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Dominating and Total Dominating Complementary Color Transversal number of Graphs

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Abstract. In this paper, we introduce two new concepts called dominating complementary color transversal number and total dominating complementary color transversal number of a graph. We determine relations between these two numbers as well their relationship with complementary chromatic number of a graph.

***Keywords*:** Complementary chromatic number, dominating complementary color transversal number, total dominating complementary color transversal number.

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1. Introduction

In [2], we defined complementary coloring of vertices of a graph. complementary coloring is coloring of vertices of a graph G in such a way that the vertices that are assigned distinct colors must be adjacent. This coloring is, in general, improper. The maximum number of colors required to color all the vertices of the graph G is the complementary chromatic number of G. It is obvious that such coloring can divide the vertex set V of G into maximum number of disjoint subsets, called color class, such that each subset contain the vertices that are assigned same color. Note that these subsets may not be independent. Such a partition of V is called χ^C - partition of the graph G**.** A transversal of a χ^c - partition of the graph g is a subset of V that meets every color class of the χ^c–partition. Dominating complementary color transversal set of G is a dominating set which is also transversal of a χ^c - partition of the graph, such a set with minimum cardinality is called dominating complementary color transversal number of G. Analogously, total dominating complementary color transversal number of G is defined. In this paper, we have discussed about some relations between these two numbers as well as their relationship with complementary chromatic number.

We begin with simple, finite, undirected, connected graph without isolated vertices. Let us first go through some definitions.

2. Definitions

Definition 2.1. [1] (Proper *n*–coloring) An *n*–coloring of a graph G is a function f : V(G) → {1, 2,……….,n} (for some n≥ 1). This is called a proper *n*- coloring if whenever u and v are adjacent then $f(u) \neq f(v)$. A graph G is said to be n – colorable if it admits n – coloring and it is called a proper n – colorable graph if it admits a proper n – coloring.

Definition 2.2. [1] (Chromatic number) The chromatic number of a graph G is the smallest value of n such that G admits a proper n – coloring. This number is denoted as χ (G).

Definition 2.3. [2] (Complementary n-coloring) A coloring of a graph G is called a complementary coloring of G if whenever two vertices u and v have distinct colors then u and *v* are adjacent. An *n*– coloring which is also complementary is called complementary *n* – coloring.

Definition 2.4. [2] (Complementary chromatic number) Complementary chromatic number of a graph G is the largest integer k such that G admits a complementary k – coloring. This number is denoted as $\chi^{C}(G)$.

Definition 2.5. (χ^{C} – partition of a graph) Complementary coloring of vertices of a graph G, by using maximum number of colors, yields maximum number of subsets (may not be independent) of vertex set of G called color classes of G. Such a partition of a vertex set of G is called a χ^{C} - Partition of the graph G.

Definition 2.6. [6] (Dominating set) Let G = (V, E) be a graph. Then a subset S of V (the vertex set of G) is said to be a dominating set of G if for each $v \in V$ either $v \in S$ or v is adjacent to some vertex in S.

Definition 2.7. [6] (Minimum dominating set/ domination number) Let G = (V, E) be a graph. Then a dominating set S is called the minimum dominating set of G if $|S|$ = minimum $\{|D|$: D is a dominating set of G$\}$. In such case S is called a γ – set of G and the cardinality of S is called domination number of the graph G denoted by γ (G) or just by γ.

Definition 2.8. (Transversal of a χ^{C} – partition of a graph) Let G = (V, E) be a graph and consider its χ^{C} – Partition $\{V_1, V_2, ..., V_{\chi^c}\}$. Then a set $S \subset V$ is called a transversal of this χ^{C} – Partition if $S \cap V_i \neq \emptyset$, $\forall\, i \in \{1, 2, 3, ..., \chi^{C}\}$.

Definition 2.9. (Dominating complementary color transversal set) Let G = (V, E) be a graph. Then a dominating set $S \subset V$ is called a dominating complementary color transversal set of G if it is transversal of a χ^{C} – partition of G.

Definition 2.10. (Minimum dominating complementary color transversal set/ dominating complementary color transversal number) Let G = (V, E) be a graph. Then $S \subset V$ is called a minimum dominating complementary color transversal set of G if $|S|$ = minimum $\{|D|$: D is a dominating complementary color transversal set of G$\}$. Here S is called γ^{C}_{std} $-$set and its cardinality, denoted by $\gamma^{C}_{std}(G)$ or just by γ^{C}_{std}, is called the dominating complementary color transversal number of G.

Definition 2.11. [6] (Total dominating set) Let G = (V, E) be a graph. Then a subset S of V (the vertex set of G) is said to be a total dominating set of G if for each v ∈ V, v is adjacent to some vertex in S.

