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# Graphs and Power Dominator Colorings

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***Abstract.*** Domination and coloring are two important concepts in the study of graphs, extensively studied for theoretical properties and applications. Power domination and dominator coloring are two notions recently introduced and investigated. Here we introduce the concept of power dominator coloring requiring each element of a set of vertices to power dominate an entire color class, thus giving rise to power dominator chromatic number which is the minimum cardinality of such sets of vertices in a graph. We derive formulae for computing this number for certain classes of graphs.

***Keywords***: Graphs; Domination; Coloring; Power domination; Dominator Coloring

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## 1. Introduction

Motivated by the problem of monitoring the state of an electric power system and modelling this problem by a graph with the vertices representing electrical nodes and the edges, the transmission lines, Haynes et al. [4] introduced a variant of domination, known as power domination. On the other hand the concept of dominator coloring [2] of a graph assigns a proper coloring to the vertices and requires every vertex to dominate a color class consisting of all the vertices with the same color. Here we combine the notions of power domination and dominator coloring and introduce a new notion of power dominator coloring which requires every vertex to power dominate all vertices in a color class. The minimum cardinality of such color classes is defined as the power dominator chromatic number $\chi_{pd}(G)$, for a given graph $G$. We obtain certain properties of $\chi_{pd}(G)$ and also compute this number for certain classes of graphs.

## 2. Basic definitions

For basic concepts in graph theory we refer to [1]. We recall certain notions on graphs needed in the sequel. We deal with only simple, undirected graphs.

A complete graph $K_n$ consists of $n$ vertices in which any two vertices are adjacent. A complete bipartite graph $K_{m,n}$ is a bipartite graph with bipartition $(V_1, V_2)$ such that every vertex of $V_1$ is adjacent to all the vertices of $V_2$ and vice versa.

Let $G(V,E)$ be a graph. A subset $S \subseteq V$ is a dominating set [5,6] of $G$ if every vertex in $V-S$ has at least one neighbor in $S$. A subset $S \subseteq V$ is a power dominating set [4] of $G(V,E)$ if all the vertices of $V$ can be observed recursively by the following rules: (i) all vertices in $N[S]$ are observed initially and (ii) if an observed vertex $u$ has all its neighbours observed except one non-observed neighbor $v$, then $v$ is observed (by $u$). We then say that $S$ power dominates the vertices of the graph $G$.

An illustration of power dominating set is now given.

Consider the graph $G(V,E)$ in Fig. 1, with vertex set $V=\{a,b,c,d,u,v,x,y,p,q\}$ and edge set $E=\{ab,bu,uv,vc,cd,ux,vy,xp,yq\}$. Here $S=\{u,v\}$ is a power dominating set. Note that initially, the vertices $\{b,c,u,v,x,y\}$ in $N[S]$ are observed. The only non-observed neighbor of the observed vertex $b$ is $a$ and hence $a$ is observed. Similarly, the only non-observed neighbor of the observed vertex $c$ is $d$ and hence $d$ is observed. Likewise, $p,q$ are also observed.

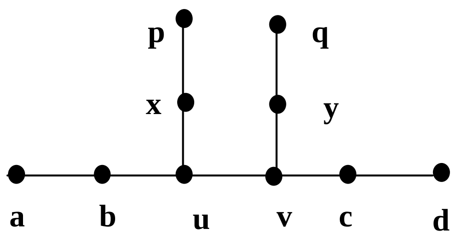


**Figure 1:** A graph with $\chi_{pd}(G)=4$.

A proper coloring [1] of a graph $G(V,E)$ is an assignment of colors to the vertices of $G$ in such a way that no two adjacent vertices receive the same color. The chromatic number $\chi(G)$, is the minimum number of colors required for a proper coloring of $G$. A color class is the set of vertices of $G$, having the same color. The color class corresponding to the color $i$ is denoted by $C_i$. A dominator coloring [2] of $G$ is a proper coloring of $G$ in which every vertex of $G$ dominates every vertex of at least one color class. The convention is that if $\{v\}$ is a color class, then $v$ dominates the color class $\{v\}$. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G.

### 3. Power dominator coloring of a graph

We first introduce the notion of power dominator coloring in a graph.

**Definition 3.1.** A power dominator coloring of a graph $G(V,E)$ is a proper coloring of $G$ such that every vertex of $V$ power dominates all vertices of at least one color class of

$G$. The power dominator chromatic number $\chi_{pd}(G)$ is the minimum number of colors required for a power dominator coloring of $G$.

For example, for the graph in Fig. 1, $\chi_{pd}(G)=4$. In fact color 1 can be assigned to the vertex $u$ while color 2 can be assigned to $v$. The vertices $b, c, x, y$ can be given color 3 while $a, d, p, q$ can be given color 4. Each of the vertices $a, b, x, p$ power dominates the vertex $u$, the only vertex with color 1. Likewise each of the vertices $c, d, y, q$ power dominates the vertex $v$, the only vertex with color 2. Each of $u, v$ self dominates itself. Thus each vertex power dominates a color class. It can be seen that no less than 4 colors will suffice to serve for a power dominator coloring of $G$. Note that the dominator chromatic number $\chi_d(G)$ for the graph in Fig. 1 is 6.

