Annals of Pure and Applied Mathematics
Vol. 10, No.1, 2015, 117-122
ISSN: 2279-087X (P), 2279-0888(online)
Published on 14 July 2015
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

q-Continuous Functions in Quad Topological Spaces

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Received 6 June 2015; accepted 5 July 2015

Abstract. The purpose of this paper is to study the properties of q-open sets and q-closed sets and introduce q-continuous function in quad topological spaces (q-topological spaces).

Keywords: quad topological spaces, q-open sets, q-interior, q-closure, q-continuous function.

AMS Mathematics Subject Classification (2010): 54A40

1. Introduction

The concept of bitopological spaces was introduced by Kelly [2] as an extension of topological spaces in 1963. A nonempty set X with two topologies is called bitopological spaces. The study of tri-topological spaces was first initiated by Kovar [3] in 2000, where a non empty set X with three topologies is called tri-topological spaces. Biswas [1] defined some mapping in topological spaces. tri α Continuous Functions and tri β continuous functions introduced by Palaniammal [5] in 2011. Mukundan [4] introduced the concept on topological structures with four topologies, quad topology (4-tuple topology) and defined new types of open (closed) sets. In year 2011, Sweedy and Hassan [6] defined δ^{**}-continuous function in tritopolgical space. In this paper, we study the properties of q-open sets and q-closed sets and q-continuous function in quad topological space (q-topological spaces).

2. Preliminaries

Definition 2.1.[4] Let X be a nonempty set and T_1, T_2, T_3 and T_4 are general topologies on X.Then a subset A of space X is said to be quad-open (q-open) set if $A \subset T_1 \cup T_2 \cup T_3 \cup T_4$ and its complement is said to be q-closed and set X with four topologies called q-topological spaces (X, T_1, T_2, T_3, T_4) .q-open sets satisfy all the axioms of topology.

Note 2.2.[4] We will denote the q-interior (resp. q-closure) of any subset, say of A by q-intA (q-clA),where q-intA is the union of all q-open sets contained in A, and q-clA is the intersection of all q-closed sets containing A.

3. Properties of q-open and q-closed sets

Theorem 3.1. Arbitrary union of q-open sets is q-open.
Proof: Let $\{A_\alpha \,/\, \alpha \in I\}$ be a family of q-open sets in X.

For each $\alpha \in I, A_\alpha \subset T_1 \cup T_2 \cup T_3 \cup T_4$.Therefore, $\cup A_\alpha \subset T_1 \cup T_2 \cup T_3 \cup T_4$. (by definition of q-open sets). Therefore $\cup A_\alpha$ is q-open.

Theorem 3.2. Arbitrary intersection of q-closed sets is q-closed.
Proof: Let $\{B_\alpha \ / \ \alpha \in I\}$ be a family of q-closed sets in X.
Let $A_\alpha = {B_\alpha}^c$. $\{A_\alpha \ / \ \alpha \in I\}$ be a family of q-open sets in X.
Arbitrary union of q-open sets is q-open .Hence $\cup A_\alpha$ is q-open and hence $(\cup A_\alpha)^c$ is q-closed i.e $\cap {A_\alpha}^c$ is q-closed i.e $\cap B_\alpha$ is q-closed. Hence arbitrary intersection of q-closed sets is q-closed.

Definition 3.3. [4] Let $(\mathrm{X}, \mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3, \mathrm{T}_4)$ be a q-topological space. Let $A \subset X$, an element $x \in A$ is called q-interior point of A ,if $\exists$ a q-open set V such that $x \in V \subset A$.

Definition 3.4 [4] The set of all q-interior points of A is called q-interior of A and is denoted as q-int A.

Note 3.5. (1) $q - int\ A \ \subset A$.
(2) q-int A is q-open.
(3) q-int A is the largest q-open set contained in A.

