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Selection of Wavelet and Thresholding Rule for Denoising the ECG Signals

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Abstract. Electrocardiogram (ECG) plays a vital role in heart disease diagnosis. Usually ECG signals are affected by various noises. Several researchers have done their works to conform the purity of ECG signals. In this work, the discrete wavelet transform (DWT) based wavelet denoising have implemented using different thresholding techniques to remove the sources of noises from the original signals. Four thresholding techniques ('Rigrsure', 'Heursure', 'Sqtwolog' and 'Minimaxi') and three wavelet functions ('db20', 'sym20' and 'coif5') have been used in this work to de-noise the original ECG signals. The significant reduction of above considered noises has been shown by the experimental result. It also retains the ECG signal morphology effectively. We have used four different performance measures to select thresholding rules and efficient wavelet functions for removal of the noises from the signals such as Root Mean Square Error (RMSE), Signal to Noise Ratio (SNR), Percentage Root Mean Square Difference (PRD) & Noise Power (Pn). The best result has been obtained with the 'Rigrsure' thresholding rule and 'coif5' wavelet function based on considered SNR for non-stationary ECG signals.

Keywords: Electrocardiogram, wavelet transform, DWT, threshold, SNR, PRD, MSE, Pn.

AMS Mathematics Subject Classification (2010): 42C40

1. Introduction

The activities of the cardiac muscles are represented by the ECG signal, while gathering and recording. It is affected by various noises. Usually ECG signal has discrete morphological properties (P-QRS-T complex). It is mostly vital than the other biological signals. Various cardiac diseases and heart abnormalities are diagnosed by using ECG morphological. Several researchers have been trying to remove the noises and to extend the morphology of ECG by different processes for last few papers [7,8,9]. Many of them have used different filters to remove the bad effect. Thresholding methods divided into two parts such, soft and hard thresholding. After a long time research many researchers has concluded that the soft thresholding is much better than the hard thresholding. So, naturally soft thresholding is used in this work. The Denoising of the signal requires thresholding methods, thresholding rules and exact wavelet functions. Various types of wavelet functions are available to de-noise the signals and extend its applications in the future. Three among them have chosen normally, 'db20' , 'sym20' and 'coif5'. In this case, proposed wavelet thresholding de-noising method based on discrete wavelet transform (DWT) up to level 5 with four threshold rules are applying for 15-ECG signals samples and each samples duration is 10second and frequency 360 Hz [2,3,4].

2. Wavelets transform

A complex valued function ψ satisfying the following conditions:

I. $\int_{-\infty}^{\infty} |\psi(t)| dt = 0$

II. $\int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty$

III. $\int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|}{|\omega|} d\omega < \infty$

where $\hat{\psi}(\omega)$ is the Fourier transform of ψ. The 2^{nd} and 3^{rd} conditions implies finite energy and admissibility condition of ψ. The function ψ is called mother wavelet [4]. Wavelets are different types, such as Haar Wavelet, Maxican hat wavelet, Daubechies Wavelet, Mayer Wavelet, Morlet wavelet, Shannon Wavelet, Symlet Wavelet &Coiflet Wavelet…etc.

3. Discrete wavelet transform

The wavelet transform is a popular technique for analysing signals. WT describes a multi-scale decomposition process in terms of expansion of signal onto a set of basic functions. The WT can be categorized into continuous and discrete [4,5]. The discrete wavelet transform (DWT) of a signal $x(t)$ can be written as

$$T_{b,a}(t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\psi\left(\frac{t-b}{a}\right)dt$$

where a and b represent the dilatation and translation along the time axis.

3.1. Wavelets analysis

The wavelet analysis is an exciting new method for solving difficult problems in mathematics. The wavelet transform is often compared with the Fourier Transform. The wavelet analysis of ECG signal is performed by MATLAB software. MATLAB allows solving many techniques for computing problem [11,13]. The MATLAB software provided a wavelet tool box. Wavelets allow filters to be constructed for stationary (a stationary signal is where there is no change in the properties of signal) and non-stationary (a non-stationary signal is where there is change in the properties of signal) signals.