**Theorem 3.1.** For any graph $G$, $\chi(G) \le \chi_{pd}(G) \le \chi_d(G)$.

**Proof:** The inequality $\chi(G) \le \chi_{pd}(G)$ is a consequence of the fact that only a proper coloring is needed in computing $\chi(G)$ while in a dominator coloring, we need a proper coloring and additionally, a vertex has to dominate all vertices in a color class which means more colors might be needed to achieve this. In proving the inequality $\chi_{pd}(G) \le \chi_d(G)$, we note that for computing dominator chromatic number, a vertex $u$ has to dominate all vertices in a color class which means that $u$ has to be adjacent to all these vertices. On the other hand, while computing power dominator coloring, again a vertex $u$ power dominates all vertices in a color class. This would mean that $u$ could "dominate" even vertices not adjacent to $u$. This could result in a reduction in the number of color classes. Hence the inequality $\chi_{pd}(G) \le \chi_d(G)$ in the statement of the Theorem holds. Equality happens, for example, for the star graph $K_{1,n}$ for which $\chi(G) = \chi_{pd}(G) = \chi_d(G) = 2$.

**Theorem 3.2.** For a path $P_n$, $n \ge 2$, on $n$ vertices, $\chi_{pd}(P_n) = 2$.

**Proof:** Let the $n$ vertices of the path $P_n$ be $v_1, \cdots, v_n$ and the $n-1$ edges be $e_1, \cdots, e_n$ where $e_i = v_i v_{i+1}, 1 \le i \le n-1$. Clearly, each vertex $v_i, 1 \le i \le n$, power dominates all the vertices of the path $P_n$. For proper coloring, assign color 1 to $v_i$, for odd $i$ and color 2 to $v_i$, for even $i$. Hence each vertex power dominates all the vertices in a color class with color either 1 or 2 and so $\chi_{pd}(P_n) = 2$.

**Remark.** Note that the dominator chromatic number is 2 only for the paths $P_2$ and $P_3$ while for $P_n, n > 3$, it follows from Theorem 3.3 in [2] that $\chi_d(P_n) > 2$.

**Theorem 3.3.** For a cycle $C_n, n \ge 3$, $\chi_{pd}(C_n) = \begin{cases} 2, & if\ n\ is\ even \\ 3, & if\ n\ is\ odd \end{cases}$

**Proof:** Case (i): Let the $2n$ vertices of the even cycle $C_{2n}$ be $v_1, \cdots, v_{2n}$, $n \geq 2$. Assign color 1 to each vertex $v_i$, for odd $i$ and color 2 to $v_i$, for even $i$. Hence each vertex power dominates all the vertices in a color class with color either 1 or 2 and so $\chi_{pd}(C_n) = 2$.

Case (ii): Let $C_{2n+1}$ be an odd cycle with vertices $v_1, \cdots, v_{2n+1}$, $n \geq 1$. Assign color 1 to each vertex $v_i$, for odd $i$, $(1 \leq i \leq 2n-1)$ and color 2 to $v_i$, for even $i$ while assign color 3 to $v_{2n+1}$. Hence each vertex power dominates all the vertices in at least one color class and so $\chi_{pd}(C_n) = 3$.

**Remark.** Note that **f**or a cycle $C_n, n \geq 6$, the dominator Chromatic number $\chi_d(C_n) > 3$ [3] while $\chi_{pd}(C_n)$ is at the most 3 for any $n$.

**Theorem 3.4.** (i) For the complete bipartite graph $K_{m,n}$, $\chi_{pd}(K_{m,n}) = 2$. (ii) If $G$ is a connected graph of order $n$, then $\chi_{pd}(G) = n$ if and only if $G = K_n, n \geq 1$.

**Proof:** The proofs of statements (i) and (ii) are similar to the corresponding results [ 3 , Propositions 3.1 and 3.2] for $\chi_d(K_{m,n})$ and $\chi_d(K_n)$. In fact, for statement (i), if $(V_1, V_2)$ is a bipartition of $K_{m,n}$, then every vertex of $V_1$ dominates every vertex of $V_2$ and vice versa so that we can assign color 1 to all the vertices in $V_1$ and color 2 to all the vertices in $V_2$ . This is a proper coloring and every vertex power dominates at least one color class. For statement (ii), the proof is exactly the same as the proof in the case of $\chi_d$ [3, Proposition 3.2].

**Theorem 3.5.** If a graph $G$ of order $n$ contains one and only one vertex of degree $n-1$, then $\chi_{pd}(G) = \chi_d(G)$.

**Proof:** Let $G$ be a graph of order $n$. Let $v_1, \cdots, v_n$ be the vertices of $G$ . Let $v_i$, for some $i$, $1 \leq i \leq n$, be the only vertex of degree $n-1$. Clearly $G$ is connected. Then $v_i$ is adjacent to every other vertex of $G$. Clearly one of the color classes is $\{v_i\}$ as $v_i$ is adjacent to all the remaining vertices and so receives a color $c$ , not received by any other vertex. Thus every vertex of $G$ dominates the color class $\{v_i\}$. Thus it is clear that a dominator coloring is also a power dominator coloring. Hence $\chi_{pd}(G) = \chi_d(G)$.

## 4. Conclusion

Several variants of the notion of domination in graphs are known [4-9]. A new notion of power dominator coloring is introduced and the power dominator chromatic number $\chi_{pd}(G)$ is computed for certain classes of graphs. Computation of $\chi_{pd}(G)$ for other classes of graphs and connections with other kinds of colorings such as total dominator coloring are for future work.

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