Theorem 3.6. Let $(\mathrm{X}, \mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3, \mathrm{T}_4)$ be a q-topological space. Let $A \subset X$ then A is q-open iff $A = q - int\ A$.
Proof: A is q-open and $A \subset A$. Therefore, $A \in \{B \ / \ B \subset A, B$ is q-open$\}$
A is in the collection and every other member in the collection is a subset of A and hence the union of this collection is A. Hence $\cup \{B \ / \ B \subset A, B$ is q-open$\} = A$ and hence $q - int\ A = A$.
Conversely, since $q - int\ A$ is q-open,
$A = q - int\ A$ implies that A is q-open.

Theorem 3.7. $q - int\ (A \cup B) \supset q - int\ A \cup q - int\ B$
Proof: $q - int\ A \subset A$ and q-int A is q-open.
$q - int\ B \subset B$ and q-int B is q-open.
Union of two q-open sets is q-open and hence q$-int\ A \cup q - int\ B$ is a q-open set. Also $q - int\ A \cup q - int\ B \subset A \cup B$.
$q - int\ A \cup q - int\ B$ is one q-open subset of $A \cup B$ and $q - int\ (A \cup B)$ is the largest q-open subset of $A \cup B$.
Hence, $q - int\ (A \cup B) \supset q - int\ A \cup q - int\ B$.

Definition 3.8. [4] Let (X, T_1, T_2, T_3, T_4) be a quad topological space and let $A \subset X$. The intersection of all q-closed sets containing A is called the q-closure of A & denoted by $q - cl\ A$. $q - cl\ A = \cap \{ B \ / \ B \supset A, B$ is tri α closed$\}$.

Note 3.9. Since intersection of q-closed sets is q-closed, q-cl A is a q-closed set.

Note 3.10. q-cl A is the smallest q-closed set containing A.

Theorem 3.11. A is q-closed iff $A = q - cl\ A$.
Proof: $q - cl\ A = \cap \{ B / B \supset A, B$ is q-closed$\}$.
If A is a q-closed then A is a member of the above collection and each member contains A. Hence their intersection is A. Hence $q - cl\ A = A$. Conversely if $A = q - cl\ A$, then A is q-closed because q-cl A is a q-closed set.

Theorem 3.12. Let (X, T_1, T_2, T_3, T_4) be a quad topological space for any $A \subset X$ $(q - intA)^C = q - clA^c$.
Proof: $(q - intA)^C = [\cup \{G/G \subset A\ \&\ G\ is\ q - open\}]^c$
$=\cap \{G^C/G^C \supset A^c\ \&\ G^c\ is\ q - closed\}$
$= \cap \{F = G^c/F \supset A^c \&\ F\ is\ q - closed\}$ whereF= G^c
$= q - clA^C$.

Definition 3.13. Let $A \subset X$, be a quad topological space. $x \in X$ is called a q-limit point of A, if every q-open set U containing x, intersects $A - \{x\}$.(ie) every q-open set containing x, contains a point of A other than x.

Example 3.14. Let X={a,b,c},
$T_1 = \{\varphi, \{a\}, \{a, b\}, X\}, T_2 = \{\varphi, \{a\}, X\}$, $T_3 = \{\varphi, \{a\}, \{a, c\}, X\}, T_4 = \{\varphi, \{a, b\}, X\}$.
q-open sets are φ ,X, {a} ,{a,b},{a,c}
Consider A={a,c}. Then b is a q-limit point of A.

Definition 3.15. Let $A \subset X$. The set of all q-limit points of A is called the q-derived set of A and is denoted as $q - D(A)$.

Theorem 3.16. $q - cl\ A = A \cup q - D(A)$.
Proof: Let $x \in q - cl\ A$. If $x \in cl\ A$, then $x \in A \cup q - D(A)$. If $x \notin A$, then we claim that x is a q-limit point of A. Let U be a q-open set containing x. Suppose $U \cap A = \emptyset$. Then $A \subset U^c$ and U^c is q-closed and hence $q - cl\ A \subset U^c$. This implies $x \in U^c \Longrightarrow\Longleftarrow$ Hence $U \cap A \neq \emptyset$. Therefore every q-open set U containing x intersects $A - \{x\}$. Hence $x \in q - D(A)$ and $x \in A \cup q - D(A)$.Therefore $q - cl\ A \subset A \cup q - D(A)$
Conversely, it is clear that $A \subset q - cl\ A$. It is enough to prove $q - D(A) \subset q - cl\ A$.
Let $x \in q - D(A)$.If $x \in A$ then it is true. So let us take $x \neq A$. Now we have to prove that $x \in$ every q-closed set containing A. Suppose not, $x \notin B$ where B is a q-closed set containing A. $B \supset A$ Now $x \in B^c$, B^C is q-open and $B^C \cap A = \emptyset$.Contradiction to the fact that x is a q-limit point of A. Hence $x \in$ every q-closed set containing A. Therefore $x \in q - clA$.
Hence $A \cup q - D(A) \subset q - cl\ A$.
Hence $q - cl\ A = A \cup q - D(A)$.

