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On Intuitionistic Fuzzy R_0−Spaces

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Abstract. In this paper, we introduce intuitionistic fuzzy topological R_0−spaces (in short, IF−R_0) in the sense of D. Coker. We define some new notions of R_0−spaces using intuitionistic fuzzy sets. We also show that R_0−spaces satisfy "good extensions" property.

Keywords: Intuitionistic set, Intuitionistic fuzzy set, Intuitionistic topological space, Intuitionistic fuzzy topological space, Intuitionistic fuzzy R_0−spaces

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I. Introduction

The concept of intuitionistic fuzzy sets was introduced by Atanassov[2, 3] as a generalization of fuzzy sets. Coker [5, 7, 8, 9, 10] and his colleagues introduced intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In this paper, we investigate the properties and features of R_0−spaces.

Definition 1.1. [10] An intuitionistic set A is an object having the form $A = (x, A_1, A_2)$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A.

Throughout this paper, we use the simpler notation $A = (A_1, A_2)$ for an intuitionistic set.

Remark 1.2. [10] Every subset A on a non-empty set X may obviously be regarded as an intuitionistic set having the form $A' = (A, A^C)$, where $A^C = X \setminus A$ is the complement of A in X.

Definition 1.3. [10] Let the intuitionistic sets A and B on X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j : j \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_j = (A_j^{(1)}, A_j^{(2)})$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $\overline{A} = (A_2, A_1)$, denotes the complement of A.
(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$.

(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$.
(f) $\phi_{\sim} = (\phi, X)$ and $X_{\sim} = (X, \phi)$.

Definition: 1.5. [7] An intuitionistic topology on a set X is a family τ of intuitionistic sets in X satisfying the following axioms:
(1) $\phi_{\sim}, X_{\sim} \in \tau$.
(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
(3) $\cup G_i \in \tau$ for any arbitrary family $G_i \in \tau$.
In this case, the pair (X, τ) is called an intuitionistic topological space (ITS, in short) and any intuitionistic set in τ is known as an intuitionistic open set (IOS, in short) in X.

Definition 1.6. [3] Let X be a non-empty set and I be the unit interval $[0, 1]$. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A=\{(x, \mu_A(x), \nu_A(x)), x \in X\}$, where $\mu_A: X \to I$ and $\nu_A: X \to I$ denote the degree of membership and the degree of non-membership respectively, and $\mu_A(x) + \nu_A(x) \leq 1$.
Let $I(X)$ denote the set of all intuitionistic fuzzy sets in X. Obviously every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form $(\mu_A, 1 - \mu_A)$.

Throughout this paper, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A=\{(x, \mu_A(x), \nu_A(x)), x \in X\}$.

Definition 1.7. [3] Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy sets in X. Then
(1) $A \subseteq B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = (\nu_A, \mu_A)$.
(4) $A \cap B = (\mu_A \cap \mu_B; \nu_A \cup \nu_B)$.
(5) $A \cup B = (\mu_A \cup \mu_B; \nu_A \cap \nu_B)$.
(6) $0_{\sim} = (0^{\sim}, 1^{\sim})$ and $1_{\sim} = (1^{\sim}, 0^{\sim})$.

Definition 1.8. [8] An intuitionistic fuzzy topology (IFT, in short) on X is a family t of IFS's in X which satisfies the following axioms:
(1) $0_{\sim}, 1_{\sim} \in t$.
(2) if $A_1, A_2 \in t$, then $A_1 \cap A_2 \in t$.
(3) if $A_i \in t$ for each i, then $\cup A_i \in t$.
The pair (X, t) is called an intuitionistic fuzzy topological space (IFTS, in short). Let (X, t) be an IFTS. Then any member of t is called an intuitionistic fuzzy open set (IFOS, in short) in X.The complement of an IFOS in X is called an intuitionistic fuzzy closed set (IFCS, in short) in X.

Definition 1.9. [3] Let X and Y be two nonempty sets and $f: X \to Y$ be a function. If $B = \{(y, \mu_B(y), \nu_B(y))/ y \in Y\}$ isan IFS in Y, then the pre image of B under f, denoted by $f^{-1}(B)$ is the IFS in X defined by $f^{-1}(B) = \{(x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)), x \in X\}$ and the image of A under f, denoted by $f(A) = \{(y, f(\mu_A), f(\nu_A), y \in Y\}$is an IFS of Y, where for each $y \in Y$

$$f(\mu_A)(y) = \begin{cases} \sup_{x\in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \phi, \\ 0 & \text{otherwise.} \end{cases}$$

$$f(\nu_A)(y) = \begin{cases} \inf_{x\in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \phi, \\ 1 & \text{otherwise.} \end{cases}$$

Theorem 1.10. [1] Let (X, τ) be an intuitionistic topological space and let $t = \{1_A : A \in \tau\}$, $1_{(A_1,\, A_2)} = (1_{A_1}, 1_{A_2})$, then (X, t) is the corresponding intuitionistic fuzzy topological space of (X, τ).

