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On Edge Regular Bipolar Fuzzy Graphs

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Abstract. In this paper, we introduce the concepts of edge regular and totally edge regular bipolar fuzzy graphs. We determined necessary and sufficient condition under which edge regular bipolar fuzzy graph and totally edge regular bipolar fuzzy graph are equivalent. Some properties of edge regular bipolar fuzzy graphs are studied.

Keywords: Edge regular fuzzy graph, totally edge regular fuzzy graph, bipolar fuzzy set, bipolar fuzzy graph, regular bipolar fuzzy graph

AMS Mathematics Subject Classification (2010): 03E72, 05C72

1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. In 1994, Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are extension of fuzzy sets whose range of membership degree is [–1, 1] . In bipolar fuzzy set, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree within (0, 1] of an element indicates that the element somewhat satisfies the property, and the membership degree within [–1, 0) of an element indicates the element somewhat satisfies the implicit counter property [1]. For example, sweetness of foods is a bipolar fuzzy set. If sweetness of foods has been given as positive membership values then bitterness of foods is for negative membership values. Other tastes like salty, sour, pungent (e.g. chili), etc are irrelevant to the corresponding property. So these foods are taken as zero membership values [9]. Akram and Dudek introduced the concept of regular and totlly regular bipolar fuzzy graphs in 2011 [1]. In this paper, we discuss some theorems of edge regular bipolar fuzzy graphs through various examples. We provide a necessary and sufficient condition under which they are equivalent.

First we go through some basic definitions which can be found in [1–9].

2. Basic concepts

Let V be a non-empty finite set and $E \subseteq V \times V$. A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(x, y) \le \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The degree of a vertex x is $d_G(x) = \sum_{x \ne y} \mu(xy)$. The minimum degree of G is $\delta(G) = \wedge\{d_G(x), \forall\ x \in V\}$ and the maximum degree of G is $\Delta(G) = \vee\{d_G(x), \forall\ x \in V\}$. The total degree of a vertex $x \in V$ is defined by $td_G(x) = \sum_{x \ne y} \mu(xy) + \sigma(x)$. If each vertex in G has same degree k, then G is said to be a regular fuzzy graph or k – regular fuzzy graph. If each vertex in G has same total degree k, then G is said to be a totally regular fuzzy graph or k – totally regular fuzzy graph.

Let $G^*:(V,E)$ be a graph and let $e = uv$ be an edge in G^*. Then the degree of an edge $e = uv \in E$ is defined by $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$. If each edge in G^* has same degree, then G^* is said to be edge regular.

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The degree of an edge $xy \in E$ is $d_G(xy) = \sum_{x \ne z} \mu(xz) + \sum_{z \ne y} \mu(zy) - 2\mu(xy)$. The total degree of an edge $xy \in E$ is $td_G(xy) = \sum_{x \ne z} \mu(xz) + \sum_{z \ne y} \mu(zy) - \mu(xy)$. If each edge in G has same degree k, then G is said to be an edge regular fuzzy graph or k – edge regular fuzzy graph. If each edge in G has same total degree k, then G is said to be a totally edge regular fuzzy graph or k – totally edge regular fuzzy graph.

The order and size of a fuzzy graph $G:(\sigma,\mu)$ are defined by $O(G) = \sum_{x \in V} \sigma(x)$ and $S(G) = \sum_{xy \in E} \mu(xy)$. A fuzzy Graph $G:(\sigma,\mu)$ is strong, if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A fuzzy Graph $G:(\sigma,\mu)$ is complete, if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.

Let X be a non-empty set. A bipolar fuzzy set A in X is an object having the form $A = \{(x, \mu_A^P(x), \mu_A^N(x)) \mid x \in X\}$, where $\mu_A^P(x): X \to [0, 1]$ and $\mu_A^N(x): X \to [-1, 0]$ are mappings.

