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0-ideals in 0-distributive Nearlattice

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Abstract. Some properties of 0-ideals in 0-distributivenearlattices are derived. It is proved that the set of all 0-ideals in a 0-distributive nearlattice forms a distributive lattice under the specially defined operations on it.

***Keywords**:* Distributive nearlattice, 0-distributive nearlattice, Prime ideals,0-ideals.

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1. Introduction

Cornish [1] introduced the concept of 0-ideal in a distributive lattice and studied their properties with the help of congruence relations. As a generalization of the concept of a distributive lattice, 0-distributive lattices are introduced by Varlet [4]. Recently in [5, 6] the authors have defined 0-distributive nearlattices. As in any abstract algebra, ideals play a vital role in nearlattices. Special types of ideals are introduced and studied in Nearlattices by various authors (See [2,7,8]). Our aim is to introduce and study 0-ideals in 0-distributive Nearlattices. A necessary and sufficient condition for a proper 0-ideal of a 0-distributive nearlattice to beprime is given.Here it is shown that every 0-ideal of a 0-distributive nearlattice is the intersection of all the minimal prime ideals containing it. We also prove that the poset of all 0-ideals under set inclusion forms a distributive nearlattice.

2. Preliminaries

In this article, we collect some basic concepts needed in the sequel for some other non-explicitly stated elementary notions please refer to [5,6].

A nearlattice is a meet semilattice together with the property that any two elements possessing a common upper bound have a supremum. This property is known as the upper bound property. A nearlattice $S$is called distributive if for all $x, y, z \in S, x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$, provided $(y \vee z)$ exists.

A nearlattice $S$with 0 is called 0-distributive if for all $x, y, z \in S$, with $(x \wedge y) = 0 = (x \wedge z)andy \vee z$ exists imply , $x \wedge (y \vee z) = 0$.Of course, every distributive nearlattice $S$with 0 is 0-distributive.A subset $I$of a nearlattice $S$is called a downset if $x \in I$ andfor $t \in S$ with $t \leq x$ imply $t \in I$. An ideal I in a nearlattice S is a non-empty subset of S such that it is down set and whenever $a \vee b$ exists for $a, b \in I$ then $a \vee b \in I$.

A proper ideal I in S is called a prime ideal if $a \wedge b \in I$ implies that either a $\in I$ or $b \in I$. A non-empty subset $F$of S is called a filter if $t \geq a, a \in F$ implies $t \in F$and if $a, b \in F$ then $a \wedge b \in F$. A proper filter F in S is called prime if $a \vee b$ exists and $a \vee b \in F$ implies either $a \in F$ or $b \in F$. It is easy to prove that F is a filter of S if and only if $S - F$is a prime down set. Moreover, a prime down set P is a prime ideal if and only if $S - P$ is a prime filter. A proper filter M of a nearlattice S is called maximal if and only if for any filter Q with $Q \supseteq M$ implies either $Q = M$ or $Q = S$. Dually we define a minimal prime ideal (down set).

Let S be a nearlattice with 0. An element a^* is called the pseudo-complement of a if $a \wedge a^* = 0$ and if $a \wedge x = 0$ for some $x \in S$, then $x \leq a^*$. A lattice L with 0 and 1 is called pseudo-complemented if its every element has a pseudo-complement. Since a nearlattice S with 1 is a lattice, so pseudo-complementation is not possible in a general nearlattice. A nearlattice S with 0 is called sectionally pseudo-complemented if the interval $[0, x]$ for each $x \in S$ is pseudo-complemented. For $A \subseteq$ S, we denote $A^{\perp} = \{x \in$ S$/x \wedge a = 0$ for all $a \in A\}$. If S is distributive then clearly $A^{\perp}$ is an ideal of S. Moreover $A^{\perp} = \bigcap_{a \in I}\{\{a^{\perp}\}\}$. If A is an ideal, then obviously $A^{\perp}$is the pseudo-complement of A in $I(S)$. Therefore, for a distributive nearlattice S with 0, $I(S)$ is pseudo-complemented. For any filter F of S define $F^0 = \{x \in S / x \wedge y = 0$, for some $y \in F\}$. An ideal I in S is called 0-ideal if $I = F^0$ for some filter F in S. For any prime ideal P of S define $0(P) = \{x \in S/x \wedge y = 0$ for some $y \notin P\}$. Note that for any prime ideal P of, $0(P) \subseteq P$. For any non empty subset A of S, the set $A^* = \{x \in S/x \wedge y = 0$ for all $y \in A\}$ is called an annihilator of A in S. An ideal I in S is called an annihilator ideal if $I = I^{**}$. An ideal I in S is called dense in Sif $I^* = \{0\}$. An element $x \in S$ is said to be dense in S if, $(x]^* = \{x\}^* = \{0\}$. An ideal I of S is called an α-ideal if $(x]^{**} \subseteq I$ for each $x \in I$.