Definition 2.12. [6] (Minimum total dominating set/total domination number) Let G = (V, E) be a graph. Then a total dominating set S is said to be a minimum total dominating set of G if |S| = minimum {|D|: D is a total dominating set of G}. Here S is called γ_t −set and its cardinality, denoted by γ_t(G) or just by γ_t, is called the total domination number of G.

Definition 2.13. (Total dominating complementary color transversal set) Let G = (V, E) be a graph. Then a total dominating set S ⊂ V is called a total dominating complementary color transversal set of G if it is transversal of a χ^C – partition of G.

Definition 2.14. (Minimum total dominating complementary color transversal set/ dominating complementary color transversal number) Let G = (V, E) be a graph. Then S ⊂ V is called a minimum total dominating complementary color transversal set of G if |S| = minimum {|D|: D is a total dominating complementary color transversal set of G}. Here S is called γ_{tstd} −set and its cardinality, denoted by γ^C_{tstd}(G) or just by γ^C_{tstd}, is called the total dominating complementary color transversal number of G.

Definition 2.15. (Pendant vertex): A vertex v of a graph G is called pendant if its degree is one. That is, it is adjacent to only one vertex of graph G.

Definition 2.16. (Support vertex): A vertex v of a graph g is called support if it is adjacent to a pendant vertex of the graph.

3. Main results

Theorem 3.1. Let G = (V, E) be a graph of order n. Then for any v ∈ V followings are equivalent:
(1) {v} is a color class.
(2) γ (G) = 1.
Proof: Obvious.

Theorem 3.2. Let G be a graph with χ^C (G) ≥ 2 then γ_t (G) = 2.

Proof: Let $\Pi = \{V_1, V_2, ..., V_{\chi^C}\}$ be a χ^C - partition of a graph G. Let u ∈ V_i and v ∈ V_j , for some i ≠ j. Then u and v are adjacent and u is adjacent to all the vertices that are not in V_i and v is adjacent to all the vertices that are not in V_j. So {u, v} is total dominating set of G. Hence γ_t (G) = 2.

Remark 3.3. Converse of above theorem is not true in general. For example consider the path graph P_4. γ_t (P_4) = 2 but χ^C (P_4) = 1.

Result 3.4. For any graph G = (V, E), $\gamma^C_{std}(G) \leq \gamma^C_{tstd}(G)$.
Proof: Obvious as a total dominating set of G is always a dominating set of G.

Theorem 3.5. For any graph G = (V, E),
(i) if χ^C (G) = 1 then γ^C_{std}(G) = γ(G) and γ^C_{tstd}(G) = γ_t(G).
(ii) if χ^C (G) ≥ 2 then γ^C_{std}(G) = γ^C_{tstd}(G) = χ^C (G).
Proof: Let χ^C(G) = 1 then definitely by the definition γ^C_{std}(G) = γ(G) and γ^C_{tstd}(G) = γ_t(G). Hence (i).

Let χ^C (G) ≥ 2. Then note that any transversal of the χ^C – partition of G will be a total dominating set of G and hence also Dominating Set of G. So χ^C (G) ≤ γ^C_{std}(G) ≤ γ^C_{tstd}(G) = χ^C(G). Hence γ^C_{std}(G) = γ^C_{tstd}(G) = χ^C (G). Hence (ii).

Corollary 3.6. For any graph G = (V, E), γ^C_{std}(G) = max {γ(G), χ^C (G)} and γ^C_{tstd}(G) = max {γ_t(G), χ^C (G)}

Theorem 3.7. Let G = (V, E) be a graph of order n. If G has two or more support vertices then χ^C (G) = 1.
Proof: Let u and v be two distinct support vertices of G. Let x and y be two distinct pendant vertices of G that are adjacent to u and v respectively. Clearly n ≥ 4. As x is not adjacent to any vertex in $V \setminus \{u\}$, all the vertices in $V \setminus \{u\}$ must be in same color class with x. Also as y is not adjacent to u, color class of y and u must be same. Therefore u is also in same color class with x. Hence all the vertices are in one color class. Hence χ^C (G) = 1.

Corollary 3.8. If a graph G = (V, E) has two or more support vertices then γ^C_{std}(G) = γ(G) and γ^C_{tstd}(G) = γ_t(G).

Remark 3.9. Converse of above theorem is not true in general. For example consider cycle graph G = C_5. C_5 does not have any support vertex, yet χ^C (G) = 1.

Result 3.10. For any graph G = (V, E), γ^C_{std}(G) ≤ γ^C_{tstd}(G) .
Proof: Obvious as any total dominating set is always a dominating set.

Theorem 3.11. [6] Let G be a graph of order n with no isolated vertices. Then $\gamma(G) \leq \frac{n}{2}$.