4. q-continuous function

Definition 4.1. Let (X, T_1, T_2, T_3, T_4) and $(Y, T_1', T_2', T_3', T_4')$ be two quad topological spaces. A function $f: X \to Y$ is called q-continuous function if $f^{-1}(V)$ is q-open in X, for every q-open set V in Y.

Example 4.2. Let $X = \{1, 2, 3,4\}$, $T_1 = \{\emptyset, \{1\}, X\}$, $T_2 = \{\emptyset, \{1\}, \{1,3\}, X\}$,
$T_3 = \{\emptyset, \{1\}, \{1,2\}, X\}$, $T_4 = \{\emptyset, \{4\}, \{1,4\}, X\}$
$Let\ Y = \{a, b, c, d\}$, $T_{1'} = \{\emptyset, \{a\}, Y\}$, $T_{2'} = \{\emptyset, \{a\}, \{a, b\}, Y\}$,
$T_{3'} = \{\emptyset, \{a\}, \{a, b\}, Y\}, T_{4'} = \{\emptyset, \{d\}, \{a, d\}, Y\}$
Let f : X → Y be a function defined as $f(1) = a; f(2) = b; f(3) = c;$
$f(4) = d$.
q-open sets in$(X, T_1, T_2, T_3, T_4)$are $\emptyset, \{1\}, \{1,2\}, \{1,3\}, \{4\}, \{1,4\}, X$.
q-open sets in $(Y, T_1', T_2', T_3', T_4')$ are $\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{d\}, \{a, d\}, Y$.
Since $f^{-1}(V)$ is q-open in X for every q-open set V in Y,
f is q-continuous.

Definition 4.3. Let X and Y be two q-topological spaces. A function f: X →Y is said to be q-continuous at a point $a \in X$ if for every q-open set V containing f(a), ∃ a q-open set U containing a, such that $f(U) \subset V$.

Theorem 4.4. $f: X \to Y$ is q-continuous iff f is q-continuous at each point of X.
Proof: Let $f: X \to Y$ be q-continuous.
Take any $a \in X$. Let V be a q-open set containing f(a).
$f: X \to Y$ is q-continuous, Since $f^{-1}(V)$ is q-open set containing a.
Let U = $f^{-1}(V)$ .Then $f(U) \subset V \Rightarrow \exists$ a q-open set U containing a and $f(U) \subset V$
Hence f is q-continuous at a.
Conversely, suppose f is q-continuous at each point of X.
Let V be a q-open set of Y. If $f^{-1}(V) = \emptyset$ then it is q-open.
Take any $a \in f^{-1}(V)$ f is q-continuous at a.
Hence ∃ Ua , q-open set containing a and $f($ Ua$) \subset V$.
Let $U = \cup\{$ Ua $/\ a \in f^{-1}(V)\ \}$.

Claim: U = $f^{-1}(V)$.
$a \in f^{-1}(V) \Rightarrow$ Ua $\subset U \Rightarrow a \in U$.
$x \in U \Rightarrow x \in$ Ua for some $a \Rightarrow f(x) \in V \Rightarrow x \in f^{-1}(V)$. Hence $U = f^{-1}(V)$
Each Ua is q-open. Hence U is q-open. ⇒ $f^{-1}(V)$ is q-open in X.
Hence f is q-continuous.