2. Intuitionistic fuzzy R_0−spaces

Definition 2.1. An intuitionistic fuzzy topological space (X, t) is called

(1) IF−R_0(i) if for all $x, y \in X, x \neq y$ whenever $\exists\ A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) = 1, \upsilon_A(x) = 0; \mu_A(y) = 0,\ \upsilon_A(y) = 1$, then $\exists\ B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) = 1, \upsilon_B(y) = 0; \mu_B(x) = 0, \upsilon_B(x) = 1$.

(2) IF−R_0(ii) if for all $x, y \in X$, $x \neq y$ whenever $\exists\ A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) = 1,\ \upsilon_A(x) = 0;\ \ \mu_A(y) = 0,\ \ \upsilon_A(y) > 0$, then $\exists\ B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) = 1, \upsilon_B(y) = 0;\ \mu_B(x) = 0, \upsilon_B(x) > 0$.

(3) IF−R_0(iii) if for all $x, y \in X$, $x \neq y$ whenever $\exists\ A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) > 0,\ \upsilon_A(x) = 0;\ \ \mu_A(y) = 0,\ \ \upsilon_A(y) = 1$, then $\exists\ B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) > 0, \upsilon_B(y) = 0;\ \mu_B(x) = 0, \upsilon_B(x) = 1$.

(4) IF−R_0(iv) if for all $x, y \in X$, $x \neq y$ whenever $\exists\ A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) > 0,\ \upsilon_A(x) = 0;\ \ \mu_A(y) = 0,\ \ \upsilon_A(y) > 0$, then $\exists\ B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) > 0, \upsilon_B(y) = 0;\ \mu_B(x) = 0, \upsilon_B(x) > 0$.

Definition 2.2. Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α −IF−R_0(i) if for all $x, y \in X$, $x \neq y$ whenever $\exists\ A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) = 1,\ \upsilon_A(x) = 0;\ \ \mu_A(y) = 0,\ \ \upsilon_A(y) \geq \alpha$, then $\exists\ B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) = 1, \upsilon_B(y) = 0;\ \mu_B(x) = 0, \upsilon_B(x) \geq \alpha$.

(b) α −IF−R_0(ii) if for all $x, y \in X$, $x \neq y$ whenever $\exists\ A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) \geq \alpha, \upsilon_A(x) = 0;\ \ \mu_A(y) = 0,\ \ \upsilon_A(y) \geq \alpha$, then $\exists\ B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) \geq \alpha,\ \upsilon_B(y) = 0;\ \mu_B(x) = 0,\ \upsilon_B(x) \geq \alpha$.

(c) α −IF−R_0(iii) if for all $x, y \in X$, $x \neq y$ whenever $\exists\ A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) > 0, \upsilon_A(x) = 0;\ \ \mu_A(y) = 0,\ \ \upsilon_A(y) \geq \alpha$, then $\exists\ B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) > 0,\ \upsilon_B(y) = 0;\ \mu_B(x) = 0,\ \upsilon_B(x) \geq \alpha$.

Theorem 2.3. The properties IF−R_0(i), IF−R_0(ii), IF−R_0(iii) and IF−R_0(iv) are all independent.

Proof: To prove the non-implications among these properties, we consider the following examples.

Example 2.3.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 1, 0), (y, 0, 0.5)} and B = {(x, 0.3, 0.2), (y, 0.1, 0.4)}. We see that the IFTS (X, t) is IF−R_0(i), but it is neither IF−R_0(ii) nor IF−R_0(iv).

Example 2.3.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 0.6, 0), (y, 0, 1)} and B = {(x, 0.2, 0.7), (y, 0.6, 0.3)}. We see that the IFTS (X, t)is $IF-R_0$(i), but it is not$IF-R_0$(iii).

