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The Semifull Graph of a Graph

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Abstract. In this paper, we introduce the concept of the semifull graph of a graph. We obtain some properties of this graph. Also we present characterizations of graphs whose semifull graphs are planar, outerplanar and minimally nonouterplanar.

Keywords: middle blict graph, semifull graph, inner point number, planar, outerplanar, minimally nonouterplanar.

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I. Introduction

By a graph we mean a finite, undirected graph without loops and multiple lines. Any undefined term in this paper may be found in [1].

If $B = \{u_1, u_2, \dots, u_r; r \geq 2\}$ is a block of a graph G, then we say that point u_1 and block B are incident with each other, as are u_2 and B, and so on. If $B = \{e_1, e_2, \dots, e_s; s \geq 1\}$ is a block of a graph G, then we say that line e_1 and block B are incident with each other, as are e_2 and B, and so on. If two distinct blocks B_1 and B_2 are incident with a common cut point, then they are adjacent blocks. This idea was introduced by Kulli in [2]. The points, lines and blocks of a graph are called its members.

The point block graph $P_b(G)$ of a graph G is the graph whose point set is the set of points and blocks of G in which two points are adjacent if the corresponding blocks are adjacent or the corresponding members are incident. This concept was studied by Kulli and Biradar in [3, 4, 5]. Many other graph valued functions in graph theory were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,41] and also graph valued functions in domination theory were studied, for example, in [18, 19, 20, 21, 22, 23, 24].

The middle blict graph $M_n(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G in which two points are adjacent if the corresponding lines of G are adjacent or the corresponding blocks of G are adjacent or the corresponding members of G are incident. This concept was introduced by Kulli and Biradar in [25]

The total graph $T(G)$ of a graph G is the graph whose point set is the union of the set of points and lines of G in which two points are adjacent if the corresponding members of G are adjacent or incident.

Let $B(G)$ and $L(G)$ be the block graph and the line graph of a graph G respectively. Let $\Delta(G)$ denote the maximum degree among the points of G.

The inner point number $i(G)$ of a planar graph G is the minimum number of points not belonging to the boundary of the exterior region in any embedding of G in the plane. A graph G is said to be k-minimally nonouterplanar if $i(G) = k$, $k \geq 1$. This concept was introduced by Kulli in [26]. A graph is outerplanar if $i(G) = 0$. A 1-minimally nonouterplanar graph is called minimally nonouterplanar, see [26]. The concepts of outerplanar and minimally nonouterplanar were studied, for example, in [27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40].

The following will be useful in the proof our results.

Theorem A [25]. If G is a (p, q) graph whose points have degree d_i and if b_i is the number of blocks to which point v_i belongs in G, then the middle blict graph $M_n(G)$ of G has $q+\sum b_i+1$ points and $q+\frac{1}{2}\sum d_i^2+\frac{1}{2}\sum b_i\left(b_i+1\right)$ lines.

Theorem B [25]. The middle blict graph $M_n(G)$ of a graph G is planar if and only if $\Delta(G) \leq 2$.

Theorem C. A graph G is outerplanar if and only if it has no subgraph homeomorphic to K_4 or $K_{2,3}$ except $K_4 - x$.

Theorem D [26]. A graph G is minimally nonouterplanar if and only if one block of G is minimally nonouterplanar and each of its remaining blocks is outerplanar.

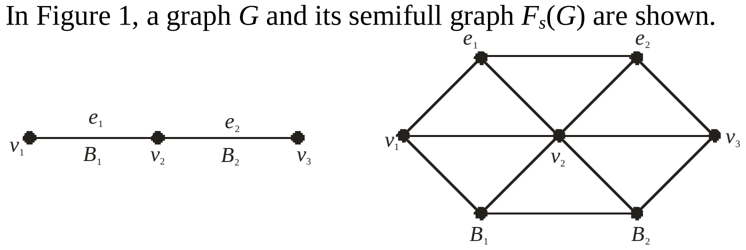
2. Semifull graphs

The definition of $M_n(G)$ of a graph G inspired us to define the following graph valued function.

