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On Multiplicative *K* Banhatti and Multiplicative *K* Hyper Banhatti Indices of *V*-Phenylenic Nanotubes and Nanotorus

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Abstract. Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The first and second multiplicative K Banhatti indices of a graph G are defined as $BII_1(G)=\prod_{ue}\left[d_G(u)+d_G(e)\right]$ and $BII_2(G)=\prod_{ue}d_G(u)d_G(e)$, where ue means that the vertex u and edge e are incident in G. The first and second multiplicative K hyper-Banhatti indices of G are

$$HBII_1(G)=\prod_{ue}\left[d_G(u)+d_G(e)\right]^2 \text{ and } HBII_2(G)=\prod_{ue}\left(d_G(u)d_G(e)\right)^2$$

respectively. In this paper, we determine multiplicative K Banhatti indices of V-Phenylenic nanotubes $VPHX[m, n]$ and V-Phenylenic nanotorus $VPHY[m, n]$ ($\forall m, n \in \mathbb{N}-\{1\}$). We also compute multiplicative K hyper-Banhatti indices of V-Phenylenic nanotubes $VPHX\,[m, n]$ and V-Phenylenic nanotorus $VPHY[m, n]$ ($\forall\, m, n \in \mathbb{N}-\{1\}$).

Keywords: Multiplicative K Banhatti indices, multiplicative K hyper-Banhatti indices, V-Phenylenic nanotubes, V-Phenylenic nanotorus

AMS Mathematics Subject Classification (2010): 05C05, 05C07, 05C35

1. Introduction

By a graph, we mean a finite, connected, undirected, without loops, multiple edges and isolated vertices. Let G be a graph with n vertices and m edges with vertex set $V(G)$ and edge set $E(G)$. Any undefined term in this paper may be found in Kulli [1].

The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v is denoted by uv. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$. The vertices and edges of a graph are called its elements.

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences.

In Chemical Science, physico-chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [2].

The first and second K Banhatti indices of a graph G are defined as

$$B_1(G) = \sum_{ue} \left[d_G(u) + d_G(e) \right]$$

$$B_2(G) = \sum_{ue} d_G(u) d_G(e)$$

where ue means that the vertex u and edge e are incident in G.

The K Banhatti indices were introduced by Kulli in [3]. Recently many other indices were studied, for example, in [4, 5, 6, 7, 8, 9, 10].

The multiplicative version of first K Banhatti index was introduced by Kulli in [11]. The first multiplicative K Banhatti index of G is defined as

$$BII_1(G) = \prod_{ue} \left[d_G(u) + d_G(e) \right].$$

The multiplicative version of second K Banhatti index was introduced by Kulli in [12]. The second multiplicative K Banhatti index of G is defined as

$$BII_2(G) = \prod_{ue} d_G(u) d_G(e).$$

The first and second K hyper-Banhatti indices of a graph G are defined as

$$HB_1(G) = \sum_{ue} \left[d_G(u) + d_G(e) \right]^2$$

$$HB_2(G) = \sum_{ue} \left(d_G(u) d_G(e) \right)^2.$$

The K hyper-Banhatti indices were introduced by Kulli in [13].

In [14], Kulli introduced the multiplicative versions of K hyper-Banhatti indices. The first and second multiplicative K hyper-Banhatti indices of G are defined as

$$HBII_1(G) = \prod_{ue} \left[d_G(u) + d_G(e) \right]^2$$

$$HBII_2(G) = \prod_{ue} \left(d_G(u) d_G(e) \right)^2.$$

Chemical structures V-Phenylenic nanotubes and V-Phenylenic nanotorus are widely used in medical science and pharmaceutical field. Thus we study multiplicative K Banhatti indices and multiplicative K hyper-Banhatti indices of these molecular structures. In this paper, we consider the structures of V-Phenylenic nanotubes $VHPX[m, n]$ and V-Phenylenic nanotorus $VPHY[m, n]$ ($\forall\ m,n \in \mathbb{N}-\{1\}$) and compute their multiplicative K-Banhatti indices and also their multiplicative K hyper-Banhatti indices.

2. Results

Molecular graphs V-Phenylenic nanotubes $VPHX[m, n]$ and V-Phenylenic nanotorus $VPHY[m, n]$ ($\forall\ m, n \in \mathbb{N} - \{1\}$) belong to two different families of nanostructures whose structures are made up of cycles with length four, six and eight. Molecular graphs of V-Phenylenic nanotubes $VPHX[m, n]$ and V-Phenylenic nanotorus $VPXY[m, n]$ ($\forall\ m, n \in \mathrm{N} - \{1\}$) are shown in Figures 1 and 2 respectively.