A 0-distributive nearlattice S with 0 is said to be quasi-complemented if for each $x \in S$, there exists $x' \in S$ such that $x \wedge x' = 0$ and $((x]^* \vee (x']^*)^* = (0]$. A 0-distributive nearlattice S with 0 is said to be normal if every prime ideal of S contains a unique minimal prime ideal.

Let $I(S)$ denote the set of all ideals of a 0-distributive nearlattice S with 0, then $(I(S), \wedge, \vee)$ is a distributive lattice where $I \wedge J = I \cap J$ and $I \vee J = < I \cup J >$ for any two ideals I and J of S **.**

3.0-ideal

We begin with the following lemma:

Lemma 3.1. In any 0-distributive nearlattice S with 0, we have

a) For any filter $F$of S , $F^0$is a down set in S and $F \cap F^0 \neq \emptyset \Longrightarrow F = S = F^0$.

b) If S contains a dense element,then F^0 = S $\Leftrightarrow F = S$, for any filter F of S.

c) For a filter $F$of S ,$F^0 = \{0\}$ if and only if S has a dense element.

d) For any prime ideal P of S, $0(P)$ is a down set in $S$and $0(P) = (S \backslash P)^0$.

e)For a proper filter F of S, F^0 is contained in some minimal prime ideal of S.

f) If M is a minimal prime ideal of S containingF^0, then $M \cap F = \emptyset$ for any filter F of S

Proof:

a) Obviously, for any filter F of S , F^0 is a down setin$S$. Let F be a filter of S such that $F \cap F^0 \neq \emptyset$. Select $x \in F \cap F^0$. $x \in F^0 \Rightarrow x \wedge y = 0$, for some $y \in F. As$ $x \in F$ and $y \in F, 0 = x \wedge y \in F \Rightarrow F = S$ and hence $F^0 = S$.

b) $F = S \Rightarrow F^0 = S$, obviously. Let $F^0 = S$ and$d$ be a dense element in S. $d \in F^0 \Rightarrow d \wedge f = 0$, for some $f \in F$. As $f \in \{d\}^{\perp} = \{0\}$, we get $f = 0$. Thus$0 \in F$and hence $F = S$.
c) Assume that there exists a filter F in S such that $\{0\} = F^0$. But then $(f]^{\perp} = \{0\}$ for some $f \in F$.This shows that S has a dense element. Conversely, assume that S has a dense element.Then the set D of all dense elements in S is a filter with $D^0 = \{0\}$. Hence the result.
d) Let P be a prime ideal of S. Then $S \setminus P$ is a filter of S .We have $x \in 0(P) \Leftrightarrow x \wedge y = 0$ for some $y \notin P \Leftrightarrow x \wedge y = 0$ for some $y \in S \setminus P \Leftrightarrow x \in (S \setminus P)^0$. Therefore $0(P) = (S \setminus P)^0$.
e) Let F be a proper filter of S. Then F must be contained in some maximal filter say Min S. Then $S \setminus M$ is a minimal prime ideal containing F^0.
f) Let M be a minimal prime ideal of S containing F^0. Assume that $M \cap F \neq \emptyset$. Select $x \in M \cap F$. M being minimal prime ideal, there exists $y \notin M$ such that $x \wedge y = 0$. As $x \wedge y = 0$ and $x \in F$ we get $y \in M$; a contradiction. Hence $M \cap F = \emptyset$ ■

Remarks. (1) In 0-distributive nearlattice S for any proper filter $F, F \cap F^0 = \emptyset$.
(2) In a 0-distributive nearlattice$S$, a proper down set F^0contains no dense elements.
(3) If S is a 0-distributive nearlattice, then for any filter F of S , $F^0$is an ideal in S and for any prime ideal in $P$of S, $0(P)$ is an ideal in S.

