Annals of Pure and Applied Mathematics
Vol. 11, No. 2, 2016, 109-116
ISSN: 2279-087X (P), 2279-0888(online)
Published on 8 June 2016
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Mean Cordial Labeling of Tadpole and Olive Tree

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Received 24February2016; accepted 15 March 2016

Abstract. Let f be a map from V(G) to {0,1,2}. For each edge uv assign the label $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called as a mean cordial labeling if $|v_f(i)- v_f(j)| \leq 1$ and $|e_{f^*}(i)- e_{f^*}(j)| \leq 1$, i, j $\in$ {0,1,2} where $v_f(x)$ and $e_{f^*}(x)$ denote the number of vertices and edges respectively labelled with x (x=0,1,2). A graph with mean cordial labeling is called mean cordial. In this paper, we prove the graphs Tadpole and Olive tree are mean cordial graphs.

Keywords: Mean cordial labeling, tadpole, olive tree

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

All graphs in this paper are finite, simple and undirected. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J. A. Gallian (2014) can be found in [2].The concept of cordial labeling was introduced by Cahit in the year 1987 in [1]. Here we introduce the notion of mean cordial labeling. We investigate the mean cordial labeling of Tadpole and Olive tree.

Definition 1.1. Let f be a map from V(G) to {0,1,2}. For each edge uv assign the label $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called as a mean cordial labeling if $|vf(i)- vf(j)| \leq 1$ and $|ef^*(i)- ef^*(j)| \leq 1$; i, j $\in${0,1,2} where vf(x) and ef*(x) denote the number of vertices and edges respectively labelled with x(x=0,1,2). A graph with mean cordial labeling is called a mean cordial graph.

Definition 1.2. Tadpole T(n,l) is a graph in which Path P_l is attached to any one vertex of cycle C_n.

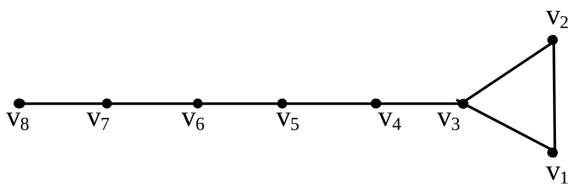

Figure 1: Tadpole (3,6)

Definition 1.3. Olive tree (T_k) is a rooted tree consisting of k branches where the i^{th} branch is a path of length "i".

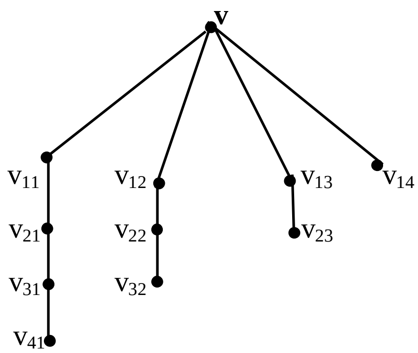

Figure 2: Olive tree $\mathbf{T_4}$

2. Results

Theorem 2.1. Tadpole T(n,l) admits a mean cordial labeling except for $n+l \equiv 1 \pmod 3$ where $(n+l-1)/3 < n$.

Proof: Let $v_1, v_2, \ldots\ldots v_n$ be vertices of cycle C_n and $v_n, v_{n+1}, \ldots\ldots v_{n+l-1}$ be vertices of path P_l

Then, $|V(T(n,l))| = n+l-1$ and $|E(T(n,l))| = n+l-1$

Case 1: $\mathbf{n+l \equiv 0 \pmod 3}$

Let $n+l = 3t$, $t = 1,2,\ldots\ldots$

Define $f : V(G) \to \{0,1,2\}$ as follows:

$f(v_i) = 0 \quad 1 \le i \le t$
$\quad\;\; = 2 \quad t+1 \le i \le 2t$
$\quad\;\; = 1 \quad 2t+1 \le i \le 3t-1$

Define induced edge labelling $f^*: E(G) \to \{0,1,2\}$ as follows:

