Annals of Pure and Applied Mathematics
Vol. 11, No. 2, 2016, 83-88
ISSN: 2279-087X (P), 2279-0888(online)
Published on 20 May 2016
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Power Dominator Coloring of Certain Special Kinds of Graphs

K. Sathish Kumar, N. Gnanamalar David* and *K.G. Subramanian
Department of Mathematics, Madras Christian College
Tambaram, Chennai 600059 India
E-mail: {sathishkumark17, ngdmcc, kgsmani1948}@gmail.com

Received 12 May 2016; accepted 19 May 2016

Abstract. Several variants of the concept of domination in graphs have been introduced and investigated. Power domination is a recently introduced variant in the study of modelling by graphs, the problem of monitoring the state of an electric power system. On the other hand coloring of graphs which has many applications, has also been extensively investigated. The authors introduced the concept of power dominator coloring requiring each vertex of a graph to power dominate an entire color class and also the associated power dominator chromatic number which is the minimum cardinality of such sets of vertices in a graph. In this paper we find the power dominator chromatic number for certain special classes of graphs.

Keywords: Graphs; Power domination; Dominator Coloring; Power dominator coloring

AMS Mathematics Subject Classification (2010): 05C15, 05C69

1. Introduction

Haynes et al. [4] introduced the concept of power domination in a graph while dealing with the problem of modelling by graphs, the activity of monitoring the state of an electric power system. In the study of coloring of graphs, the concept of dominator coloring considered in [2] assigns a proper coloring to the vertices, requiring every vertex to dominate a color class which consists of all the vertices with the same color. Combining the notions of power domination and dominator coloring, a new notion, called power dominator coloring which requires every vertex to power dominate all vertices in a color class was introduced in [11]. The power dominator chromatic number $\chi_{pd}(G)$, for a given graph G is the minimum cardinality of such color classes. Certain properties of $\chi_{pd}(G)$ were derived in [11], besides computing this number for certain classes of graphs. Here we compute $\chi_{pd}(G)$ for certain special kinds of graphs that are of interest in various contexts in the study of different properties of graphs.

2. Basic definition

We recall some notions on graphs needed in the subsequent sections. We are concerned here with only simple, undirected graphs. We refer to [1] for basic concepts in graph theory.

Let $G(V,E)$ be a graph. A subset $S \subseteq V$ is a dominating set [5] of G if every vertex in $V-S$ has at least one neighbor in S. A subset $S \subseteq V$ is a power dominating set [4,5] of $G(V,E)$ if all the vertices of V can be observed recursively by the following rules: (i) all vertices in $N[S]$ are observed initially and (ii) if an observed vertex u has all its neighbors observed except one non-observed neighbor v, then v is observed (by u). We then say that S power dominates the vertices of the graph G.

A power dominator coloring [8] of a graph $G(V,E)$ is a proper coloring of G such that every vertex of V power dominates all vertices of at least one color class of G. The power dominator chromatic number $\chi_{pd}(G)$ is the minimum number of colors required for a power dominator coloring of G.
We give an example graph in Fig. 1 with $\chi_{pd}(G) = 4$. The numbers indicated in the vertices stand for the colors assigned to the vertices. The vertices *y, z, r* power dominate the color class $\{x\}$. In fact *y* dominates (and hence power dominates) the color class $\{x\}$ and so the vertex *z* power dominates the color class $\{x\}$. Likewise, the vertices *u, v, p, q* power dominate the color class $\{w\}$. The vertex *w* power dominates itself and *x* also dominates itself. It can be seen with a little reflection that $\chi_{pd}(G)$ cannot be less than 4.

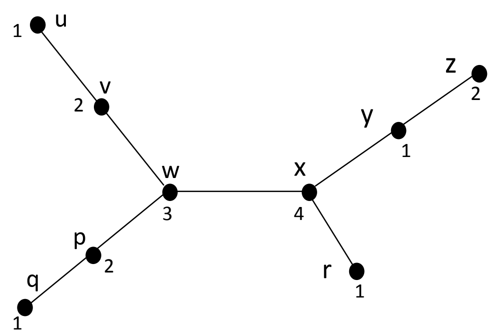

Figure 1: A graph *G* with $\chi_{pd}(G) = 4$

We now recall certain special kinds of graphs.
Definition 1. (i) [6] Given a path P_n on n vertices $u_1, \cdots, u_n$, centipede is a graph obtained from P_n adding n new vertices $v_1, \cdots, v_n$ and joining u_i with v_i, for $1 \leq i \leq n$.
(ii) [3,6] The n-barbell graph $B(K_n, K_n)$ is a simple graph obtained by connecting two copies of the complete graph K_n by an edge joining any one vertex in one copy with any other vertex in the other copy.
(iii) [6] The n-sunlet graph S_n is a graph on $2n$ vertices with a cycle C_n and each vertex of the cycle being joined to a new pendant vertex.

(iv) [6, 9] The crown graph S_n^o for an integer $n > 2$ is the graph with the vertex set $\{v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n\}$ and the edge set $\{(v_i, w_j): 1 \leq i, j \leq n, i \neq j\}$

v) [9] The windmill graph $w_n^{(m)}$ is the graph obtained by taking m copies of the complete graph K_n with a vertex in common.