Consider the 0-distributive nearlattice $S = \{0, a, b, c, d, e\}$as shown by Hasse diagram of Figure 1.The ideal $(a]$ is not a 0-ideal of S. Hence the set Ω of all 0-ideals of S is a subset of the set of all ideals of S. The ideal (0] is a 0-ideal of S which is not prime. The ideals $(b]$ and $(d]$ are prime 0-ideals of S.

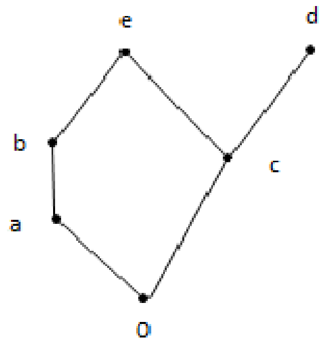

Figure 1:

In general about the 0-ideals of a 0-distributive nearlattice we have

Theorem 3.2. For any 0-distributive nearlattice S, the following statements hold.
(a) A proper 0-ideal contains no dense elements.
(b) Every prime 0-ideal in S is minimal prime.
(c) Every minimal prime ideal in S is a 0-Ideal.
(d) Every non dense prime ideal in$S$ is a 0-ideal
(e) Every 0-ideal in S is an α-ideal.

(f) If S is quasi-complemented nearlattice, then every prime ideal P not containing any dense element is a 0-ideal

Proof:

(a) Let a proper 0-ideal I contains a dense element d in S.As I is an 0-ideal $I = F^0$ for some filter F in S. But then $d \in F^0 \Rightarrow d \wedge f = 0$ for some $f \in F$. As $f \in \{d\}^{\perp} = \{0\}$, we get $f = 0$. As $0 \in F, F = S$. Hence $F^0 = S$ (by lemma 3.1(b)). This contradicts the fact that I is proper and the result follows.

(b) Let P be prime 0-ideal in S.Then $P = F^0$ for some proper filter F in S. Select $x \in P = F^0$. Hence $x \wedge f = 0$,for some $f \in F$. If $f \in P$, then $f \in F \cap F^0$. Hence $F \cap F^0 \neq \emptyset$. Then by lemma 3.1(a), $P = F^0 = F = S$ which is not true. Hence $f \notin P$. Therefore P is minimal prime.

(c) Let P be a minimal prime ideal in S, then $S\backslash F$ is a filter of S. Since $P$is minimal prime ideal in S, we get $P = 0(P)$. Hence $P = (S\backslash P)^0$(by lemma 3.1(c)). Hence P is a 0-ideal.

(d) Let P be a non-dense prime ideal of S. As $\{P\}^{\perp} \neq \{0\}$, there exists $0 \neq x \in \{P\}^{\perp}$. Hence $P \subseteq P^{\perp\perp} \subseteq (x]^{\perp}$. Now let $y \in (x]^{\perp}$. Then x$\wedge y = 0 \in P$ and $x \notin P$ imply $y \in P$. Thus $(x]^{\perp} \subseteq P$. From both the inclusions we get $P = (x]^{\perp}$. As $(x]^{\perp} = ([x))^0$, we get $P = ([x))^0$. Therefore P is a 0-ideal of S.

(e) Let I be a 0-ideal in S. Hence there exists a filter F in S such that $I = F^0$. Let $x \in F^0$. Then $x \in (f]^{\perp}$for some $f \in F$. Hence$(x]^{\perp\perp} \subseteq (f]^{\perp} \subseteq F^0$. This shows that the 0-ideal I in S is an α-ideal.