3.2. Wavelet thresholding

Wavelet thresholding de-noising methods deals with wavelet coefficients using a suitable chosen threshold value in advance. The wavelet coefficient at different scales could be obtained by taking DWT of the noisy signal. Thresholding methods divided into two types such as hard thresholding and soft thresholding [8]. Hard thresholding de-noising method may lead to the oscillation of the reconstructed ECG signal and the soft thresholding do-noising method may reduce the amplitudes of ECG wave. The noisy ECG signal can be assume with finite length as follows $x_{j-1}(t) = x_j(t) + d_j(t)$ $j = 1,2,3,......N$ where $x_j(t)$ is the original ECG signal, $d_j(t)$ is the Gaussian white noise with zero mean and constant variance and $x_{j-1}(t)$ is the noisy ECG signal. Applying DWT the noisy signal is decomposed, at the decomposition level of 5. So, approximation coefficients a_j and detail coefficients d_j are obtained [7,8].

Hard thresholding: $d^{*}{}_j = \begin{cases} d_j, |d_j| \geq t_j \\ 0,, |d_j| \leq t_j \end{cases}$

Soft thresholding: $d^{**}{}_j = \begin{cases} sign(d_j)(|d_j| - t), |d_j| \geq t_j \\ 0,, |d_j| \leq t_j \end{cases}$

Soft thresholding method is much stable than hard thresholding. The value of threshold (t) is $t = \delta\sqrt{2\log\|d_j\|}$, where $\delta = \left(median\left(|d_j|\right)\right)/0.6745$.

4. Thresholding rules

There are four types of thresholding rules such as

Global thresholding

Global thresholding or fixed threshold computed as:

$t_g = \sqrt{2\log(d_j)}$ where d_j is the total number of wavelet coefficients.

Rigrsurethresholding

Rigrsure is an adaptive thresholding method like as threshold t.

Heursurethresholding

Heursurethresholding is combination of Rgrsure and global thresholding method. If the signal-to-noise ratio of the signal is very small, then the Rgrsurethresholding method estimation will have more amounts of noises.

Minimaxthresholding

Minimaxthresholding method like as a global threshold value and proposed minimum performance for Mean Square Error (MSE) against the required signal.

Performance Estimation

Root mean squre error (RMSE)

The equation $\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}\left(x(n)-\hat{x}(n)\right)^2}$ is called Root Mean Square Error,

where $\hat{x}(n)$ is reconstructed Signal, $x(n)$ is row signal.

$$RMS=\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}\left(x(n)-\hat{x}(n)\right)^2}$$

Signal to noise ratio (SNR)

Signal to noise ratio (SNR) is the power ratio between a signal and noise. It is expressed in terms of the logarithmic decibel scale.

$$SNR=10\log_{10}\left(\frac{X_{signal}}{X_{noise}}\right)^2$$

where X_{signal} **,** Root mean square amplitude of the signal

X_{noise} **,** Root mean square amplitude of the noise

Percentage root mean square difference (PRD)

The most conspicuously used performance is the Percentage Root mean square Difference $PRD=\sqrt{\frac{\sum_{n=0}^{N}\left(x(n)-\hat{x}(n)\right)^2}{\sum_{n=0}^{N}\left(x(n)\right)^2}}*100$

where $\hat{x}(n)$ is reconstructed Signal, $x(n)$ is row signal of length N, respectively. It provides point wise comparison with the original data.

Noise power (P_n)

The Noise power P_n is the difference between original and de-noised signal. The noise power expressed as: $P_n = X(i)_{original}{}^2 - X(i)_{denoised}{}^2$

Figure 1: This diagram shows ECG signal decomposition at level 5 using “coif5” wavelet with detail and approximation coefficients of ECG signal.