$f^*(v_iv_{i+1}) = 0 \quad 1 \le i \le t-1$
$f^*(v_tv_{t+1}) = 1$
$f^*(v_iv_{i+1}) = 2 \quad t+1 \le i \le 2t$
$f^*(v_iv_{i+1}) = 1 \quad 2t+1 \le i \le 3t-2$
$f^*(v_nv_1) = 0 \quad$ if $n \le (n+l)/3$
$\quad\quad\quad\; = 1 \quad$ if $n > (n+l)/3$

<u>Subcase 1</u>: **$n \le (n+l)/3$**

Then,

$v_f(0) = t$, $v_f(1) = t-1$, $v_f(2) = t$

$e_{f^*}(0) = t$, $e_{f^*}(1) = t-1$, $e_{f^*}(2) = t$

Thus,

$|v_f(i) - v_f(j)| \le 1 \quad \forall\, i,j \in \{0,1,2\}$

$|e_{f^*}(i) - e_{f^*}(j)| \le 1 \quad \forall\, i,j \in \{0,1,2\}$

Hence f is a mean cordial labeling of T(n,l)

<u>Subcase 2</u>: **$n > (n+l)/3$**

Then,

$v_f(0) = t$, $v_f(1) = t-1$, $v_f(2) = t$

$e_{f^*}(0) = t-1$, $e_{f^*}(1) = t$, $e_{f^*}(2) = t$

Thus,

$|v_f(i) - v_f(j)| \le 1 \quad \forall\, i,j \in \{0,1,2\}$

$|e_{f^*}(i) - e_{f^*}(j)| \le 1 \quad \forall\, i,j \in \{0,1,2\}$

Hence f is a mean cordial labeling of T(n,l)

Case 2: $n+l \equiv 2 \pmod 3$

Let $n+l = 2+3t$, $t = 1,2,\ldots\ldots$

Define f: $V(G) \rightarrow \{0,1,2\}$ as follows:

$f(v_i) = 0 \quad 1 \le i \le t+1$

$\quad = 2 \quad t+2 \le i \le 2t+1$

$\quad = 1 \quad 2t+2 \le i \le 3t+1$

Define induced edge labeling f^*: $E(G) \rightarrow \{0,1,2\}$ as follows:

$f^*(v_i v_{i+1}) = 0 \quad 1 \le i \le t$

$f^*(v_{t+1} v_{t+2}) = 2 \quad t+2 \le i \le 2t+1$

$f^*(v_i v_{i+1}) = 1 \quad 2t+2 \le i \le 3t$

$f^*(v_n v_1) = 0 \quad \text{if } n < l$

$\quad = 1 \quad \text{if } n \ge l$

Subcase 1: $n < l$

Then,

$v_f(0) = t+1$, $v_f(1) = t$, $v_f(2) = t$

$e_{f^*}(0) = t+1$, $e_{f^*}(1) = t$, $e_{f^*}(2) = t$

Thus,

$|v_f(i) - v_f(j)| \le 1 \quad \forall\, i,j \in \{0,1,2\}$

$|e_{f^*}(i) - e_{f^*}(j)| \le 1 \quad \forall\, i,j \in \{0,1,2\}$

Hence f is a mean cordial labeling of T (n,l).

Subcase 2: $n \ge l$

Then,

$v_f(0) = t+1$, $v_f(1) = t$, $v_f(2) = t$

$e_{f^*}(0) = t$, $e_{f^*}(1) = t+1$, $e_{f^*}(2) = t$

Thus,

$|v_f(i) - v_f(j)| \le 1 \quad \forall\, i,j \in \{0,1,2\}$

$|e_{f*}(i) - e_{f*}(j)| \leq 1 \quad \forall\, i, j \in \{0,1,2\}$

Hence f is a mean cordial labeling of T(n,l)

Case 3: n+l ≡ 1(mod3) where (n+l-1)/3 ≥ n

Let n+l-1=3t

Define f : V(G) →{0,1,2} as follows:

$$f(v_i) = \begin{cases} 0 & 1 \leq i \leq t \\ 2 & t+1 \leq i \leq 2t \\ 1 & 2t+1 \leq i \leq 3t \end{cases}$$