(f) Let S be a quasi-complemented nearlattice. P be a prime idealof S with $P \cap D = \emptyset$. Let $x \in P$. Since S is quasi-complemented, there exists $y \in S$ such that $x \wedge y = 0$ and $x \vee y \in D$. But then $y \notin P$ as $P \cap D = \emptyset$. Thus $x \in (S\backslash P)^0$ shows that P $\subseteq (S\backslash P)^0$. As $(S\backslash P)^0 \subseteq$ P always, we get P $= (S\backslash P)^0$ and the result follows. ■

Converse of theorem 3.2(b) need not be true, i.e. every 0-ideal need not be a minimal prime ideal in S. For this consider 0-distributive nearlattice represented in Figure 1. $\{0\}$is a 0-ideal, but not a prime ideal in S and hence not a minimal prime ideal in S.

Necessary and sufficient condition for a proper 0-ideal of a 0-distributive nearlattice to be prime is proved in the following theorem.

Theorem 3.3. Let I be a proper 0-ideal of a 0-distributive nearlattice S. Then I is prime if and only if it contains aprime ideal.

Proof: If I is a prime ideal, then obviously it contains a minimal prime ideal. Now assume that I contains a prime ideal P but I is not prime.Select $a \notin I, b \notin I$such that $a \wedge b \in I$. As $P \subseteq$ I and P is prime, we have $a \notin P, b \notin P$ with $a \wedge b \notin P$. Thus $(a \wedge b]^{\perp} \subseteq$ P $\subseteq I$. As I is a 0-ideal of, there exists a filter F in S such that $I = F^0$. Now $a \wedge b \in I = F^0 \Rightarrow a \wedge b \wedge y = 0$ for some $y \in F$. Hence $y \in (a \wedge b]^{\perp} \subseteq$ I $= F^0 \Rightarrow y \in F \cap F^0 \Rightarrow F \cap F^0 \neq \emptyset$. By Lemma 3.1(a), $F = F^0 = S$. Hence $I = S$, which is absurd. Hence I is prime. I being a prime 0- ideal of S, it is minimal prime, by Theorem 3.2(b). Hence the result. ■

It is well known that every ideal of a 0-distributive nearlattice S cannot be expressed as the intersection of all prime ideals containing it. (a] is an ideal but it cannot be expressed as the intersection of all the prime ideals containing it (ref. Figure1), but for 0-ideals of a 0-distributive nearlattice S we have the following theorem.

Theorem 3.4. Every 0-Ideal of a 0-distributive nearlattice is the intersection of all minimal prime ideals containing it.

Proof. Let I be a 0-ideal of S. Hence there exists a filter F in S such that $I = F^0$. Define $J = \cap\{M/M$ is a minimal prime ideal containing $I\}$. Clearly $I \subseteq$ J. Suppose $I \nsubseteq$ J. Choose $x \in J$ such that $x \notin I = F^0$. Hence $x \wedge y \neq 0$ for each $y \in F$. Fix up any $y \in F$. $x \wedge y \neq 0 \Rightarrow x \wedge y \in G$, for some maximal filter G of S. $S \setminus G$ is a minimal prime ideal, $y \notin S \setminus G \Rightarrow (y]^{\perp} \subseteq \mathrm{S} \setminus G$. Again we know that $F^0 = \cup \{(f]^{\perp}/f \in F\}$. Hence $I = F^0 \subseteq S \setminus G$.

This in turn shows that $x \in S \setminus G$, which is absurd. Hence $I = J$ and the result follows. ■

Theorem 3.5. Every proper 0-ideal of a 0-distributive nearlattice S is contained in a minimal prime ideal.

Proof: Let I be a prime 0-ideal in S. Then $I = F^0$ for some proper filter F of S. Clearly $I \cap F = F^0 \cap F = \emptyset$.

Let $\varphi = \{$G/G is a filter of S such that $F \subseteq$ G and $I \cap G = \emptyset\}$. Clearly $F \in \varphi$ and φ satisfies Zorn's Lemma. Let M be a maximal element of φ. We claim that M is a maximal filter of S. Suppose K is a proper filter of S such that $M \subset K$. By maximality of M and $F \subseteq M \subseteq$ K, we get $I \cap K \neq \emptyset$. Select $x \in I \cap K$. As $x \in I = F^0, x \wedge y = 0$, for some $y \in F$ but then $x \wedge y = 0 \in K$; a contradiction. Hence F is a maximal filter of S. S being a 0-distributive nearlattice, M is a prime filter. Therefore I is a minimal prime ideal of S such that $I \subseteq$ S\M. ■

Immediately by Theorem 3.5 we have the following result.

