(G)$ of a graph G as the sum of the distance between all pair of degree d vertices. This concept was defined and discussed in the ICDM 2013 with late Dr. B. D. Acharya. In this paper we determined the $AI_\lambda(G)$ for some classes of graphs.

Keywords: Wiener index, Terminal Wiener index, Terminal distance matrix

AMS Mathematics Subject Classification (2010): 72C05

1. Introduction

The Wiener index is considered as one of the most classic and applicable topological indices in the molecular graph theory, which was introduced by chemist Harold Wiener in 1947 and used it to determine physico-chemical properties of alkanes known as paraffin. Alkanes are chemical compounds exclusively composed of carbon and hydrogen [1,2]. For more information about Wiener index in chemical graph theory and its mathematical applications see [3,4, 5, 6, 7]. Throughout this paper we considered only simple connected graphs. For undefined terms one can see [5]. One can find distance concept in graphs in [8,9,10]

The distance between two vertices u and v is denoted by $d(u,v)$ and n be the order of graph, $\Delta(G)$ denotes maximum degree of graph G and p=diam(G), is the diameter of a graph G.

The **Wiener Index** W(G) is defined as sum of distances of all pair of vertices of graph G.

$$W = W(G) = \sum_{1<i<j<n} d(v_i, v_j)$$

The **Terminal Wiener index** TW(G) of a graph G is defined as the sum of the distances between all the pair of pendent vertices [11] ,

$$TW = TW(G) = \sum_{1<i<j<k} d(v_i, v_j / G)$$

where $d(v_i, v_j / G)$ is the distance between pendent vertices v_i and v_j in graph G.

The **Hosoya polynomial of graph** is defined as, $H(G,\lambda) = \sum_{\{u,v\}\subseteq v(G)} \lambda^{d(u,v)}$. The first derivate $H(G,\lambda)$ at $\lambda = 1$ is equal to the wiener index of G.

The **Terminal Hosoya Polynomial** is defined in [12] as $TH(G,\lambda) = \sum_{k\geq 1} d_T(G,k)\lambda^k$ .The first derivate $TH(G,\lambda)$ at $\lambda = 1$ is equal to the Terminal Wiener index of G.

Motivated by the research on terminal Wiener index and its chemical applications and mathematical properties, we define

Definition 1. Let G be a connected graph of order n and degree d, the **Acharya Index AI_λ(G)** of a graph G as the sum of the distance between all pair of degree d vertices, i.e.,

$$AI_\lambda(G) = \sum_{\substack{1\leq d\leq n-1 \\ 1\leq k\leq p}} \mu(d,G)\cdot k$$

where $\mu(d,G)$ denotes pair of vertices of degree d at distance k, p=diam(G).

For λ=1 then the Acharya index reduces to terminal wiener index i.e. AI_λ(G)=TW(G)

2. Main results

In this section we present Acharya index AI_λ(G) of some important class of graph.

Theorem 1. Let G=P_n be the path on n vertices then

$AI_\lambda(P_n) = W(P_n) - (n^2 - 3n + 2)$

Theorem 2. For a regular graph G with regularity r, $AI_r(G) = W(G)$

Corollary 2.1. If C_n be a cycle on n vertices then $AI_2(C_n) = W(C_n)$

Corollary 2.2. If K_n be a complete graph on n vertices then $AI_{n-1}(K_n) = W(K_n)$

Theorem 3. If $K_{1,n}$ is star on n vertices then $AI_\lambda(K_{1,n}) = TW(K_{1,n})$ where TW(G) is the terminal Wiener index.

Proof: If $G = K_{1,1}$ then there exist one pair of vertices with degree one (terminal vertices) at a distance one. Therefore $AI(G) = 1$. If $n > 1$ then there exist only one vertex with degree n and remaining vertices are of degree one. Hence $AI_\lambda(K_{1,n}) = TW(K_{1,n})$.

Theorem 4. If P is Petersen graph then $AI_\lambda(P) = W(G) = 75$

Theorem 5. If $G = K_{m,n}$ is bipartite graph then $AI_\lambda(K_{m,n}) = m^2 + n^2 - m - n$

Proof: Let G=$K_{m,n}$ be the bipartite graph with vertex sets $|V_1| =$ m and $|V_2| =$ n, then every edge in G has one vertex in V_1 and other in V_2.

The $\deg(v_i)=n$, $1\leq i\leq m$ and $\deg(v_j)=m$, $1\leq j\leq n$ then distance between the vertices of V_1 and V_2 is 2. There are $\binom{m}{2}$ pair of vertices in V_1 and $\binom{n}{2}$ pair of vertices in V_2 which are at distance 2.

Therefore the number of pair of vertices at distance two in G are $\binom{m}{2}+\binom{n}{2}$.

$$AI_{\lambda}(K_{m,n})=\left[\binom{m}{2}+\binom{n}{2}\right]\cdot 2$$

$$= m^2+n^2-m-n$$

Theorem 6. If $G=K_{m,n}$ is bipartite graph with m=n, then $AI_{\lambda}(K_{n,n})=W(K_{n,n})=3n^2-2n$

Proof: Let $G=K_{m,n}$ be the complete bipartite graph with m=n.

If m=n then $K_{m,n}$ is n regular graph.

Let $|V_1|=|V_2|=n$,The number pair of vertices at distance 1 are mn .

From above Theorem 5 the pair of vertices at distance 2 are $\binom{n}{2}+\binom{n}{2}$.

$$\therefore\ AI_{\lambda}(K_{m,n}) = n^2+\left[\binom{n}{2}+\binom{n}{2}\right].2$$

$$= 3n^2-2n$$

$$= W(K_{n,n})$$

Theorem 7. Let G be a connected graph then**,** $AI_{\lambda}(G)\leq W(G)$,the equality holds if G is regular.