Define induced edge labeling f^*:E(G)→{0,1,2} as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 0 & 1 \leq i \leq t-1 \\ 2 & t+1 \leq i \leq 2t \\ 1 & 2t+1 \leq i \leq 3t-1 \end{cases}$$

$f^*(v_n v_1) = 0$

Then,

$v_f(0) = t, \quad v_f(1) = t, \quad v_f(2) = t$

$e_{f*}(0) = t, \quad e_{f*}(1) = t, \quad e_{f*}(2) = t$

Thus,

$|v_f(i) - v_f(j)| \leq 1 \quad \forall\, i, j \in \{0,1,2\}$

$|e_{f*}(i) - e_{f*}(j)| \leq 1 \quad \forall\, i, j \in \{0,1,2\}$

Hence f is a mean cordial labeling of T(n,l).

Case 4: n+l ≡1 (mod3) where (n+l-1)/3 < n

Let n+l-1 = 3t , t = 1,2,…..

Then , |V(T(n,l))| = 3t

Hence,

$v_f(0) = v_f(1) = v_f(2) = t$

But then,

$e_{f*}(0) < t$ and hence

$|e_{f*}(0)-e_{f*}(i)| > 1$ for some i ∈ {1,2}

Hence T(n,l) is not a mean cordial graph for n+l ≡1(mod 3) where (n+l-1)/3 < n.

Illustration 2.2. Mean cordial labeling of T(3,8) is shown in Figure 3.

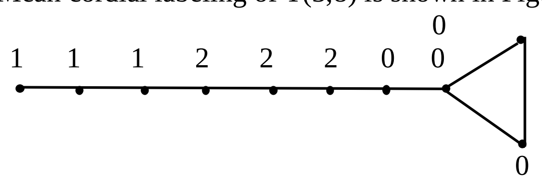

Figure 3: Mean cordial labeling of T(3,8) (n+l ≡ 2(mod3), n < l)

Illustration 2.3. Mean cordial labeling of T(4,4) is shown is shown in Figure 4

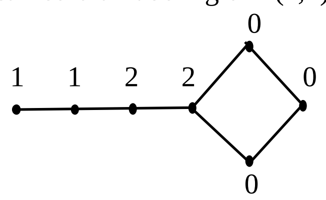

Figure 4: Mean cordial labeling of T(4,4)
(n+l≡2(mod3), n=l)

Illustration 2.4. Mean cordial labeling of T (4,5) is shown in Figure 5

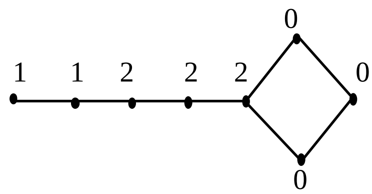

Figure 5: Mean cordial labeling of T(4,5)
(n+l ≡ 0(mod3), n > (n+l)/3)

Illustration 2.5. Mean cordial labeling of T(3,7) is shown in Figure 6

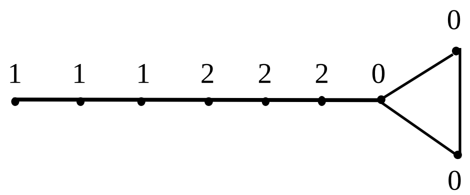

Figure 6: Mean cordial labeling of T(3,7)
(n+l ≡1(mod3), (n+l-1)/3 = n)

Illustration 2.6. Mean cordial labeling of T(3,10) is shown in Figure 7

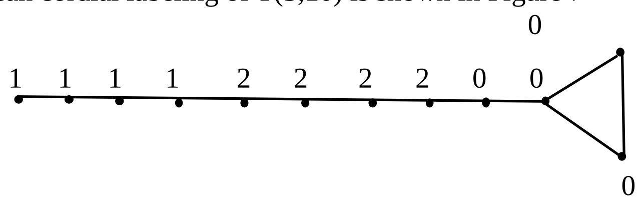

Figure 7: Mean cordial labeling of T(3,10)
(n+l ≡1(mod3), (n+l-1)/3 > n)