Corollary 3.6. A proper filter of a 0-distributive nearlattice S is maximal if and only if $S \backslash F$ is a 0-ideal.

Proof: Let F be a maximal filter of S. Then $S \backslash F$ is a minimal prime ideal of S and hence a 0-ideal. Conversely, let $S \backslash F$ be a 0-ideal. Then $S \backslash F$ being a proper 0-ideal, it must be contained in some minimal prime ideal say $M$of S (by Theorem 3.5). Thus, $S \backslash M \subseteq$ F. ■

Theorem 3.7. Intersection of any two 0-ideals in a 0-distributive nearlattice S is a 0-ideal of S.

It is enough to prove that for any two filters $F$and G of $S, F^0 \cap G^0 = (F \cap G)^0$.

Obviously, $(F \cap G)^0 \subseteq F^0 \cap G^0$. Let $x \in F^0 \cap G^0$, then $x \wedge f = 0$, for some $f \in F$

And $x \wedge g = 0$, for some $g \in G$. As S is 0-distributive $x \wedge (f \vee g) = 0$. As $f \vee g \in (F \cap G)$, we get $x \in (F \cap G)^0$. Thus, $F^0 \cap G^0 \subseteq (F \cap G)^0$. Combining both the results $F^0 \cap G^0 = (F \cap G)^0$. ■

Corollary 3.8. Intersection of any family of 0-ideals in a 0-distributive nearlattice S is a 0-ideal of S.

4. The set Ω of all 0-ideals

Let S be a 0-distributive nearlattice and let Ω denote the poset $(\Omega, \subseteq)$ of all 0-ideals of S.In this article we prove that the poset $(\Omega, \subseteq)$ need not be a sub lattice of the lattice $(I(S), \wedge, \vee)$ of all ideals in S in general. But under the condition of normality of S the poset $(\Omega, \subseteq)$ will be sub lattice of $(I(S), \wedge, \vee)$.Consider the 0-distributive nearlattice $S = \{0, a, b, c, d, e, f, g, h\}$ as shown in Hasse Diagram of Figure 2.

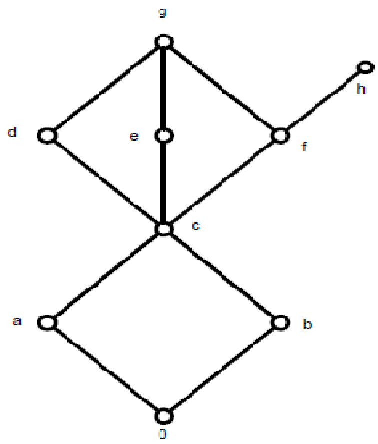

Figure 2:

For the filters $F = [a)$ and $G = [b)$, $[F)^0 \vee [G)^0 = \{0, a, b, c\}$ which is not a $0-$ideal of S. As join of two 0-ideals of a 0-distributive nearlattice S is not a 0-ideal of S, the set Ω of all 0-ideals is not the sub-lattice of the lattice $(I(S), \wedge, \vee)$. ■
In the following theorems we prove some properties of 0-ideals in a nearlattice.

Theorem 4.1.The poset $(\Omega, \subseteq)$ is a sublattice of the lattice $I(S)$ provided S is a normal 0-distributive nearlattice.
Proof: For any two filters F and G of S, $(F^0 \wedge G^0) = F^0 \cap G^0 = (F \cap G)^0$. (See Theorem 3.7) $F^0 \cap G^0 \in \Omega$. Now we prove that $F^0 \vee G^0 = (F \vee G)^0$. Obviously, $F^0 \vee G^0 \subseteq (F \vee G)^0$. Let $x \in (F \vee G)^0$. Then $x \wedge t = 0$ for some $t \in F \vee G$. Hence $t \geq f \wedge g$ for some $f \in F$ and $g \in G$. Hence, $x \wedge f \wedge g = 0 \Rightarrow x \in (f \wedge g]^\perp \Rightarrow x \in (f]^\perp \vee (g]^\perp$(Since S is normal) $\Rightarrow x \in F^0 \vee G^0$. Since $(f]^\perp \subseteq F^0$ and $(g]^\perp \subseteq G^0$). Thus, $(F \vee G)^0 \subseteq F^0 \vee G^0$. Combining both the inclusions we get $F^0 \vee G^0 = (F \vee G)^0$. Hence $(\Omega, \wedge, \vee)$ is a sub lattice of the lattice $(I(S), \wedge, \vee)$. ■