Definition 1. The semifull graph $F_s(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G in which two points are adjacent if the corresponding members of G are adjacent or one corresponds to a point and the other to a point v of G with it or one corresponds to a block of G and the other to a point v of G and v is in B.

If G has an isolated point v, then the corresponding point of v is an isolated point of $F_s(G)$. Hence we consider graphs without isolated points.

Example 2. In Figure 1, a graph G and its semifull graph $F_s(G)$ are shown.



G $\quad$ $F_s(G)$

Figure 1:

Remark 3. If G is a connected graph, then $F_s(G)$ is also connected and conversely.

Remark 4. The middle blict graph $M_n(G)$ is a spanning subgraph of the semifull graph $F_s(G)$ of G.

Remark 5. For any graph G, $L(G)$ and $B(G)$ are point and also line disjoint induced subgraphs of $F_s(G)$.

Proposition 6. For any graph G, $F_s(G) = T(G) \cup P_b(G)$.

Theorem 7. For any graph G, $F_s(G) = M_n(G) \cup G$.

The following theorem determines the number of points and lines in the semifull graph of a graph.

Theorem 8. If G is a connected (p, q) graph whose points have degree d_i and if b_i is the number of blocks to which point v_i belongs in G, then semifull graph $F_s(G)$ of G has $q+\sum b_i+1$ points and $2q+\frac{1}{2}\sum b_i^2+\frac{1}{2}\sum b_i(b_i+1)$ lines.

Proof: By Remark 4, the middle blict graph $M_n(G)$ is a spanning subgraph of the semifull graph $F_s(G)$ of G. Thus the number of points of $M_n(G)$ equals the number of points of $F_s(G)$. By Theorem A, $M_n(G)$ has $q+\sum b_i+1$ points. Hence the number of points in $F_s(G)$ $= q + \sum b_i + 1$.

By Theorem 7, the number of lines in $F_s(G)$ is the sum of the number of lines in $M_n(G)$ and the number of lines in G. By Theorem A, $M_n(G)$ has $q+\frac{1}{2}\sum b_i^2+\frac{1}{2}\sum b_i(b_i+1)$ lines and G has q lines. Thus the number of lines in $F_s(G) = 2q+\frac{1}{2}\sum b_i^2+\frac{1}{2}\sum b_i(b_i+1)$.

3. Planarity of semifull graphs

We now present a characterization of graphs whose semifull graphs are planar.

Theorem 9. The semifull graph $F_s(G)$ of a connected graph G is planar if and only if $\Delta(G)\leq 2$.

Proof: Suppose $F_s(G)$ is planar. We now prove that $\Delta(G)\leq 2$. On the contrary, assume $\Delta(G)\geq 3$. By Theorem B, $M_n(G)$ is nonplanar. Since $M_n(G)$ is a subgraph of $F_s(G)$, it implies that $F_s(G)$ is nonplanar, which is a contradiction. Hence $\Delta(G) \leq 2$.

Conversely suppose $\Delta(G)\leq 2$. Then G is either a path or a cycle. Clearly G is either P_p, $p \leq 1$ or C_p, $p\geq 3$. It is easy to observe that $F_s(G)$ is planar see Figure 2.

Corollary 10. Let G be a graph. Then $F_s(G)$ is planar if and only if every component of G is either a path or a cycle.

We characterize graphs whose semifull graphs are outerplanar.

Theorem 11. The semifull graph $F_s(G)$ of a connected graph G is outerplanar if and only if G is P_2.

Proof: Suppose $G = P_2$. Then $F_s(G) = K_4 - e$. Since $K_4 - e$ is outerplanar, $F_s(G)$ is outerplanar.

Conversely suppose $F_s(G)$ is outerplanar and G is connected. We now prove that $G=P_2$. On the contrary, assume $G\neq P_2$. Then G has two lines e_1 and e_2. We consider the following cases.