3. Power dominator chromatic number of special graph classes

We compute the power dominator chromatic number of the special graphs described in Definition 1.

Theorem 1. For a centipede G, $\chi_{pd}(G) = n,\ n > 1$.

Proof: Let the vertices of the path P_n in G, be $u_1, \cdots, u_n$ in this order with u_1 and u_n as the ends of the path and the remaining n vertices be $v_1, \cdots, v_n$. Assign color 1 to the vertices $v_1, \cdots, v_n$, and color 2 to u_1 and u_n. Assign a distinct color i+1 to the vertex v_i, $2 \leq i \leq n-1$. Due to power domination, the vertices u_1 and v_1 power dominate the color class $\{u_2\}$ while u_n and v_n power dominate the color class $\{u_{n-1}\}$. The vertex v_i power dominates the color class $\{u_i\}$, for $2 \leq i \leq n$-1. Also, each u_i, $2 \leq i \leq n$-1 power dominates itself. Note that the number n of colors cannot be reduced. Hence $\chi_{pd}(G) = n$.

Remark. (i) Note that for the centipede G, the dominator chromatic number $\chi_d(G) = n + 1$ [6] while $\chi_{pd}(G) = n$. (ii) A centipede graph G with $\chi_{pd}(G) = 4$ is shown in Fig. 2. The numbers in brackets are the colors assigned to the vertices.

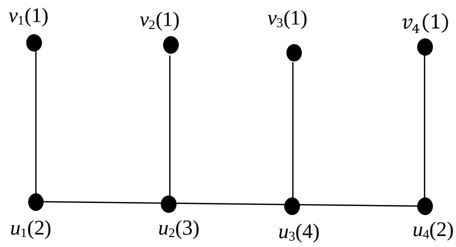

Figure 2: A Centipede Graph G with $\chi_{Pd}(G) = 4$

Theorem 2. For the n-barbell graph $B(K_n, K_n), n > 1, \chi_{pd}(B(K_n, K_n)) = n$.

Proof: Let $V = \{v_1, v_2, \ldots, v_n\}$ be the vertex set of one copy X of K_n and $W = \{w_1, w_2, w_3, \ldots, w_n\}$ be the vertex set of another copy Y of K_n. Let e be the edge joining a vertex v_i with w_j. We note that the power dominator chromatic number of K_n is n [11]. So we color the vertices of X as well as the vertices of Y with colors

$1, 2, \ldots, n$ so that v_i and w_j receive different colors. Note that only one of the vertices of Y is adjacent to only one of the vertices of X. Also there will be a vertex in X, say v_k, which will have the color assigned to w_j. Likewise there will be a vertex in Y, say w_l, which will have the color assigned to v_i. Now each vertex of X power dominates the color class $\{v_k, w_j\}$ while each vertex of Y power dominates the color class $\{v_i, w_l\}$. Hence $\chi_{pd}(B(K_n, K_n)) = n$.

Remark: (i) Note that for n-barbell graph $B(K_n, K_n)$, the dominator chromatic number $\chi_d(B(K_n, K_n) = n + 1$ [6] while $\chi_{pd}(B(K_n, K_n)) = n$.

(ii) A barbell graph $B(K_4, K_4)$, with $\chi_{pd}\big(B(K_4, K_4)\big) = 4$ is shown in Fig. 3.

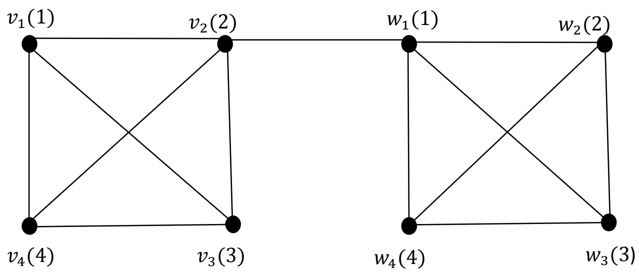

Figure 3: A Barbell Graph $B(K_4, K_4)$ with $\chi_{Pd}(B(K_4, K_4)) = 4$

Theorem 3. For n- sunlet graph S_n, $n \geq 3$, $\chi_{pd}(S_n) = n + 1$.

Proof: Let the vertices in cycle C_n of the n-sunlet graph S_n be $w_1, w_2, \ldots, w_n$ and the remaining n pendant vertices be $v_1, v_2, \ldots, v_n$ with v_i adjacent to w_i. Assign color 1 to all the vertices $v_1, v_2, \ldots, v_n$ and assign a distinct color i +1 to the vertex w_i, $1 \leq i \leq n$ in the cycle. The vertex v_i power dominates the color class w_i, $1 \leq i \leq n$. Each w_i, $1 \leq i \leq n$ power dominates itself. Hence $\chi_{pd}(S_n) = n + 1$.

Remark: A sunlet graph S_4, with $\chi_{pd}(S_4) = 5$ is shown in Fig. 4.