Proof: If G is connected graph with n vertices, the Wiener index of G is the sum of distance between of all pair of vertices where as the Acharya Index is the sum of distance between pair of vertices of same degree. Therefore $AI_{\lambda}(G)\leq W(G)$. The second part of the theorem is obevious if G is regular.

Theorem 8. Let T be a tree on n vertices then**,** $TW(T)\leq AI_{\lambda}(T)\leq W(T)$.

Proof : The inequalities are obvious as we are partitioning the vertices with their degrees and calculate the distance between them .The equality holds if $n\leq 2$.

Definition 2. Let G a connected n-vertex graph with vertex set $V(G)=\{v_1, v_2, v_3, \ldots, v_n\}$, let $p=(p_1, p_2, p_3 \ldots p_n)$ be an n-tuple of non negative integers .The thorn graph G_p is the graph obtained by attaching p_i pendent vertices to the vertex v_i of G for i=1,2,……….,n. The p_i pendent vertices attached to the vertex v_i will be called the thorn of v_i [9-10]

Theorem 9. Let G be the connected non regular graph with vertices $v_1, v_2, v_3, \ldots\ldots v_n$ .Let G_p be the thorn graph obtained by attaching p_i pendent vertices to the vertex v_i of G for i=1,2,……….,n.If $p_i>0$ for all i=1,2,……….,n, then $AI_{\lambda}(G_p)=TW(G_p)+\sum_{1\leq i\leq j\leq n} d(v_i,v_j\mid G)$

Where $\deg(v_i)+p_i=\deg(v_j)+p_j$

Proof: Considering the thorn attached to the given vertex v_i ,are pendent vertices are all of same degree and their summation of the distance leads to first term in the right side of above equation. For the second term consider the vertices p_i thorn $v^i_1, v^i_2, v^i_{3,\text{--------}}$ v^i_{pi} attached to the vertex v_i and $v^j_1, v^j_2, v^j_{3,\text{--------}}$ v^j_{pj} attached to the vertex v_j of G ,i≠j for all i,j=1,2,……….,n. In G_p ,if $deg(v_i)+p_i= deg(v_j)+p_j$ $d(v^i_k,v^j_l|G_p)=d(v_i,v_j|G)$,k=1,2,……….,.p_i l=1,2,……….,. p_j. there contribution to AI is $d(v^i_k,v^j_l|G_p)=d(v_i,v_j|G)$ which leads to second term.

Observation 1. The above formula remains valid if p_i's are equal to zero, provided that the corresponding vertices of graph G are not pendent.

Corollary 9.2. Let G is regular graph, If $p_i \neq p_j$ for all i,j=1,2,……….,n. then $AI_\lambda(G_p)=TW(G_p)$

Corollary 9.3. If G is regular, $p_i=p>0$ for all i=1,2,……….,n. then $AI(G_p)=W(G_p)$.

Corollary 9.4. If G is complete graph on n vertices ,If $p_i=p>0$, for all i=1,2,……….,n. then $AI_\lambda (G_p)=W(G_p)$.

Corollary 9.5. If G is complete graph on n vertices. If $p_i \neq p_j$ for all i,j=1,2,……….,n. then $AI_\lambda (G_p)=TW(G_p)$.

Corollary 9.6. For a thorn ring C^*_n, $p_i=p>0$ where i=1,2,……….,n. then $AI_\lambda (C^*_n)=W(C^*_n)$.

Corollary 9.7. For a thorn ring C^*_n, If $p_i \neq p_j$ where i,j=1,2,……….,n. then $AI_\lambda (C^*_n)=TW(C^*_n)$

Bonchev and Klein [11-12] proposed the concept of thorn trees, where the parent graph is a tree, then it is called as thorn trees. In this paper we consider the cases when the parent tree is a path. Then the respective thorn tree is caterpillar and "thorn rod" is understood a caterpillar obtained so that new vertices are attached only to the two terminal vertices of the underlying path. Another thorn tree is the "thorn star", obtained by attaching pendent vertices to the vertices of a star, except to its central vertex. If the parent graph is a cycle, then we speak of "thorn cycles".

Theorem 10. Let the thorn star $K^*_{1,n}$ is the graph obtained by attaching $p_i=p>0$,i=1,2,……….,n. pendent vertices to the i-th pendent vertex of a star $K_{1,n}$,n≥2 then $AI_\lambda(K^*_{1,n})= TW(K^*_{1,n})+n^2-n$

Proof: Considering the distance between p_i's is the first term and since all p_i's are same, $deg(v_i)=p_i+1$,for all v_i, i=1,2,……….,n. .Thus there are $\binom{n}{2}$ pair of vertices at distance 2.which is $\binom{n}{2}.2= n^2-n$,which leads to second term.

Corollary 10.1. Let the thorn star $K^*_{1,n}$ .If $p_i \neq p_j$ for all i,j=1,2,……….,n. then

$AI_\lambda(K^*_{1,n}) = TW(K^*_{1,n})$

3. Conclusion and scope

Acharya index is new Distance –Degree based index. We observe that $TW(G) \le AI_\lambda(G) \le W(G)$. The equality holds if and only if G=K_2 and also $AI_\lambda(G)$=W(G) for regular graph. Obtaining class of graphs with A(G)=W(G), TW(G)=AI(G) and TW(G)=W(G) interesting problem to compute.

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