5. Results and discussion

For analysing the required performance of de-noising the ECG signals have considered three wavelet function and four threshold rules. DWT based thresholding has been tested over the 15 ECG datasets [2] and each with duration of (10sec) from the stress assessment experiment.

SNR												
	db20				sym20				coif5			
	Heur-sure	Rigr-sure	Sqtwo-log	Mini-maxi	Heursure	Rigr-sure	Sqtwo-log	Mini-maxi	Heur-sure	Rigr-sure	Sqtwo-log	Mini-maxi
1	41.6504	41.6503	41.6165	41.6304	41.6532	41.6530	41.6278	41.6377	**41.6536**	41.6535	41.6291	41.6383
2	41.4022	41.4024	41.3732	41.3842	41.4023	**41.4026**	41.3811	41.3892	41.4022	41.4024	41.3826	41.3901
3	40.3704	40.3705	40.3285	40.3446	40.3688	40.3690	40.3307	40.3461	40.3713	**40.3715**	40.3309	40.3461
4	40.1251	40.1251	40.0751	40.0955	40.1296	40.1296	40.0908	40.1056	40.1305	**40.1305**	40.0903	40.1050
5	40.8884	40.8884	40.8876	40.8877	40.8885	**40.8886**	40.8879	40.8880	40.8885	40.8885	40.8873	40.8875
6	40.6093	40.6093	40.5802	40.5915	40.6075	40.6075	40.5853	40.5944	40.6106	**40.6107**	40.5860	40.5950
7	39.5771	39.5570	39.4312	39.4832	**39.5879**	39.5874	39.447	39.4935	39.5823	39.5828	39.4197	39.4755
8	39.2894	39.2894	39.2626	39.2726	39.2897	39.2897	39.2686	39.2766	39.2900	**39.2900**	39.2712	39.2783
9	40.3921	40.3921	40.3773	40.3873	40.3930	40.3833	40.3764	40.3866	40.4008	**40.4009**	40.3781	40.3876
10	**38.7946**	38.7945	38.7770	38.7834	38.7942	38.7941	38.7814	38.7862	38.7941	38.7942	38.7828	38.7870
11	40.3635	40.3636	40.2731	40.3074	40.3644	40.3635	40.2774	40.3099	**40.3839**	40.3642	40.2712	40.3061
12	39.8608	39.8608	39.8102	39.8291	39.8626	39.8626	39.8246	39.8384	39.8632	**39.8633**	39.8248	39.8387
13	39.9950	39.9950	39.8923	39.9317	39.9991	39.9990	39.9058	39.9402	**40.0003**	40.0001	39.9076	39.9412
14	40.4581	40.4573	40.4371	40.4497	40.4693	40.4693	40.4433	40.4534	40.4693	**40.4693**	40.4459	40.5552
15	39.4016	39.4016	39.3753	39.3852	39.3968	39.3968	39.3799	39.3881	39.4025	**39.4025**	39.3799	39.3883
Ratio of thresholding Rules in numbers(%)												
	1 6.67%				1 6.67%	2 13.33%			3 20%	**8 53.33%**		
Ratio of wavelets in numbers (%)												
1 6.67%					3 20%				11 73.33%			

Table 1: Selection of bestthresholding rule and wavelet function for de-noising the ECG based on SNR

Bold letters indicates the best value between Thresholding rules and wavelet functions of 15- ECG signals.

The SNR performance on denoising ECG signals is given in Table 1 and it allows finding out the bestthresholding rule(rigrsure) which is performing well over other thresholding rules and "coif5" wavelet function gives the best SNR rate while comparing with other three-wavelet functions.

Bold letters indicates the best value between Thresholding rules and wavelet functions of 15- ECG signals.

The PRD performance on denoising ECG signals is given in Table 2 and it allows finding out the best thresholding rule (rigrsure) which is performing well over other thresholding rules and "coif5" wavelet function gives the best PRD rate while comparing with other three-wavelet functions.