Example 2.3.3. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 0.5, 0), (y, 0, 1)} and B = {(x, 0.2, 0.4), (y, 0.1, 0.6)}. We see that the IFTS (X, t)is $IF-R_0$(ii), but it is neither $IF-R_0$(iii) nor$IF-R_0$(iv)**.**

Example 2.3.4. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 1, 0), (y, 0, 1)} and B = {(x, 0, 0.2), (y, 1, 0)}. We see that the IFTS (X, t)is $IF-R_0$(ii), but it is not$IF-R_0$(i).

Example 2.3.5. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 0.6, 0), (y, 0, 0.4)} and B = {(x, 0.3, 0.1), (y, 0.4, 0.2)}. We see that the IFTS (X, t)is $IF-R_0$(iii), but it is not$IF-R_0$(iv)**.**

Example 2.3.6. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 1, 0), (y, 0, 1)} and B = {(x, 0, 1), (y, 0.5, 0)}. We see that the IFTS (X, t)is $IF-R_0$(iii), but it is not $IF-R_0$(i)**.**

Example 2.3.7. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 0.2, 0), (y, 0, 1)} and B = {(x, 0, 0.5), (y, 0.6, 0)}. We see that the IFTS (X, t)is $IF-R_0$(iv), but it is not$IF-R_0$(iii)**.**

Theorem 2.4. The properties $\alpha-IF-R_0$(i), $\alpha-IF-R_0$(ii) and $\alpha-IF-R_0$(iii) are all independent.

Proof: To prove the non-implications among these properties, we consider the following examples.

Example 2.4.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A= {(x, 0.6, 0), (y, 0, 0.8)} and B= {(x, 0.4, 0.3), (y, 0.5, 0.2)}. For $\alpha = 0.2$, we see that the IFTS(X, t)is $\alpha-IF-R_0$(i), but it isneither $\alpha-IF-R_0$(ii) nor $\alpha-IF-R_0$(iii).

Examples 2.4.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$ where A={(x, 1, 0), (y, 0, 0.5)} and B={(x, 0, 0.6), (y, 0.4, 0)}. For $\alpha = 0.3$, we see that the IFTS (X, t) is $\alpha-IF-R_0$(ii), but it is not $\alpha-IF-R_0$(i).

Examples 2.4.3. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$ where A={(x, 0.2, 0), (y, 0, 0.5)} and B={(x, 0, 0.1), (y, 0.3, 0)}. For $\alpha = 0.5$, we see that the IFTS (X, t) is $\alpha-IF-R_0$(ii), but it is not $\alpha-IF-R_0$(iii).

Examples 2.4.4. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$ where A={(x, 1, 0), (y, 0, 0.7)} and B={(x, 0, 0.3), (y, 0.2, 0)}. For $\alpha = 0.3$, we see that the IFTS (X, t)is $\alpha-IF-R_0$(iii), but it is neither $\alpha-IF-R_0$(i) nor $\alpha-IF-R_0$(ii).

Theorem 2.5. Let $(X,\ t)$ be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{\, A|U : A \in t\}$ and $\alpha \in (0,\ 1)$, then

(a) $(X,\ t)$ is IF$-R_0$(i) $\Longrightarrow (U,\ t_U)$ is IF$-R_0$(i).
(b) $(X,\ t)$ is IF$-R_0$(ii) $\Longrightarrow (U,\ t_U)$ is IF$-R_0$(ii).
(c) $(X,\ t)$ is IF$-R_0$(iii) $\Longrightarrow (U,\ t_U)$ is IF$-R_0$(iii).
(d) $(X,\ t)$ is IF$-R_0$(iv) $\Longrightarrow (U,\ t_U)$ is IF$-R_0$(iv).
(e) $(X,\ t)$ is $\alpha-$IF$-R_0$(i) $\Longrightarrow (U,\ t_U)$ is $\alpha-$IF$-R_0$(i).
(f) $(X,\ t)$ is $\alpha-$IF$-R_0$(ii) $\Longrightarrow (U,\ t_U)$ is $\alpha-$IF$-R_0$(ii).
(g) $(X,\ t)$ is $\alpha-$IF$-R_0$(iii)$\Longrightarrow (U,\ t_U)$ is $\alpha-$IF$-R_0$(iii).

The proofs (a), (b), (c), (d), (e), (f), (g) are similar. As an example we proved (e).