Theorem 2.7. An olive tree T_n admits a mean cordial labeling for $n \geq 2$

Proof:

Case 1: n=2,

Let V (T_2) = {v ,v_{11}, v_{12}, v_{21} } and E(T_2) = {vv_{11},vv_{12}, $v_{11}v_{21}$ }

Define f: V $(T_2) \rightarrow \{0,1,2\}$ as follows :

$f(v) = 1$, $f(v_{11}) = 0$, $f(v_{21}) = 0$, $f(v_{12}) = 2$
Define induced edge labeling $f^*: E(T_2) \to \{0,1,2\}$ as follows:
$f^*(vv_{11}) = 1$, $f^*(vv_{12}) = 2$, $f^*(v_{11}v_{21}) = 0$
Then,
$v_f(0) = 2$, $v_f(1) = 1$, $v_f(2) = 1$
$e_{f^*}(0) = 1$ $e_{f^*}(1) = 1$ $e_{f^*}(2) = 1$
Thus
$|v_f(i) - v_f(j)| \leq 1 \quad \forall\, i,j \in \{0,1,2\}$
$|e_{f^*}(i) - e_{f^*}(j)| \leq 1 \quad \forall\, i,j \in \{0,1,2\}$
Hence f is a mean cordial labelling.

Case 2: n =3
Let $V(T_3) = \{v, v_{11}, v_{12}, v_{13}, v_{21}, v_{22}, v_{31}\}$
$E(T_3) = \{vv_{1j} : 1 \leq j \leq 3\} \cup \{v_{11}v_{21}, v_{21}v_{31}, v_{12}v_{22}\}$
Define $f: V(T_3) \to \{0,1,2\}$ as follows :
$f(v) = 1$, $f(v_{11}) = 0$, $f(v_{21}) = 0$, $f(v_{31}) = 0$, $f(v_{12}) = 1$, $f(v_{13}) = 2$, $f(v_{22}) = 2$
Define induced edge labeling $f^*: E(T_3) \to \{0,1,2\}$ as follows:
$f^*(vv_{11}) = 1$, $f^*(vv_{12}) = 1$, $f^*(v_{11}v_{21}) = 0$
$f^*(vv_{13}) = 2$, $f^*(v_{12}v_{22}) = 2$, $f^*(v_{21}v_{31}) = 0$
Then,
$v_f(0) = 3$, $v_f(1) = 2$, $v_f(2) = 2$
$e_{f^*}(0) = 2$ $e_{f^*}(1) = 2$ $e_{f^*}(2) = 2$
Thus
$|v_f(i) - v_f(j)| \leq 1 \quad \forall\, i,j \in \{0,1,2\}$
$|e_{f^*}(i) - e_{f^*}(j)| \leq 1 \quad \forall\, i,j \in \{0,1,2\}$
Hence f is a mean cordial labelling.

Case 3: n ≥ 4
Let $V(T_n) = \{v, v_{ij} : 1 \leq i \leq n,\ 1 \leq j \leq n+1-i\}$
$E(T_n) = \{vv_{1j} : 1 \leq j \leq n;\ v_{ij}v_{i+1,j} : 1 \leq j \leq n,\ 1 \leq i \leq n-j\}$
Then,
$|V(T_n)| = n(n+1)/2 + 1$, $E(T_n) = n(n+1)/2$.