PRD												
	db20				sym20				coif5			
	Heur-sure	Rigr-sure	Sqtwo-log	Mini-maxi	Heur-sure	Rigrsure	Sqtwo-log	Mini-maxi	Heur-sure	Rigrsure	Sqtwo-log	Mini-maxi
1	0.00024	0.00024	0.00059	**0.00007**	0.00021	0.00021	0.0036	0.0026	0.00034	0.00034	0.0016	0.0011
2	0.0001	0.0001	0.0063	0.0045	0.0004	0.0004	0.0017	0.0008	0.0001	**0.0001**	0.0050	0.0033
3	0.0001	**0.00009**	0.0019	0.0015	0.0005	0.0002	0.0016	0.0012	0.0005	0.0002	0.0054	0.0037
4	0.0001	**0.0001**	0.0002	0.0002	0.0008	0.0008	0.0018	0.0014	0.0005	0.0005	0.0036	0.0026
5	0.0012	0.0012	0.0136	0.0097	0.0003	**0.0002**	0.0137	0.0097	0.0013	0.0013	0.0213	0.0175
6	0.0001	0.0001	0.0031	0.0017	0.00007	**0.00004**	0.0011	0.0006	0.0004	0.0003	0.0049	0.0035
7	0.0018	0.0021	0.0026	0.0017	0.0024	0.0039	0.0057	0.0036	0.0006	0.0003	**0.00001**	0.00005
8	0.0002	**0.0002**	0.0011	0.0003	0.0004	0.0004	0.0123	0.0069	0.0006	0.0006	0.0151	0.0090
9	0.0002	0.0002	0.0003	0.0001	0.0004	0.0004	0.0104	0.0058	0.0001	**0.0001**	0.0140	0.0083
10	0.0046	0.0050	0.0034	0.0019	**0.0002**	0.0002	0.0038	0.0033	0.0008	0.0008	0.0011	0.0008
11	0.0002	0.0002	0.0026	0.0017	0.0001	0.0001	0.0055	0.0032	0.00009	**0.00009**	0.0051	0.0031
12	0.0003	0.0003	0.0118	0.0083	0.00004	0.00004	0.0039	0.0032	0.00002	**0.00002**	0.0057	0.0040
13	0.0005	0.0005	0.0104	0.0079	**0.0003**	0.00003	0.0030	0.0009	0.0021	0.0020	0.0036	0.0042
14	0.0252	0.0351	0.0252	0.0254	0.0794	**0.0213**	0.0893	0.0575	0.0448	0.0362	0.0447	0.0449
15	0.00005	0.00005	0.00014	0.0003	0.00007	0.00007	0.0028	0.0019	0.00002	**0.00002**	0.0036	0.0028
Ratio of thresholding Rules in numbers(%)												
		3 20%		1 6.67%	2 13.33 %	3 20%				**5 33.33 %**	1 6.67%	
Ratio of wavelets in numbers(%)												
	4 26.67%				5 33.33%				**6 40%**			

Table 2: Selection of bestthresholding rule and wavelet function for de-noising the ECG based on PRD

RMS												
	db20				sym20				coif5			
	Heursure	Rigrsure	Sqtwolog	minimaxi	Heursure	Rigrsure	Sqtwolog	minimaxi	Heursure	Rigrsure	Sqtwolog	Minimaxi
1	0.0033	**0.0029**	0.0094	0.0071	0.0034	0.0031	0.0097	0.00740	0.0033	0.0031	0.0095	0.0072
2	0.0032	0.0031	0.0089	0.0068	0.0032	**0.0030**	0.0088	0.0068	0.0033	0.0031	0.0089	0.0070
3	0.0034	**0.0033**	0.0106	0.0079	0.0034	0.0034	0.0107	0.0079	0.0035	0.0034	0.0108	0.0080