Theorem 4.2. Let S be a normal lattice. Then for any ideal I which contains a 0-ideal K, there exist the largest 0-ideal containing K and contained in I.
Proof: Define $\beta = \{J/J$ is an 0-ideal such that $K \subseteq J \subseteq I\}$. Clearly, $K \in \beta$. Let $\{J_i/i \in \Delta\}$ be a chain in β. Then $\cup \{J_i/i \in \Delta\}$ is a 0-ideal and $K \subseteq \cup J_i \subseteq I$. So by Zorn's lemma β contains a maximal element, say M. We now prove that M is unique. Suppose there exists a maximal element $M_1 \neq M$ in β. Then we have $\subseteq M_1 \vee M \subseteq I$. As S is a normal lattice. $M_1 \vee M \in \beta$ (See Theorem 5.1). But, then $M_1 = M_1 \vee M = M$; and hence the uniqueness. Thus in a normal lattice S, for any ideal I which contains a 0-ideal K, there exists a largest 0-ideal containing K and contained in I. ■

We know that $\{0\}$ is an ideal in a nearlattice S with 0, if it contains a dense element. Hence by Theorem 5.2 it follows the following.

Corollary 4.3. In a normal lattice S containing dense elements, there exists largest 0-ideals in S.

Corollary 4.4. In a normal nearlattice S, there exist largest 0-ideal/ideals in S.

Theorem 4.5. Let S be a normal nearlattice. If $\{I\alpha/\alpha \in \Delta\}$ is a family of 0-ideals in S, then $\vee\, I\alpha$ is a 0 - ideal of S.

Proof: As $I\alpha$ is a 0-ideal of S, let $I\alpha = F_\alpha{}^0$ for some filter F_α of S, for each $\alpha \in \Delta$.$F_\alpha{}^0 \subseteq (\vee F_\alpha)^0$for each $\alpha \in \Delta \Rightarrow \vee F_\alpha{}^0 \subseteq (\vee F_\alpha)^0$. Conversely, let $x \in (\vee F_\alpha)^0$. Then $x \wedge t = 0$ for some $t \in (\vee F_\alpha)$. But then $t \geq f_1 \wedge f_2 \wedge f_3 \wedge \ldots \wedge f_n$ for some $f_i \in F_i (1 \leq i \leq n)$, n finite.Hence, $x \wedge (f_1 \wedge f_2 \wedge f_{3\,\wedge} \ldots \wedge f_n) = 0 \Rightarrow (x \wedge f_1) \wedge (x \wedge f_2) \wedge (x \wedge f_3\,) \wedge \ldots (x \wedge f_n) = 0$. As S is normal lattice $\left((x \wedge f_1)\right]^\perp \vee \left((x \wedge f_2)\right]^\perp \vee \ldots . . \vee \left((x \wedge f_n)\right]^\perp = S. x \in S \Rightarrow x \in \left((x \wedge f_1)\right]^\perp \vee \left((x \wedge f_2)\right]^\perp \vee \ldots . . \vee \left((x \wedge f_n)\right]^\perp \Rightarrow$ $x \leq a_1 \vee a_2\, \vee \ldots . \vee a_n$ where $a_i \in \left((x \wedge f_i)\right]^\perp (1 \leq i \leq n)$. Thus $x \wedge f_i \wedge a_i = 0$. $(1 \leq i \leq n)$. Hence $x \wedge (\wedge_{i=1}^n f_i) \wedge a_i = 0$. As S is 0-distributive, $x \wedge (\wedge_{i=1}^n f_i) \wedge (\vee_{i=1}^n a_i) = 0$. But then $x \wedge (\wedge_{i=1}^n f_i) = 0$ $(as\ x \leq (\vee_{i=1}^n a_i) \Rightarrow x \in ((\wedge_{i=1}^n f_i)]^\perp \Rightarrow x \in ((f_1)]^\perp \vee ((f_2)]^\perp \vee \ldots \vee ((f_n)]^\perp$ (Since S is a normal lattice) $\Rightarrow x \in F_1{}^0 \vee F_2{}^0 \vee \ldots \vee F_n{}^0$as $(f_i] \subseteq F_i{}^0 (1 \leq i \leq n)$. This shows that $(\vee F_\alpha) \subseteq \vee\, (F_\alpha{}^0)$. Combining both the inclusions we get $\vee\, I\alpha = \vee\, F_\alpha{}^0 = (\vee F_\alpha)^0$, and the result follows. ■