Subcase 1: $\mathbf{n \equiv 0, 2 (mod 3)}$
Let $t = n(n+1)/6$ and $r_{ij} = n(i-1) + j - \sum_1^{i-2} r$
Define $f: V(T_n) \to \{0,1,2\}$ as follows:
$f(v) = 0$
$f(v_{ij}) = 0 \quad 1 \leq r_{ij} \leq t$
$= 1 \quad t+1 \leq r_{ij} \leq 2t$
$= 2 \quad 2t+1 \leq r_{ij} \leq 3t.$

Define induced edge labeling $f^*: E(T_n) \to \{0,1,2\}$ as follows:
$f^*(vv_{1j}) = 0 \quad 1 \leq j \leq n$
$f^*(v_{ij}v_{i+1,j}) = 0 \quad n+1 \leq r_{i+1,j} \leq t$
$= 1 \quad t+1 \leq r_{i+1,j} \leq 2t$
$= 2 \quad 2t+1 \leq r_{i+1,j} \leq 3t$

Then
$v_f(0) = 1+t$, $v_f(1) = t$, $v_f(2) = t$
$e_{f^*}(0) = t$, $e_{f^*}(1) = t$, $e_{f^*}(2) = t$
Thus,
$|v_f(i) - v_f(j)| \leq 1 \quad \forall\, i,j \in \{0,1,2\}$
$|e_{f^*}(i) - e_{f^*}(j)| \leq 1 \quad \forall\, i,j \in \{0,1,2\}$
Hence f is a mean cordial labeling of T_n.

Illustration 2.8. Mean cordial labeling of T_{10} is shown in Figure 8.

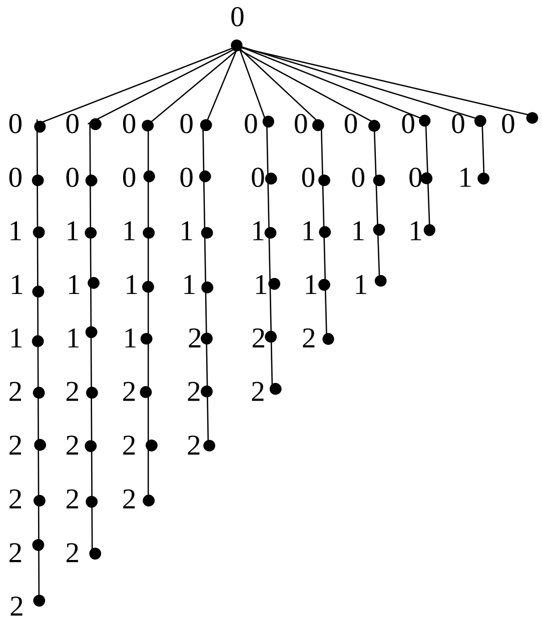

Figure 8: Mean cordial labeling of T_{10}

Subcase 2: n ≡ 1(mod3)
Let $t = (n(n+1)-2)/6$ and $r_{ij} = n(i-1) + j - \sum_1^{i-2} r$.
Define f: $V(T_n) \rightarrow \{0,1,2\}$ as follows:
$f(v) = 0$
$f(v_{ij}) = 0 \quad 1 \leq r_{ij} \leq t$
$= 1 \quad t+1 \leq r_{ij} \leq 2t+1$
$= 2 \quad 2t+2 \leq r_{ij} \leq 3t+1$
Define induced edge labeling f*: $E(T_n) \rightarrow \{0,1,2\}$ as follows:
$f^*(vv_{1j}) = 0 \quad 1 \leq j \leq n$
$f^*(v_{ij}\, v_{i+1,j}) = 0 \quad n+1 \leq r_{i+1,j} \leq t$
$= 1 \quad t+1 \leq r_{i+1,j} \leq 2t+1$
$= 2 \quad 2t+2 \leq r_{i+1,j} \leq 3t+1$
Then
$v_f(0) = t+1$, $v_f(1) = t+1$, $v_f(2) = t$
$e_{f^*}(0) = t$, $e_{f^*}(1) = t+1$, $e_{f^*}(2) = t$

Thus,

$|v_f(i) - v_f(j)| \leq 1 \qquad \forall\, i, j \in \{0,1,2\}$

$|e_{f*}(i) - e_{f*}(j)| \leq 1 \qquad \forall\, i, j \in \{0,1,2\}$

Hence f is a mean cordial labeling of T_n

Hence Olive tree T_n is a mean cordial graph.

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