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On the Acharya Polynomial of a Graph

Shailaja S. Shirkol[1] **and** ***Shobha V. Patil***[2]

[1]Department of Mathematics, SDMCET, Dharwad-580002, Karnataka, India
Email: shailajashirkol@gmail.com
[2]Department of Mathematics, KLE Dr. MSSCET, Belgavi-590008, Karnataka, India
Corresponding author Email: shobhap49@gmail.com

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Abstract. For connected graph G, of order n and degree d, the ***Acharya Polynomial*** is defined as $AP(G,\lambda) = \sum_{\substack{1\le d\le n-1 \\ 1\le k\le p}} \mu(d,G)\cdot\lambda^k$, where $\mu(d,G)$ denotes pair of vertices of degree d at distance k and p *is diam*(G). In the present paper some elementary properties of Acharya polynomial are studied and compute it for some common graphs. We given relation between Hosoya polynomial and Acharya polynomial of common graphs.

Keywords: Hosoya polynomial, Acharya index, Terminal Wiener index, Wiener Index

AMS Mathematics Subject Classification (2010): 05C12

1. Introduction

The graphs in this paper are taken as finite and connected. The degree of a vertex v in G is the number of edges incident to vertex v. A vertex of degree 1 is called terminal vertex. $d(u,v)$ denote the distance between u and v in graph G. **[1]** Wiener index was first proposed by Harold Wiener as an aid to determining the boiling point of paraffin. Since then, the index has been used to build a correlation model between the chemical structures of various chemical compounds. Wiener index is the most celebrated topological index that identifies the characteristics chemical compounds.

In June 2013 at ICDM -2013, B. D. Acharya, in discussion with the first author defined the distance degree parameter, Acharya Index and Acharya polynomial. Here we find the polynomial for Acharya index as Acharya polynomial and discussed for class of graphs **[2].**

Definition 1.1. [3] The ***Wiener index*** is a graph invariant based on distance in graphs. It is denoted by $W(G)$ and defined as sum of distances of all pair of vertices in G:

$$W(G) = \sum_{u<v} d(u,v)$$

Definition 1.2. [4] The ***Hosoya polynomial*** of graph is a polynomial introduced by Hosoya [5] in 1988. Hosoya polynomial (also called Wiener polynomial) of G is defined as

$$H(G,\lambda) = \sum_{k\ge 1} d(G,k)\lambda^k$$

where *d(G,k)* is the number of pair of vertices of *G* that at a distance *k* and λ is a parameter.

It is clear that, $W(G) = \frac{d}{d\lambda} H(G,\lambda)$ at $\lambda = 1$.

Definition 1.3. [5] The Terminal Wiener index is denoted by $TW(G)$ and defined as sum of distances between all pair of terminal vertices in G.

$$TW(G) = \sum_{1<i<j<k} d(v_i, v_j / G)$$

Definition 1.4. [6] The Terminal Hosoya Polynomial of graph G is defined as,

$$TH(G,\lambda) = \sum_{k \geq 1} d_T(G,k)\lambda^k$$

The first derivative of Terminal Hosoya Polynomial of a graph G is Terminal Wiener index of a graph.

Definition 1.5. [7] Let *G* be a connected graph of order *n* and degree *d*, the ***Acharya Index*** $AI_\lambda(G)$ of a graph *G* as the sum of the distance between all pair of degree *d* vertices, denote as

$$AI_\lambda(G) = \sum_{\substack{1 \leq d \leq n-1 \\ 1 \leq k \leq p}} \mu(d,G) \cdot k$$

where $\mu(d,G)$ denotes pair of vertices of degree *d* at distance *k*, $p = diam(G)$.

When λ=1 then the above index reduces to Terminal Wiener index i.e. $AI_\lambda(G) = TW(G)$

Theorem 1.6. For a connected graph $G, TW(G) \leq AI_\lambda(G) \leq W(G)$

Theorem 1.7. [7] Following are the Acharya indices of common graphs,

1. $AI_\lambda(P_n) = \binom{n+1}{3} - (n^2 - 3n + 2)$

2. If G be a r-regular graph then $AI_r(G) = W(G)$

3. $AI_3(P) = W(P) = 75$, where P is Petersen graph

4. $AI_{n-1}(K_n) = W(K_n) = \binom{n}{2}$

5. $AI_\lambda(K_{1,n}) = TW(K_{1,n})$

6. $AI_\lambda(K_{m,n}) = m^2 + n^2 - m - n, \quad m \neq n$

7. $AI_\lambda(K_{n,n}) = 3n^2 - 2n$

8. $AI_2(C_{2n}) = W(C_{2n}) = (2n)^3/8$

9. $AI_2(C_{2n+1}) = W(C_{2n+1}) = (2n+2)(2n+1)(2n)/8$

2. Main results

Let *Acharya Polynomial $AP(G,\lambda)$ is defined as follows*

Definition 2.1. Let G be a connected graph of order n and degree d, the ***Acharya Polynomial $AP(G,\lambda)$*** of a graph G is defined as $\sum_{\substack{1\le d\le n-1\\1\le k\le p}} \mu(d,G)\cdot\lambda^k$, where $\mu(d,G)$ denotes pair of vertices of degree d at distance k and p *is diam*(G).