Theorem 3.12. Let G = (V, E) be a graph of order n. Then γ^C_{std}(G) = n if and only if χ^C(G) = n.
Proof: Assume that γ^C_{std}(G) = n. If χ^C(G) ≥ 2 then γ^C_{std}(G) = n = χ^C(G) and we are done.
Claim: χ^C(G) ≠ 1.
Suppose χ^C (G) = 1. Then γ^C_{tstd} (G) = max {γ (G), χ^C (G)} = γ(G) = n, which is contradiction to theorem 3.11, $\gamma(G) \leq \frac{n}{2} < n$. So χ^C(G) ≠ 1.
Hence we have χ^C(G) = n.
Converse is obvious.

Theorem 3.13. Let G = (V, E) be a graph of order n. Then γ^C_{std}(G) = n - 1 if and only if χ^C(G) = n – 1.

Proof: The proof is analogous to above theorem.

Theorem 3.14. Let G = (V, E) be a graph of order n. If $\chi^C(G) = n - 2$ then $\gamma^C_{std}(G) = n - 2$.

Remark 3.15. Converse of above theorem is not true in general. For example consider G = P_4, the path graph with four vertices. $\gamma^C_{std}(G) = 2 = n - 2$ but $\chi^C(G) = 1 \neq 2 = n - 2$.

Theorem 3.16. [6] Let G = (V, E) be a connected graph of order n ≥ 3 with no isolated vertices. Then $\gamma_t(G) \leq \frac{2n}{3}$.

Theorem 3.17. Let G = (V, E) be a graph of order n ≥ 3. Then $\gamma^C_{tstd}(G) = n$ if and only if $\chi^C(G) = n$.

Proof: Assume that $\gamma^C_{tstd}(G) = n$. If $\chi^C(G) \geq 2$ then $\gamma^C_{tstd}(G) = n = \chi^C(G)$ and we are done.

Claim: $\chi^C(G) \neq 1$.

Suppose $\chi^C(G) = 1$. Then $\gamma^C_{tstd}(G) = \max\{\gamma_t(G), \chi^C(G)\} = \gamma_t(G) = n$, which is contradiction to theorem 3.16 ,$\gamma_t(G) \leq \frac{2n}{3} < n$. So $\chi^C(G) \neq 1$.

Hence we have $\chi^C(G) = n$.

Converse is obvious.

Remark 3.18. If G = (V, E) is a graph of order 2 then G = P_2, the path graph with two vertices and hence $\gamma^C_{tstd}(G) = 2 = \chi^C(G)$.

Theorem 3.19. Let G = (V, E) be a graph of order n ≥ 3. Then $\gamma^C_{tstd}(G) = n - 1$ if and only if $\chi^C(G) = n - 1$.

Proof: The proof is analogous to above theorem.

Theorem 3.20. Let G = (V, E) be a graph of order n. If $\chi^C(G) = n - 2$ then $\gamma^C_{tstd}(G) = n - 2$.

Remark 3.21. Converse of above theorem is not true in general. For example consider the graph G = P_4, the path graph with four vertices. $\gamma^C_{tstd}(G) = 2 = n - 2$ but $\chi^C(G) = 1 \neq 2 = n - 2$.

4. Concluding remarks

By the results it seems that both dominating color transversal number and total dominating color transversal number loses their identity as they are actually domination number, total domination number or complementary chromatic number of a graph. This is due to the fact that for the graphs with complementary chromatic number greater than or equal to two, a transversal itself becomes total dominating set. This property actually is eye - catching. It was our effort to relate these two numbers with each other as well as with complementary chromatic number. This has actually produced beautiful results.

REFERENCES

1. D.B.West (Second Edition), Introduction to Graph Theory, Pearsen Education, INC., 2005.
2. D.K.Thakkar and A.B.Kothiya, Complementary coloring of graphs, *PRAJNA – Journal of Pure and Applied Sciences*, 20 (2012) 79 – 80.
3. D.K.Thakkar and A.B.Kothiya, Total dominating color transversal number of graphs, *Annals of Pure and Applied Mathematics*, 11 (2) (2016) 39 – 44.
4. R.L.J.Manoharan, Dominating colour transversals in graphs, Bharathidasan University, September, 2009.
5. G.Bacak, *Vertex Color of a Graph*, Master of Science Thesis, ˙IZM˙IR, December, 2004.
6. D.Amos, On total domination in graphs, University of Houston Downton, December 2012 .
7. D.K.Thakkar and K.N.Kalariya, Strong vertex covering in hypergraph, *Annals of Pure and Applied Mathematics*, 11(1) (2016) 123- 131.
8. D.K.Thakkar and A.A.Prajapat, Vertex covering and independence in semigraphs, *Annals of Pure and Applied Mathematics,* 4(2) (2013) 172-181.
9. E.J.Cockayne, R.M.Dawes, and S.T.Hedetniemi, Total domination in graphs, *Networks,* 10 (1980) 211- 219.