Proof (e): Suppose $(X,\ t)$is the intuitionistic fuzzy topological space and is also $\alpha-$IF$-R_0$(i). We shall prove that $(U,\ t_U)$ is $\alpha-$IF$-R_0$(i). Let$x,\ y \in U, x \neq y$with $A_U = (\mu_{A_U},\ \nu_{A_U}) \in t_U$ such that $\mu_{A_U}(x) = 1,\ \nu_{A_U}(x) = 0$ and $\mu_{A_U}(y) = 0,\ \nu_{A_U}(y) \geq \alpha$. Suppose $A = (\mu_A,\ \nu_A) \in t$is the extension IFS of $A_U$on X, then $\mu_A(x) = 1,\ \nu_A(x) = 0$ and $\mu_A(y) = 0,\ \nu_A(y) \geq \alpha$. Since $x,\ y \in U \subseteq X$. That is,$x,\ y \in X$. Again, since $(X,\ t)$is $\alpha-$IF$-R_0$(i), then $\exists\ B = (\mu_B,\ \nu_B) \in t$ such that $\mu_B(y) = 1,\ \nu_B(y) = 0$ and$\mu_B(x) = 0, \nu_B(x) \geq \alpha \Longrightarrow (\mu_B|U)(y) = 1,\ \ (\nu_B|U)(y) = 0$ and $(\mu_B|U)(x) = 0,\ \ (\nu_B|U)(x) \geq \alpha$. Hence $(\mu_B|U, \nu_B|U) \in t_U$. Therefore $(U,\ t_U)$ is $\alpha-$IF$-R_0$(i).

Definition 2.7. An intuitionistic topological space $(X,\ \tau)$is called intuitionistic R_0-space ($I-R_0$ space) if for all$x,\ y\ \in X,\ \ x \neq y$whenever $\exists\ C = (\,C_1,\ C_2\,) \in \tau$with $x \in C_1$,$x \notin C_2$ and $y \notin C_1,\ y \in C_2$then $\exists\ D = (\,D_1,\ D_2) \in \tau$ such that$y \in D_1,\ \ y \notin D_2$ and $x \notin D_1$, $x \in D_2$.

Theorem 2.8. Let $(X,\ \tau)$ be an intuitionistic topological space and let $(X,\ t)$ be an intuitionistic fuzzy topological space. Then we have the following implications:

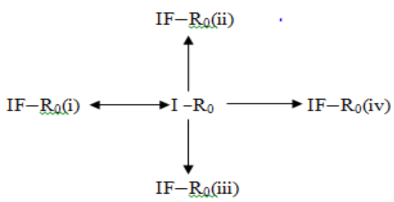

Figure 1:

Proof: Let $(X,\ \tau)$ be$I-R_0$ space. We shall show that $(X,\ t)$ isIF$-R_0$(i). Suppose $(X,\ \tau)$is $I-R_0$. Let $x,\ y \in X,\ \ x \neq y$ with $(1_{C_1},\ 1_{C_2}) \in t$ such that $1_{C_1}(x) = 1,\ \ 1_{C_2}(x) = 0$; $1_{C_1}(y) = 0,\ \ 1_{C_2}(y) = 1 \Longrightarrow x \in C_1, x \notin C_2;\ y \notin C_1,\ \ y \in C_2$. Hence$(\,C_1,\ C_2\,) \in \tau$. Since $(X,\ \tau)$ is $I-R_0$, then $\exists\ (\,D_1,\ D_2) \in \tau$ such that $y \in D_1$,$y \notin D_2$ and $x \notin D_1,\ \ x \in D_2 \Longrightarrow 1_{D_1}(y) = 1, 1_{D_2}(y) = 0$and $1_{D_1}(x) = 0, 1_{D_2}(x) = 1 \Longrightarrow (1_{D_1},\ 1_{D_2}) \in t$. Hence $(X,\ t)$ is IF$-R_0$(i). Therefore $I-R_0 \Longrightarrow$ IF$-R_0$(i).

Conversely, let$(X,\ t)$beIF$-R_0$(i). We shall show that $(X,\ \tau\,)$is $I-R_0$. Suppose $(X,\ t)$ is IF$-R_0$(i).Let $x,\ y\ \in X,\ \ x \neq y$ with $C = (C_1,\ C_2) \in \tau$ such that $x \in C_1$,

$x \notin C_2$ and $y \notin C_1$, $y \in C_2 \Rightarrow 1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$ and $1_{C_1}(y) = 0$, $1_{C_2}(y) = 1$.Hence $(1_{C_1}, 1_{C_2}) \in t$. Since (X, t)is IF$-R_0$(i), then $\exists\ (1_{D_1}, 1_{D_2}) \in t$ such that$1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $1_{D_1}(x) = 0$, $1_{D_2}(x) = 1 \Rightarrow y \in D_1$, $y \notin D_2$ and $x \notin D_1$, $y \in D_2 \Rightarrow (D_1, D_2) \in \tau$. Hence (X, τ) is I$-R_0$. Hence IF$-R_0(i)\Rightarrow$I$-R_0$. Therefore I$-R_0 \Leftrightarrow$IF$-R_0$(i).