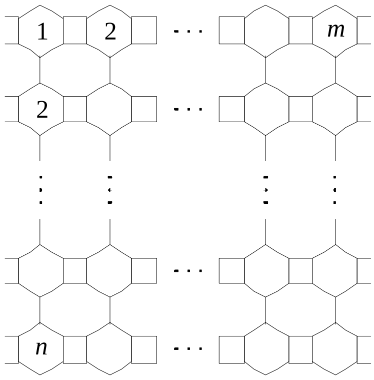

Figure 1:

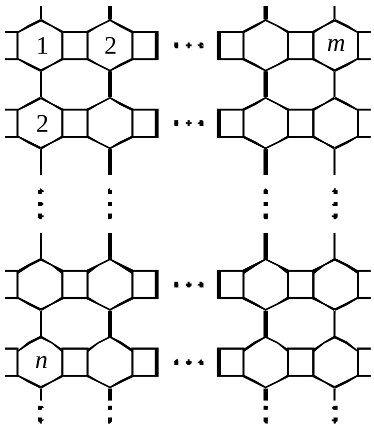

Figure 2:

2.1. *V*- Phenylenic nanotubes

We consider the structure of molecular graph V-Phenylenic nanotubes and compute their multiplicative K Banhatti indices and multiplicative K hyper-Banhatti indices.

Theorem 1. Let G be V-Phenylenic nanotubes $VPHX[m, n]$ ($\forall\, m, n \in \mathbb{N} - \{1\}$). Then

(1) $BII_1(G) = (5)^{4m} \times (6)^{4m} \times (7)^{2m(9n-5)}$.
(2) $BII_2(G) = (6)^{4m} \times (9)^{4m} \times (12)^{2m(9n-5)}$.
(3) $HBII_1(G) = (5)^{8m} \times (6)^{8m} \times (7)^{4m(9n-5)}$.
(4) $HBII_2(G) = (6)^{8m} \times (9)^{8m} \times (7)^{4m(9n-5)}$.

Proof: Let G be V-Phenylenic nanotubes $VPHX[m, n]$ where m and n are the number of hexagons in the first row and colu*mn* in G, see Figure 1. By algebraic method, we get $|V(G)| = 6mn$ and $|E(G)| = 9mn - m$. We have two partitions of the vertex set $V(G)$ as follows:

$V_2 = \{v \in V(G) : d_G(v) = 2\}$, $|V_2| = 2m$.
$V_3 = \{v \in V(G) : d_G(v) = 3\}$, $|V_3| = 6mn - 2m$.

Also we have two partitions of the edge set $E(G)$ as follows:

$E_5 = E^*_6 = \{uv\ E(G) : d_G(u) = 2, d_G(v) = 3\}$, $|E_5| = |E^*_6| = 4m$.

$E_6 = E^*_9 = \{uv\ E(G) : d_G(u) = 3, d_G(v) = 3\}$, $|E_6| = |E^*_9| = 9mn - 5m$.

The edge degree partition of G is given in Table 1.

Table 1:

$(d_G(u), d_G(v) \backslash e=uv \in E(G)$	(2, 3)	(3, 3)
$d_G(e)$	3	4
Number of edges	$4m$	$9mn - 5m$

Now

(1) $BII_1(G) = \prod_{ue}\left[d_G(u) + d_G(e)\right]$

$$= \prod_{e=uv\in E_5}\left[d_G(u)+d_G(e)\right]\left[d_G(v)+d_G(e)\right]\times\prod_{e=uv\in E_6}\left[d_G(u)+d_G(e)\right]\left[d_G(v)+d_G(e)\right]$$

$$=(2+3)^{4m}\times(3+3)^{4m}\times(3+4)^{9mn-5m}\times(3+4)^{9mn-5m}$$

$$=(5)^{4m}\times(6)^{4m}\times 7^{2m(9n-5)}.$$

(2) $BII_2(G) = \prod_{ue} d_G(u)\, d_G(e)$

$$= \prod_{e=uv\in E_6^*}\left[d_G(u)d_G(e)\right]\left[d_G(v)d_G(e)\right]\times\prod_{e=uv\in E_9^*}\left[d_G(u)d_G(e)\right]\left[d_G(v)d_G(e)\right]$$

$$=(2\times3)^{4m}\times(3\times3)^{4m}\times(3\times4)^{9mn-5m}\times(3\times4)^{9mn-5m}$$

$$=(6)^{4m}\times(9)^{4m}\times(12)^{2m(9n-5)}.$$

(3) $HBII_1(G) = \prod_{ue}\left[d_G(u) + d_G(e)\right]^2$

$$= \prod_{e=uv\in E_5}\left[d_G(u)+d_G(e)\right]^2\left[d_G(v)+d_G(e)\right]^2\times\prod_{e=uv\in E_6}\left[d_G(u)+d_G(e)\right]^2\left[d_G(u)+d_G(e)\right]^2$$

