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First Multiplicative *K* Banhatti Index and Coindex of Graphs

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Abstract. The vertices and edges of a graph G are called the elements of G. If $e = uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. For a (molecular) graph, the first multiplicative K Banhatti index $BII_1(G)$ is equal to the product of the sums of degrees of the pairs of incident elements of G. Also we define the first multiplicative K Banhatti coindex of graphs. In this paper, we initiate a study of the first multiplicative K Banhatti index of graphs.

Keywords: Banhatti indices, Banhatti coindices, first multiplicative K Banhatti index, first multiplicative K Banhatti coindex

AMS Mathematics Subject Classification (2010): 05C05, 05C07

1. Introduction

By a graph, we mean a finite, undirected, without loops, multiple edges and isolated vertices. Let G be a graph with n vertices and m edges with vertex set $V(G)$ and edge set $E(G)$. Any undefined term in this paper may be found in Kulli [1].

The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v is denoted by uv. If $e = uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$.

The vertices and edges of a graph are called its elements.

The first K Banhatti index is defined as the sum of the sums of the degrees of the pairs of incident elements:

$$B_1(G) = \sum_{ue}\left[d_G(u) + d_G(e)\right]$$

where ue means that the vertex u and edge e are incident in G.

The first K Banhatti coindex is defined as the sum of the sums of the degrees of the pairs of nonincident elements:

$$\overline{B_1}(G) = \sum_{u*e}\left[d_G(u) + d_G(e)\right]$$

where $u*e$ means that the vertex u and edge e are nonincident in G.

The first K Banhatti index and coindex were introduced by Kulli in [2]. Recently many other indices and coindices of graphs were studied, for example, in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

In this paper, we consider the multiplicative variants of the first K Banhatti index and the first K Banhatti coindex of graphs. Recently many other multiplicative indices and coindices of graphs were studied, for example in [14, 15, 16, 17, 18, 19].

2. First multiplicative *K* Banhatti index

We introduce the first multiplicative K Banhatti index of a graph in terms of incident vertex-edge degrees.

Definition 1. The first multiplicative K Banhatti index of a graph G is defined as

$$BII_1(G) = \underset{ue}{II}\left[d_G(u) + d_G(e)\right]$$

where ue means that the vertex u and edge e are incident in G.

Proposition 2. Let C_n be a cycle with $n\geq 3$ vertices .Then

$$BII_1(C_n) = 4^{2n}.$$

Proof: Let C_n be a cycle with $n\geq 3$ vertices. Then C_n has n edges. Every edge of C_n is incident with exactly two vertices. Consider

$$BII_1(C_n) = \underset{ue}{II}\left[d_{C_n}(u) + d_{C_n}(e)\right] = \overset{n}{II}(2+2)^2 = 4^{2n}.$$

Proposition 3. Let K_n be a complete graph with n vertices and m edges. Then

$$BII_1(K_n) = (3n-5)^{n(n-1)}.$$

Proof: Let K_n be a complete graph with n vertices and $m = \dfrac{n(n-1)}{2}$ edges. Every edge of K_n is incident with exactly two vertices. Consider

$$BII_1(K_n) = \underset{ue}{II}\left[d_{K_n}(u) + d_{K_n}(e)\right] = \overset{m}{II}\left[(n-1)+(2n-4)\right]^2$$
$$= (3n-5)^{2m} = (3n-5)^{n(n-1)}.$$

Proposition 4. Let $K_{m,n}$ be a compete bipartite graph. Then

$$BII_1(K_{m,n}) = (m+2n-2)^{mn}(2m+n-2)^{mn}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m+n$ vertices, mn edges, $|V_1|=m$, $|V_2|=n$, $V(K_{m,n}) = V_1 \cup V_2$. Every edge of $K_{m,n}$ is incident with one vertex of V_1 and another vertex of V_2. Let $V_1 = \{v_1, v_2,\ldots,v_m\}$ and $V_2 = \{w_1, w_2, \ldots, w_n\}$. Clearly every vertex v_i of V_1 is incident with e_{ij} edges, $j = 1, 2, \ldots, n$ and every vertex w_j of V_2 is incident with e_{ij}, edges, $i = 1, 2, \ldots, m$. Consider

$$BII_1(K_{m,n}) = \underset{ue}{II}\left[d_{K_{m,n}}(u) + d_{K_{m,n}}(e)\right]$$

$$= \prod_{v_i \in V_1} \left[d_{K_{m,n}}(v_i) + d_{K_{m,n}}(e_{ij}) \right] \prod_{w_j \in V_2} \left[d_{K_{m,n}}(w_j) + d_{K_{m,n}}(e_{ij}) \right]$$

$$= \prod^{mn} \left[n + (m+n-2) \right] \prod^{mn} \left[m + (m+n-2) \right]$$

$$= (m + 2n - 2)^{mn} (2m + n - 2)^{mn}.$$

Corollary 5. Let $K_{n,n}$ be a complete bipartite graph. Then

$$BII_1(K_{n,n}) = (3n-2)^{2n^2}.$$

Corollary 6. Let $K_{1,n}$ be a star. Then

$$BII_1(K_{1,n}) = (2n-1)^n n^n.$$

Theorem A[1, p, 13]. Let *G* be an *r*-regular graph with *n* vertices. Then *G* has $\frac{nr}{2}$ edges.

Theorem 7. Let *G* be an *r*-regular graph with *n* vertices. Then

$$BII_1(G) = (3r-2)^{nr}.$$

Proof: Let *G* be an *r*-regular graph with *n* vertices. By Theorem A, *G* has $\frac{nr}{2}$ edges. Every edge of *G* is incident with exactly two vertices. Consider

$$BII_1(G) = \prod_{ue} \left[d_G(u) + d_G(e) \right] = \prod^{\frac{nr}{2}} \left[r + (2r-2) \right]^2$$

$$= (3r-2)^{2 \cdot nr/2} = (3r-2)^{nr}.$$

3. First multiplicative *K* Banhatti coindex

We define the first multiplicative *K* Banhatti coindex of a graph in terms of nonincident vertex edge degrees.

Definition 8. The first multiplicative *K* Banhatti coindex of a graph *G* is defined as

$$\overline{BII}_1(G) = \prod_{u*e} \left[d_G(u) + d_G(e) \right]$$

where *u**e* means that the vertex *u* and edge *e* are nonincident in *G*.

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