Furthermore, it can verify that I$-R_0 \Rightarrow$IF$-R_0$(ii), I$-R_0$ $\Rightarrow$IF$-R_0$(iii) and I$-R_0$ $\Rightarrow$IF$-R_0$(iv).
None of the reverse implications is true in general which can be seen from the following examples.

Examples 2.8.1. Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by{A, B} where A= {(x, 1, 0), (y, 0, 0.4)} and B= {(x, 0, 0.5), (y, 1, 0)}, we see that the IFTS (X, t) is IF$-R_0$(ii), but not corresponding I$-R_0$.

Examples 2.8.2. Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by{A, B} where A= {(x, o.5, 0), (y, 0, 1)} and B= {(x, 0, 1), (y, 0.6, 0)}, we see that the IFTS (X, t) isIF$-R_0$(iii), but not corresponding I$-R_0$**.**

Examples 2.8.3. Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by{A, B} where A= {(x, 1, 0), (y, 0, 0.3)} and B= {(x, 0, 0.4), (y, 0.6, 0)}, we see that the IFTS (X, t) is IF$-R_0$(iv), but not corresponding I$-R_0$.

Theorem2.9. Let (X, τ)be an intuitionistic topological space and let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:

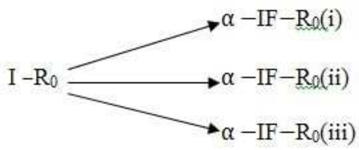

Figure 2:

Proof: Let (X, τ) be I$-R_0$ space. We shall show that (X, t) is α $-$IF$-R_0$(i). Let$\alpha \in (0, 1)$. Suppose (X, τ) is I$-R_0$.Letx, $y \in X$, $x \neq y$ with $(1_{C_1}, 1_{C_2}) \in t$ such that $1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$; $1_{C_1}(y) = 0$, $1_{C_2}(y) \geq \alpha \Rightarrow 1_{C_1}(x) = 1$,$1_{C_2}(x) = 0$; $1_{C_1}(y) = 0$, $1_{C_2}(y) = 1$ for any $\alpha \in (0, 1) \Rightarrow x \in C_1$, $x \notin C_2$; $y \notin C_1$, $y \in C_2$. Hence$(C_1, C_2) \in \tau$. Since (X, τ) is I$-R_0$, then $\exists\ (D_1, D_2) \in \tau$ such that $y \in D_1$, $y \notin D_2$ and $x \notin D_1$, $x \in D_2 \Rightarrow 1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $1_{D_1}(x) = 0$, $1_{D_2}(x) = 1 \Rightarrow 1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $1_{D_1}(x) = 0$, $1_{D_2}(x) \geq \alpha$for $\alpha \in (0, 1) \Rightarrow (1_{D_1}, 1_{D_2}) \in t$. Hence (X, t) is α $-$IF$-R_0$(i). Therefore I$-R_0 \Rightarrow$ α $-$IF$-R_0$(i).
Furthermore, it can verify that I$-R_0 \Rightarrow$ α $-$IF$-R_0$(ii) and I$-R_0 \Rightarrow$ α $-$IF$-R_0$(iii).
None of the reverse implications is true in general which can be seen from the following examples.

Example 2.9.1. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A= {(x, 1, 0), (y, 0, 0.7)} and B= {(x, 0, 0.8), (y, 1, 0)}. For $\alpha = 0.7$, we see that the IFTS (X, t) is $\alpha-$IF$-R_0$(i) but not corresponding I $-R_0$.

Example 2.9.2. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A= {(x, 1, 0), (y, 0, 0.5)} and B= {(x, 0, 0.6), (y, 0.6, 0)}. For $\alpha = 0.5$, we see that the IFTS (X, t) is $\alpha-$IF$-R_0$(ii) but not corresponding I $-R_0$.

Example 2.9.3. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A= {(x, 0.3, 0), (y, 0, 0.5)} and B= {(x, 0, 0.5), (y, 1, 0)}. For $\alpha = 0.4$, we see that the IFTS (X, t) is $\alpha-$IF$-R_0$(iii) but not corresponding I $-R_0$.