$$=\left[(2+3)^2\right]^{4m}\times\left[(3+3)^2\right]^{4m}\times\left[(3+4)^2\right]^{9mn-5m}\times\left[(3+4)^2\right]^{9mn-5m}$$

$$=(5)^{8m}\times(6)^{8m}\times(7)^{4m(9n-5)}.$$

(4) $HBII_2(G) = \prod_{ue}\left(d_G(u)\, d_G(e)\right)^2$

$$= \prod_{e=uv\in E_6^*}\left[d_G(u)d_G(e)\right]^2\left[d_G(v)d_G(e)\right]^2\times\prod_{e=uv\in E_9^*}\left[d_G(u)d_G(e)\right]^2\left[d_G(v)d_G(e)\right]^2$$

$$=\left[(2\times3)^2\right]^{4m}\times\left[(3\times3)^2\right]^{4m}\times\left[(3\times4)^2\right]^{9mn-5m}\times\left[(3\times4)^2\right]^{9mn-5m}$$

$$=(6)^{8m}\times(9)^{8m}\times(12)^{4m(9n-5)}.$$

2.2. *V*- Phenylenic nanotorus

We consider the structure of molecular graph *V*-Phenylenic nanotorus and compute their multiplicative *K* Banhatti indices and multiplicative *K* hyper Banhatti indices.

Theorem 2. Let G_1 be *V*-Phenylenic nanotorus *VPHY[m, n]* ($\forall\, m, n \in \mathbb{N} - \{1\}$). Then,

(1) $BII_1(G_1) = (7)^{18mn}$.
(2) $BII_2(G_1) = (12)^{18mn}$.
(3) $HBII_1(G_1) = (7)^{36mn}$.
(4) $HBII_2(G_1) = (12)^{36mn}$.

Proof: Let G_1 be *V*-Phenylenic nanotorus *VPHY[m, n]* ($\forall\, m, n \in \mathbb{N} - \{1\}$), where m and n are the number of hexagons in the first row and column in G_1, see Figure 2. By algebraic method, we get $|V(G_1)| = 6mn$ and $|E(G_1)| = 9mn$. We have only one partition of the vertex set $V(G_1)$ as follows:

$$V_3 = \{v \in V(G_1) : d_{G_1}(v) = 3\}, \qquad |V_3| = 6mn.$$

Also we have only one partition of the edge set $E(G_1)$ as follows :

$$E_6 = E_9^* = \{uv \in E(G_1) : d_{G_1}(u) = d_{G_1}(v) = 3\}, \quad |E_6| = |E_9^*| = 9mn.$$

Clearly, $d_{G_1}(e) = 4$, for every edge e in G_1.

Now

(1)
$$\begin{aligned} BII_1(G_1) &= \prod_{ue}\left[d_{G_1}(u) + d_{G_1}(e)\right] \\ &= \prod_{e=uv\in E_6}\left[d_{G_1}(u) + d_{G_1}(e)\right]\left[d_{G_1}(v) + d_{G_1}(e)\right] \\ &= (3+4)^{9mn} \times (3+4)^{9mn} \\ &= (7)^{18mn}. \end{aligned}$$

(2)
$$\begin{aligned} BII_2(G_1) &= \prod_{ue} d_{G_1}(u)\, d_{G_1}(e) \\ &= \prod_{e=uv\in E_9^*}\left[d_{G_1}(u)\, d_{G_1}(e)\right]\left[d_{G_1}(v)\, d_{G_1}(e)\right] \\ &= (3\times 4)^{9mn} \times (3\times 4)^{9mn} \\ &= (12)^{18mn}. \end{aligned}$$

(3)
$$\begin{aligned} HBII_1(G_1) &= \prod_{ue}\left[d_{G_1}(u) + d_{G_1}(e)\right]^2 \\ &= \prod_{e=uv\in E_6}\left[d_{G_1}(u) + d_{G_1}(e)\right]^2\left[d_{G_1}(v) + d_{G_1}(e)\right]^2 \\ &= \left[(3+4)^2\right]^{9mn} \times \left[(3+4)^2\right]^{9mn} \\ &= (7)^{36mn}. \end{aligned}$$

(4)
$$\begin{aligned} HBII_2(G_1) &= \prod_{ue}\left[d_{G_1}(u)\, d_{G_1}(e)\right]^2 \\ &= \prod_{e=uv\in E_9^*}\left[d_{G_1}(u)\, d_{G_1}(e)\right]^2\left[d_{G_1}(v)\, d_{G_1}(e)\right]^2 \end{aligned}$$

$$=\left[(3\times 4)^2\right]^{9mn}\times\left[(3\times 4)^2\right]^{9mn}$$
$$=(12)^{36mn}.$$

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