Though the poset $(\Omega, \subseteq)$ need not be a sublattice of $I(S)$, interestingly we prove that the poset $(\Omega, \subseteq)$ forms a distributive lattice under the special operations $\sqcap$ and $\sqcup$ defined on it.

Theorem 4.6. Let S be a 0-distributivenearlattice with 0. The poset $(\Omega, \subseteq)$forms a distributive lattice on its own.

Proof. Let F and G be any two filters in S.

Claim I. The poset $(\Omega, \subseteq)$ is a lattice.

(1)$(F \cap G)^0$ is an infimum of $F^0$and G^0in Ω.Let $K^0 \subseteq G^0$ and $K^0 \subseteq F^0$ for some filter K in S. Hence K^0 is in Ω. Let $x \in K^0$. Then $x \in F^0$ and $x \in G^0$ implies $x \wedge y = 0$ for some $y \in F$ and $x \wedge z = 0$ for some $z \in G$. As S is 0-distributive, $x \wedge (y \vee z) = 0$ but then $x \in (F \cap G)^0$ as $(y \vee z) \in F \cap G$. Hence $K^0 \subseteq (F \cap G)^0$. So $(F \cap G)^0$ is infimum of F^0 and G^0 in Ω. If we denote infimum of F^0 and G^0 by $F^0 \sqcap G^0$ then we have $F^0 \sqcap\ G^0 = (F \cap G)^0$.

(2) $(F \vee G)^0$ is supremum of $F^0$and G^0 in Ω. Let $F^0 \subseteq K^0$ and $G^0 \subseteq K^0$for some filter K in S. Let $x \in (F \vee G)^0$.$x \wedge f \wedge g = 0$ for some $f \in F$ and $g \in G$. As $x \wedge f \in G^0 \subseteq K^0$, we get $x \wedge k \wedge f = 0$ for some $k \in K$. But then $x \wedge k \in G^0 \subseteq K^0$. Hence $x \wedge k \wedge s = 0$ for some $s \in K$. As $k \wedge s \in K$, we get $x \in K^0$. Therefore $(F \vee G)^0$ is supremum of F^0 and G^0 in Ω. If we denote supremum of F^0 and G^0in Ω by $F^0 \sqcup\ G^0$, then we have $F^0 \sqcup\ G^0 = (F \vee G)^0$From **(1)** and **(2).**

We get the poset $(\Omega, \subseteq)$ is a lattice under the binary operations $\sqcup$,$\sqcap$ defined on it.

Claim II. The lattice$(\Omega, \sqcap, \sqcup)$ is a distributive lattice.