Theorem2.10. Let (X, t) and (Y, s) be two intuitionistic fuzzy topological spaces and $f : X \to Y$ be one-one, onto, continuous open mapping, then
(1) (X, t) is IF$-R_0$(i) $\Leftrightarrow$ (Y, s) is IF$-R_0$(i).
(2) (X, t) is IF$-R_0$(ii) $\Leftrightarrow$ (Y, s) is IF$-R_0$(ii).
(3) (X, t) is IF$-R_0$(iii) $\Leftrightarrow$ (Y, s) is IF$-R_0$(iii).
(4) (X, t) is IF$-R_0$(iv) $\Leftrightarrow$ (Y, s) is IF$-R_0$(iv).
(5) (X, t) is $\alpha-$IF$-R_0$(i) $\Leftrightarrow$ (Y, s) is $\alpha-$IF$-R_0$(i).
(6) (X, t) is $\alpha-$IF$-R_0$(ii) $\Leftrightarrow$ (Y, s) is $\alpha-$IF$-R_0$(ii).
(7) (X, t) is $\alpha-$IF$-R_0$(iii) $\Leftrightarrow$ (Y, s) is $\alpha-$IF$-R_0$(iii).
The proofs (1), (2), (3), (4), (5), (6), (7) are similar. As an example we proved (1).

Proof (1) Suppose the intuitionistic fuzzy topological space (X, t) is IF$-R_0$(i). We shall prove that the intuitionistic fuzzy topological space (Y, s) is IF$-R_0$(i). Let $y_1, y_2 \in Y$, $y_1 \neq y_2$ with $A = (\mu_A, \nu_A) \in s$ such that $\mu_A(y_1) = 1$, $\upsilon_A(y_2) = 1$. Since f is onto, then $\exists\, x_1, x_2 \in X$ such that $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$. Hence $x_1 \neq x_2$. We have $(f^{-1}(\mu_A), f^{-1}(\nu_A)) \in t$, as f is IF-continuous. Now, $(f^{-1}(\mu_A))(x_1) = \mu_A(f(x_1)) = \mu_A(y_1) = 1$ and $(f^{-1}(\nu_A))(x_2) = \nu_A(f(x_2)) = \nu_A(y_2) = 1$. Therefore, since (X, t) is IF$-R_0$(i), then $\exists\, B = (\mu_B, \nu_B) \in t$ such that $\mu_B(x_2) = 1$, $\nu_B(x_1) = 1$. Now, $(f(\mu_B))(y_2) = \mu_B(f^{-1}(y_2)) = \mu_B(x_2) = 1$ and $(f(\nu_B))(y_1) = \nu_B(f^{-1}(y_1)) = \nu_B(x_1) = 1$ as f is one-one and onto. Hence $(f(\mu_B), f(\nu_B)) \in s$. Therefore (Y, s) is IF$-R_0$(i).

Conversely, Suppose the intuitionistic fuzzy topological space (Y, s) is IF$-R_0$(i). We shall prove that the intuitionistic fuzzy topological space (X, t) is IF$-R_0$(i). Let $x_1, x_2 \in X$, $x_1 \neq x_2$ with $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x_1) = 1$, $\upsilon_A(x_2) = 1$. Since f is one-one, then $\exists\, y_i \in s$ such that $y_i = f(x_i)$, i = 1, 2. Hence $f(x_1) \neq f(x_2)$ implies $y_1 \neq y_2$ as f is one-one. We have $(f(\mu_A), f(\nu_A)) \in s$ as f is IF-continuous. Now, $(f(\mu_A))(y_1) = (f(\mu_A))(f(x_1)) = \mu_A(f^{-1}(f(x_1))) = \mu_A(x_1) = 1$ and $(f(\nu_A))(y_2) = \nu_A(f^{-1}(f(x_2))) = \nu_A(x_2) = 1$. Therefore, since (Y, s) is IF$-R_0$(i), then $\exists\, D = (\mu_D, \nu_D) \in s$ such that $\mu_D(y_2) = 1$, $\nu_D((y_1) = 1$. Now, $(f^{-1}(\mu_D))(x_2) = \mu_D(f(x_2)) = \mu_D(y_2) = 1$ and $(f^{-1}(\nu_D))(x_1) = \nu_D(f(x_1)) = \nu_D(y_1) = 1$, as f is one-one and onto. Hence $(f^{-1}(\mu_D), f^{-1}(\nu_D)) \in t$. Therefore (X, t) is IF$-R_0$(i).

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