Case 1. Assume e_1 and e_2 are adjacent and each is a block. Then $G = P_3$. Then clearly $F_s(G)=W_7$ and hence $F_s(G)$ is not outerplanar, a contradiction.

Case 2. Assume e_1 and e_2 lie in a block. Then e_1 and e_2 lie on a cycle of G. From Figure 2, it is easy to observe that $F_s(C_p)$ is not outerplanar and hence $F_s(G)$ is not outerplanar, a contradiction.

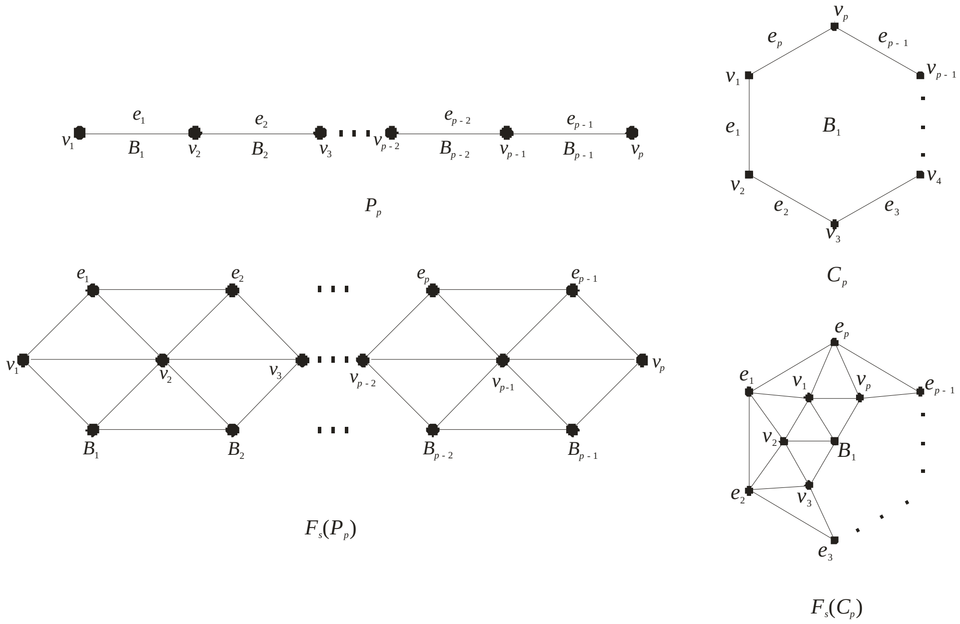


Figure 2:

From the above two cases, we conclude that $G = P_2$.

Corollary 12. Let G be a graph. Then $F_s(G)$ is outerplanar if and only if $G = mP_2$, $m\geq 1$.

We now establish a characterization of graphs whose semifull graphs are minimally nonouterplanar.

Theorem 13. The semifull graph $F_s(G)$ of a connected graph G is minimally nonouterplanar if and only if $G = P_3$.

Proof: Suppose $G = P_3$. Then $F_s(G)$ is W_7, see Figure 1 and hence $F_s(G)$ is minimally nonouterplanar.

Conversely suppose $F_s(G)$ is minimally nonouterplanar. We now prove that $G=P_3$. On the contrary, assume $G \neq P_3$. We consider the following cases.

Case 1. Assume $G = P_2$. By Theorem 11, $F_s(G)$ is outerplanar, a contradiction.

Case 2. Assume G has at least 3 lines. Since $F_s(G)$ is minimally nonouterplanar, it implies that $F_s(G)$ is planar. Thus by Theorem 9, G is a path or a cycle.

If G is a path containing at least 3 lines, then $F_s(G)$ has at least two inner points, a contradiction. If G is a cycle containing at least 3 lines, then $i(F_s(G)) >1$, a contradiction.

From the above two cases, we conclude that $G = P_3$.

Corollary 14. The semifull graph $F_s(G)$ of a graph G is minimally nonouterplanar if and only if $G = mP_2 \cup P_3$, $m \geq 0$.

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