Now $x \in F^0 \cap (K \sqcup G)^0 \Rightarrow x \in F^0 \cap (K \vee G)^0 \Rightarrow x \wedge f = 0, x \wedge k = 0$ and $x \wedge g = 0$ for some $f \in F, k \in K$and $g \in G \Rightarrow x \wedge (f \vee k) = 0$and $x \wedge (f \vee g) = 0$(as $S\ is$ 0-

distributive) $\Rightarrow x \in (F^0 \sqcap K^0)$ and $x \in (F^0 \sqcap G^0)$(as $f \vee k \in (F^0 \sqcap K^0)$and $f \vee g \in (F^0 \sqcap G^0)$ $\Rightarrow x \in (F^0 \sqcap K^0) \sqcup (F^0 \sqcap G^0)$ $(as\ x \wedge (f \vee k \vee g) = 0) \Rightarrow F^0 \sqcap (K^0 \sqcup G^0) \subseteq (F^0 \sqcap K^0) \sqcup (F^0 \sqcap G^0)$. As $(F^0 \sqcap K^0) \sqcup (F^0 \sqcap G^0) \subseteq F^0 \sqcap (K^0 \sqcup G^0)$ always, we get $F^0 \sqcap (K^0 \sqcup G^0) = (F^0 \sqcap K^0) \sqcup (F^0 \sqcap G^0)$. Hence $(\Omega,\sqcap,\sqcup)$ is a distributive lattice. ■

Corollary 4.7. If a 0-distributive lattice S contains dense elements, thenthe lattice $(\Omega,\sqcap,\sqcup)$ is a bounded, complete distributive lattice.

Proof. The lattice $(\Omega,\sqcap,\sqcup)$ is a distributive lattice (by Theorem 4.6). Clearly, $\{0\}$and S are bounds of the poset $(\Omega, \subseteq)$. Let $\{F_i/i \in \Delta\}$ be any family of filters of S. Then $(\cap\ Fi\ )^0 = \cap \left(F_i{}^0\right)$. (See corollary 3.8). Hence the poset$(\Omega, \subseteq)$ is a complete lattice. Thus it follows that the lattice $(\Omega,\sqcap,\sqcup)$ is a bounded, complete distributive lattice. ■

Corollary 4.8. For a 0-distributive lattice S, the lattice $(\Omega,\sqcap,\sqcup)$ is bounded complete distributive lattice.

Any two distinct 0 – ideals I and J are said to be $\sqcup$ co-maximal if $I \sqcup J = S$.

Lemma 4.9. Let S be a 0-distributive lattice. $x \wedge y = 0 \Rightarrow (x]^\perp \sqcup (y]^\perp = S$ for $x, y \in S$. i.e$(x]^\perp, (y]^\perp$are $\sqcup$ co-maximal.

Proof. As $(x]^\perp = \left([x)\right)^0$and $(y]^\perp = \left([y)\right)^0$, we get $(x]^\perp, (y]^\perp \in \Omega$**.** Now $(x]^\perp \sqcup (y]^\perp = ([x))^0 \sqcup ([y))^0 = ([x) \vee [y))^0 = (x \wedge y)^0 = (x \wedge y])^\perp = (0]^\perp = S$. ■

In the following theorem we show that any two distinct prime 0-idealsof a 0-distributive lattice S are $\sqcup$ co-maximal.

Theorem 4.10. Any two prime 0-ideals P and Q of a 0-distributive nearlattice S are $\sqcup$co-maximal.

Proof. Let P and Q be two distinct prime 0-ideals of a nearlattice $S \Rightarrow P$ and Q be two distinct minimal prime 0-ideals of S. Select $a \in P\backslash Q$ and $b \in Q\backslash P$. As $a \in P$ and P is minimal there exists $x \notin P$ such that $x \wedge a = 0$. Similarly for $b \in Q\backslash P$ and there exists $y \notin Q$ such that $y \wedge b = 0$. Now P being a prime ideal, $x \notin P$ and $b \notin P \Rightarrow x \wedge b \notin P$. Similarly, $y \notin Q$ and$a \notin Q \Rightarrow y \wedge a \notin Q$. But then $(x \wedge b]^\perp \subseteq$ P and $(y \wedge a]^\perp \subseteq$ Q. Again $(x \wedge b) \wedge (y \wedge a) = (x \wedge a) \wedge (y \wedge b) = 0 \Rightarrow (x \wedge b]^\perp \sqcup (y \wedge a]^\perp = S$ by Lemma 3.12. As S= $(x \wedge b]^\perp \sqcup (y \wedge a]^\perp \subseteq P \sqcup Q$, we get $P \sqcup Q = S$. ■

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