Theorem 4.5. Let $(X, T_1, T_2, T_3, T_4)$and$(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces. Then f: X → Y is q-continuous function iff $f^{-1}(V)$ is q-closed in X whenever V is q-closed in Y.
Proof: Let f : X → Y be q-continuous function.
Let V be any q-closed in Y.
⇒ V^c is tri α open in Y ⟹ $f^{-1}(V^c)$ is q-open in X.
⇒ $[f^{-1}(V)]^c$ is q-open in X.
⇒ $f^{-1}(V)$is q-closed in X.
Hence $f^{-1}(V)$is q- closed in X whenever V is q-closed in Y.
Conversely, suppose $f^{-1}(V)$ is q-closed in X whenever V is q-closed in Y.
V is a q-open set in Y.
⇒ V^c is q-closed in Y.

$\Rightarrow f^{-1}(V^c)$is tri α closed in X.
$\Rightarrow [f^{-1}(V)]^c$ is q-closed in X.
$\Rightarrow f^{-1}(V)$ is q- open in X.
Hence f is q-continuous.

Theorem 4.6. Let $(X, T_1, T_2, T_3, T_4)$and$(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces. Then, f : X → Y is q-continuous iff $f[q - cl\ A] \subset q - cl\ [f(A)] \quad \forall A \subset X$.
Proof: Suppose f: X → Y is q-continuous. Since $q - cl\ [f(A)]$ is q-closed in Y. Then by theorem (4.5) $f^{-1}(\ q - cl\ [f(A)]\ )$ is q-closed in X,
$q - cl\ [f^{-1}(q - cl(f(A))] = f^{-1}(q - cl(f(A))$. (1)
Now :$f(A) \subset q - cl\ [f(A)],\ A \subset f^{-1}(f(A)) \subset f^{-1}(q - cl(f(A))$.
Then $q - cl(A) \subset\ q - cl\ [f^{-1}(q - cl(f(A))] = f^{-1}(q - cl(f(A))$ by (1)
Then $f(q - cl(f(A)) \subset\ q - cl(f(A))$.
Conversely, let $f(q - cl(A)) \subset q - cl(f(A)) \forall A \subset X$.
Let F be q-closed set in Y ,so that $q - cl(F) = F$. Now $f^{-1}(F) \subset X$,by hypothesis,
$f(q - cl(f^{-1}(F)) \subset\ q - cl\left(f(f^{-1}(F))\right) \subset q - cl(F) = F$.
Therefore $q - cl(f^{-1}(F)) \subset f^{-1}(F)$. But $f^{-1}(F) \subset q - cl(f^{-1}(F))$ always.
Hence $q - cl(f^{-1}(F)) = f^{-1}(F)$ and so $f^{-1}(F)$ is q-closed in X.
Hence by theorem (4.5) f is q-continuous.

5. q-Homomorphism

Definition 5.1. Let (X, T_1, T_2, T_3, T_4) and $(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces. A function f: X → Y is called q-open map if f (V) q-open in Y for every q-open set V in X.

Example 5.2. In example 4.2 f is q- open map also.

Definition 5.3. Let $(X, T_1, T_2, T_3, T_4)$and $(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces .Let f: X → Y be a mapping . f is called q-closed map if f(F) is q-closed in Y for every q-closed set F in X.

Example 5.4. The function f defined in the example 4.2 is q-closed map.

Result 5.5. Let X & Y be two q-topological spaces. Let f: X → Y be a mapping.f is q-continuous iff f^{-1} : Y → X is q-open map.

Definition 5.6. Let $(X, T_1, T_2, T_3, T_4)$and $(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces. Let f: X → Y be a mapping . f is called a q-homeomorphism.
If (i) f is a bijection.
(ii) f is q-continuous.
(iii) f^{-1} is q-continuous.

Example 5.7. The function f defined in the example 4.2 is
(i) a bijection. (ii) f is q-continuous. (iii) f^{-1} is q-continuous.
Therefore f is a q-homeomorphism.

6. Conclusion

In this paper, the idea of q-continous function in quad topological spaces were introduced and studied. Also properties of q-open and q-closed sets were studied.

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