4	0.0034	0.0034	0.0109	0.0080	0.0034	0.0034	0.0108	0.079	0.0034	**0.0033**	0.0110	0.0081
5	0.0083	0.0080	0.0297	0.0204	0.0078	**0.0070**	0.0285	0.0196	0.0091	0.0078	0.0336	0.0233
6	0.0033	0.0033	0.0097	0.0074	0.0033	0.0031	0.0097	0.074	0.0034	**0.0031**	0.0097	0.0075
7	0.0037	0.0035	0.0093	0.0067	0.0038	0.0054	0.0098	0.0071	0.0030	**0.0027**	0.0095	0.0069
8	0.0037	0.0036	0.0116	0.0086	0.0037	0.0036	0.0114	0.0085	0.0037	**0.0036**	0.0115	0.0086
9	0.0036	0.0036	0.0105	0.0079	0.0036	0.0035	0.0104	0.0080	0.0036	**0.0035**	0.0104	0.0079
10	0.0041	0.0042	0.0085	0.0064	0.0034	0.0034	0.0085	0.0064	0.0033	**0.0032**	0.0086	0.0064
11	0.0036	**0.0036**	0.0135	0.0098	0.0037	0.0037	0.0132	0.0096	0.0039	0.0039	0.0134	0.0098
12	0.0036	0.0035	0.0110	0.0082	0.0036	0.0034	0.0109	0.0081	0.0036	**0.0033**	0.0110	0.0082
13	0.0035	**0.0031**	0.0116	0.0083	0.0035	0.0034	0.0117	0.0084	0.0036	0.0032	0.0121	0.0087
14	0.0031	0.0031	0.0086	0.0063	0.0034	0.0034	0.0087	0.0065	0.0031	**0.0030**	0.0086	0.0063
15	0.0034	0.0031	0.0093	0.0072	0.0032	0.0030	00.90	0.0069	0.0033	**0.0029**	0.0089	0.0070
Ratio of thresholding Rules in numbers (%)												
		4 26.67 %				2 13.33 %				**9 60%**		
Ratio of wavelets in numbers(%)												
	4 26.67%				**2 13.33%**				**9 60%**			

Table 3: Selection of bestthresholding rule and wavelet function for de-noising the ECG based on RMS.

Bold letters indicates the best value between Thresholding rules and wavelet functions of 15- ECG signals.

The RMS performance on denoising ECG signals is given in Table 3 and it allows finding out the best thresholding rule (rigrsure) which is performing well over other thresholding rules and "coif5" wavelet function gives the best RMS rate while comparing with other three-wavelet functions.

Bold letters indicates the best value between Thresholding rules and wavelet functions of 15- ECG signals.

The rigrsure gives the minimum performance on all three wavelet functions "db20", "sym20" and "coif5". Based on the Pn value rigrsure of "coif5" wavelet is better. It’s shows that, the noise power is very less for thresholding rule rigrsure and wavelet function "coif5". The minimum noise power(Pn) and perfect morphology show the excellent de-noising performance.

Through, ECG morphology based analysis shows “coif5” wavelet function and “rigrsure” gives the excellent de-noising performance rather than other functions and rules.