Theorem 2.2. The Acharya polynomial satisfies the following conditions

1. $\deg AP(G,\lambda)$ equals to the diameter of a graph G
2. $[\lambda^{(0)}]AP(G,\lambda) = \sum_{\substack{1\le d\le n-1\\1\le k\le p}} \mu(0,G)\cdot\lambda^0 = 0$

Example 1. In order to illustrate, we show how the Acharya Polynomial computed for following graph, $V_1=\{v_1, v_2, v_4, v_3\}$, $V_2=\phi$, $V_4=\phi$, $V_3=\{v_5, v_6, v_7, v_8\}$

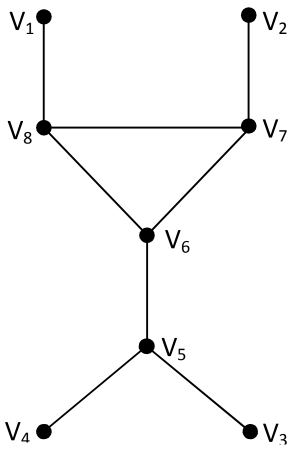

Figure 1:

For the graph in Figure 1.

$$AP(G,\lambda) = \sum_{1\le d\le 3, 1\le k\le 4} \sum \mu(d,G)\lambda^k = 4\lambda + 3\lambda^2 + 2\lambda^2 + \lambda^3 + 4\lambda^4$$

$$AI_\lambda(G) = 29 = AP^1(G,1)$$

$$W(G) = 63$$

Theorem 2.3. If G is a regular graph then $AP(G,\lambda) = H(G,\lambda)$

Corollary 2.4. For a complete graph G on n vertices

$AP(K_n,\lambda)=H(K_n,\lambda)=\binom{n}{2}\lambda$

Corollary 2.5. For a cycle on n vertices

i) $AP(C_{2n},\lambda)=H(C_{2n},\lambda)=(2n)(\lambda+\lambda^2+\cdots+\lambda^{n-1})+n\lambda^n$

ii) $AP(C_{2n+1},\lambda)=H(C_{2n+1},\lambda)=(2n+1)(\lambda+\lambda^2+\cdots+\lambda^n)$

Corollary 2.6. If P is Petersen graph. Then

$AP(P,\lambda)=H(P,\lambda)=15\lambda+30\lambda^2$

Theorem 2.7. If $G=K_{m,n}$ is bipartite graph with $m\neq n$, then Acharya Polynomial

$$AP(K_{m,n},\lambda)=\left[\binom{m}{2}+\binom{n}{2}\right]\cdot\lambda^2$$

Proof: Let $V_1=\{u_1, u_2,\ldots\ldots u_m\}$ and $V_2=\{v_1, v_2,\ldots\ldots v_n\}$ are the set of vertices of $K_{m,n}$ such that $V_1\cup V_2=V$. Then deg(V_1)= n, $\forall u_i\in V_1$ and deg(V_2)= m, $\forall v_j\in V_2$. Therefore there are no vertices at distance and *diam*($K_{m,n}$)= 2. Hence number of vertices at distance 2 are $\binom{m}{2}+\binom{n}{2}$. Thus Acharya polynomial of bipartite graph is given by

$$AP(K_{m,n},\lambda)=\left[\binom{m}{2}+\binom{n}{2}\right]\cdot\lambda^2$$

Theorem 2.8. If $G=K_{m,n}$ is bipartite graph with $m=n$, then Acharya Polynomial

$AP(K_{n,n},\lambda)=H(K_{n,n},\lambda)=n^2\lambda+(n^2-n)\lambda^2$.

Proof: Consider the set of vertices as in the previous theorem with $m = n$, then the distance between vertices of V_1 and V_2 is 1, which are n^2 in number. And the distance between the vertices of V_1 or V_2 is 2. The number of vertices at distance 2 are $\binom{n}{2}+\binom{n}{2}$.

Thus Acharya polynomial of bipartite graph is given by

$$AP(K_{n,n},\lambda)=n^2\lambda+\left[\binom{n}{2}+\binom{n}{2}\right]\lambda^2=n^2\lambda+(n^2-n)\lambda^2$$

Theorem 2.9. For Path on n vertices,

$$AP(P_{n,}\lambda)=\lambda^{n-1}+\lambda^{n-3}+2\lambda^{n-4}+3\lambda^{n-5}+4\lambda^{n-6}+\ldots\ldots+(n-3)\lambda$$

Example 2.

$$AP(P_6,\lambda)=\lambda^5+\lambda^3+2\lambda^2+3\lambda$$

$$AP(P_7,\lambda)=\lambda^6+\lambda^4+2\lambda^3+3\lambda^2+4\lambda$$

$AP(P_8,\lambda) = \lambda^7 + \lambda^5 + 2\lambda^4 + 3\lambda^3 + 4\lambda^2 + 5\lambda$

$AP(P_9,\lambda) = \lambda^8 + \lambda^6 + 2\lambda^5 + 3\lambda^4 + 4\lambda^3 + 5\lambda^2 + 6\lambda$ and so on.

Theorem 2.10. If W_n and Q_n are the wheel and hypercube graphs then

i) $AP(W_{2n+1},\lambda) = (2n)(\lambda + \lambda^2 + \cdots + \lambda^{n-1}) + n\lambda^n$

$AP(W_{2n},\lambda) = (2n-1)(\lambda + \lambda^2 + \cdots + \lambda^n)$

ii) $AP(Q_n,\lambda) = H(G,\lambda) = 2^{n-1}[(1+\lambda)^n - 1]$

Proof: (*i*) From definition of wheel $W_n = K_1 \vee C_{n-1}$, then the degree of all vertices of C_{n-1} is 3 and the degree of vertex K_1 is n-1. Hence Acharya index is calculated only for vertices of cycle C_{n-1} the first result.

(*ii*)From definition of cube, its 3 regular graph. Hence the second result direct from theorem 2.

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