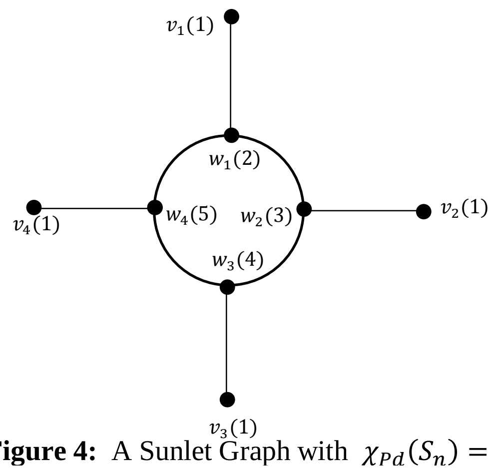

Figure 4: A Sunlet Graph with $\chi_{Pd}(S_n) = 5$

Theorem 4. Let S_n^o be a crown graph. Then $\chi_{pd}(S_n^o) = 4$, where $n \geq 4$.
Proof: Let the vertices of the crown graph S_n^o be $v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n$. Each v_i is adjacent to all the vertices $w_j, j \neq i$. Assign color 1 to $v_2, v_3, \ldots, v_n$ and color the vertex v_1 by 2. Now assign color 3 to the vertices $w_2, w_3, \ldots, w_n$ and color the vertex w_1 by 4. It can be seen that every vertex power dominates at least one color class. Hence $\chi_{pd}(S_n^o) = 4$ when $n \geq 4$.

Remark. (i) Note that the dominator chromatic number is 4 for the crown graph S_3^o, while the power dominator chromatic number $\chi_{pd}(S_3^o) = 2$, although for $n \geq 4$, both are equal. (ii) A crown Graph with $\chi_{Pd}\ (S_n^0\ ) = 4$ is shown in Fig.5

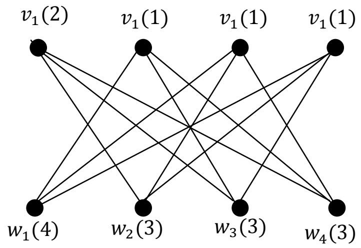

Figure 5: A crown Graph with $\chi_{Pd}(S_n^0) = 4$

Theorem 5. Let $G = w_n^{(m)}$ be a windmill graph. Then $\chi_{pd}(G) = n$
Proof: Let v be the common vertex of the m copies K_n of the windmill graph. Assign color 1 to v and use $n-1$ distinct colors for a proper coloring of vertices in each copy.

The vertex v power dominates itself. Every other vertex in G being adjacent to v power dominates the color class $\{v\}$. Hence $\chi_{pd}(G) = n$.

4. Conclusion

The study of domination in graphs has been of great interest (see, for example, [4,5,7,8,10]). The notion of power dominator coloring of a graph was introduced in [11] and this number $\chi_{pd}(G)$ is computed here for several kinds of graphs *G*. It remains to explore other properties of $\chi_{pd}(G)$.

Acknowledgements. The third author acknowledges support from the Emeritus Fellowship of UGC (2016-17), India. The authors thank the referees for useful comments.

REFERENCES

1. J.A.Bondy, U.S.R.Murty, *Graph Theory with Applications*, Elsevier, North Holland, 1986.
2. R.Gera, S.Horton, C.Rasmussen, Dominator colorings and safe clique partitions, *Congressus Numerantium*, 181 (2006) 19-32.

3. A.Ghosh, S.Boyd and A.Saberi, Minimizing effective resistance of a graph, Proc. 17th International Symposium on Mathematical Theory of Networks and Systems, Japan, 2006.
4. T.W.Haynes, S.M.Hedetniemi, S.T.Hedetniemi and M.A.Henning, Domination in graphs applied to electric power networks. *SIAM J. Discrete Math.,* 15(4) (2002) 519-529.
5. T.W.Haynes, S.M.Hedetniemi and P.J.Slater, *Fundamentals of Domination in Graphs*, Marcel Decker, New York , 1998.
6. L. Jethruth E. Mary and P.Ananthi, Dominator Coloring of Some Special Graphs, *Int. J. Innovative Research in Technology, Science & Engineering*, 1(4) (2015).
7. V.R.Kulli, The Disjoint Total Domination Number of a Graph, *Annals of Pure and Applied Mathematics*, 11(2) (2006) 33-38.
8. V.R.Kulli, The total dominating graph, *Annals of Pure and Applied Mathematics*, 10(1) (2015) 123-128.
9. T.Ramachandran, D.Udayakumar and A.N.Ahmed, Dominator coloring number of some graphs, *Int. J Scientific and Research Publications*, 5(10) (2015) 1-4.
10. N.Saradha and V.Swaminathan, Strong Complementary Acyclic Domination of a Graph, *Annals of Pure and Applied Mathematics*, 8(2) (2014) 83-89.
11. K.Sathish Kumar, N.Gnanamalar David and K.G.Subramanian, Graphs and power dominator colorings, *Annals of Pure and Applied Mathematics*, 11(2) (2016) 67-71.