Noise Power(Pn)												
	db20				sym20				coif5			
	Heursure	Rigrsure	Sqtwolog	minimaxi	Heursure	Rigrsure	Sqtwolog	minimaxi	Heursure	Rigrsure	Sqtwolog	Minimaxi
1	0.00008	0.00008	0.00190	0.00120	0.00007	0.00007	0.0015	0.0009	0.00007	**0.00007**	0.0014	0.00091
2	0.0002	0.0002	0.0026	0.0017	0.0002	0.0002	0.0020	0.0013	0.0001	**0.0001**	0.0019	0.0012
3	0.00005	0.00005	0.0009	0.0006	0.00006	0.00006	0.0008	0.0005	0.00004	**0.00004**	0.0008	0.0006
4	0.0001	0.0001	0.0022	0.0014	0.0002	0.0002	0.0018	0.0012	0.00006	**0.00006**	0.0018	0.0012
5	0.0009	0.0009	0.0144	0.0076	0.0002	**0.0002**	0.0097	0.0064	0.0004	0.0004	0.0110	0.0074
6	0.00007	0.00007	0.0010	0.0006	0.00006	**0.00006**	0.0008	0.0005	0.00006	0.00006	0.0008	0.0005
7	0.0018	0.0018	0.0047	0.0030	0.0014	0.0025	0.0041	0.0026	0.0004	**0.0004**	0.0037	0.0024
8	0.0001	0.0001	0.0025	0.0016	0.00009	0.00009	0.0018	0.0012	0.00007	**0.00007**	0.0017	0.0012
9	0.0001	0.0001	0.0014	0.00009	0.00009	0.00009	0.0011	0.0007	0.00007	**0.00007**	0.0011	0.0007
10	0.0009	0.0010	0.0032	0.0021	0.0001	**0.0001**	0.0025	0.0016	0.0001	0.0001	0.0023	0.0015
11	0.0002	0.0002	0.0019	0.0012	0.0001	0.0001	0.0014	0.0008	0.00006	**0.00006**	0.0013	0.0008
12	0.0004	0.0004	0.0041	0.0027	0.0002	0.0002	0.0032	0.0021	0.0001	**0.0001**	0.0032	0.0021
13	0.0001	**0.0001**	0.0028	0.0018	0.0001	0.00001	0.0022	0.0014	0.0001	0.0001	0.0021	0.0014
14	0.0001	0.0001	0.0026	0.0017	0.0001	0.00001	0.0024	0.0015	0.0001	**0.0001**	0.0022	0.0014
15	0.0001	**0.0001**	0.0035	0.0023	0.0006	0.00006	0.0028	0.0018	0.0001	0.0001	0.0026	0.0017
Ratio of thresholding Rules in numbers (%)												
		2 13.33%				3 20%				**10 66.67%**		
Ratio of wavelets in numbers (%)												
	2 13.33%				3 20%				**10 66.67%**			

Table 4: Selection of best thresholding rule and wavelet function for de-noising the ECG based on Pn.

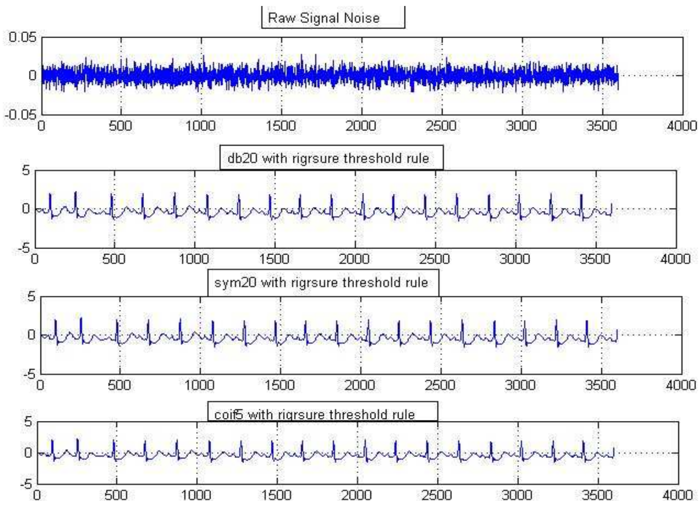

Figure 2: Shows the excellent de-noising performance between ECG morphologies.

6. Conclusion

The practical benefit of the wavelet based ECG signal analysis using DWT based denoising were devoted by using three wavelet function and four thresholding rules. The performance of denoising, four simple measures (SNR, PRD, RMS and Pn) were research and results are discussed. Overall, ECG morphology based analysis shows "coif5" wavelet function and "rigrsure" gives the excellent de-noising performance rather than other wavelet based on morphological characteristics. The conclusion can be drawn from the study of four simple measures (SNR, PRD, RMS &Pn) that the "coif5" wavelet and "rigrsure" threshold rule gives the best performance for ECG signal de-noising.Toimprove more suitable representation for other biological signals de-noising, it also a valuable in future study.

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