A bipolar fuzzy graph is a pair $G:(A, B)$ with an underlying graph $G^*:(V, E)$, where $A = (\sigma^P, \sigma^N)$ is a bipolar fuzzy set on V and $B = (\mu^P, \mu^N)$ is a bipolar fuzzy set on $E \subseteq V \times V$ such that $\mu^P(xy) \le \sigma^P(x) \wedge \sigma^P(y)$ and $\mu^N(xy) \ge \sigma^N(x) \vee \sigma^N(y)$ for all $xy \in E$. Here A is called bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E.

The order and size of a bipolar fuzzy graph $G:(A, B)$ are defined by $O(G)=\left(O^P(G), O^N(G)\right)$, where $O^P(G)=\sum_{x\in V}\sigma^P(x)$, $O^N(G)=\sum_{x\in V}\sigma^N(x)$ and $S(G)=\left(S^P(G), S^N(G)\right)$, where $S^P(G)=\sum_{xy\in E}\mu^P(xy)$, $S^N(G)=\sum_{xy\in E}\mu^N(xy)$. A bipolar fuzzy Graph $G:(A, B)$ is strong, if $\mu^P(xy)=\sigma^P(x)\wedge\sigma^P(y)$ and $\mu^N(xy)=\sigma^N(x)\vee\sigma^N(y)$ for all $xy\in E$. A bipolar fuzzy Graph $G:(A, B)$ is complete, if $\mu^P(xy)=\sigma^P(x)\wedge\sigma^P(y)$ and $\mu^N(xy)=\sigma^N(x)\vee\sigma^N(y)$ for all $x, y\in V$.

3. Edge regular bipolar fuzzy graphs

3.1. Edge regular bipolar fuzzy graph

Let $G:(A, B)$ be a bipolar fuzzy graph on $G^*:(V, E)$. If all the edges have the same degree (k_1, k_2), then G is called a (k_1, k_2) – edge regular bipolar fuzzy graph. The degree of an edge $e=xy$ in G is defined by $d_G(xy)=(d_G^P(xy), d_G^N(xy))$, where $d_G^P(xy)=\sum_{\substack{zx\in E\\ z\neq y}}\mu^P(zx)+\sum_{\substack{yz\in E\\ z\neq x}}\mu^P(yz)=\sum_{zx\in E}\mu^P(zx)+\sum_{yz\in E}\mu^P(yz)-2\mu^P(xy)$ and

$$d_G^N(xy)=\sum_{\substack{zx\in E\\ z\neq y}}\mu^N(zx)+\sum_{\substack{yz\in E\\ z\neq x}}\mu^N(yz)=\sum_{zx\in E}\mu^N(zx)+\sum_{yz\in E}\mu^N(yz)-2\mu^N(xy).$$

3.2. Totally edge regular bipolar fuzzy graph

Let $G:(A, B)$ be a bipolar fuzzy graph on $G^*:(V, E)$. If all the edges have the same total degree, then G is called a totally edge regular bipolar fuzzy graph. The total degree of an edge $e=xy$ in G is defined by $td_G(xy)=(td_G^P(xy), td_G^N(xy))$, where $td_G^P(xy)=d_G^P(xy)+\mu^P(xy)$ and $td_G^N(xy)=d_G^N(xy)+\mu^N(xy)$.

Remark 3.3.

1. G is a (k_1, k_2) – edge regular fuzzy graph if and only if ${}^B\delta_E(G)={}^B\Delta_E(G)=(k_1, k_2)$, where ${}^B\delta_E(G)$ is the minimum total edge degree of G and ${}^B\Delta_E(G)$ is the maximum total edge degree of G.
2. G is a (k_1, k_2) – totally edge regular fuzzy graph if and only if ${}^B\delta_{tE}(G)={}^B\Delta_{tE}(G)=(k_1, k_2)$, where ${}^B\delta_{tE}(G)$ is the minimum total edge degree of G and ${}^B\Delta_{tE}(G)$ is the maximum total edge degree of G.

Remark 3.4. In crisp graph theory, any complete graph is edge regular. But this result does not carry over to the fuzzy case. A complete bipolar fuzzy graph need not be edge regular. For example, in fig.3.2, G is not edge regular, but it is a complete bipolar fuzzy graph.

Example 3.5.

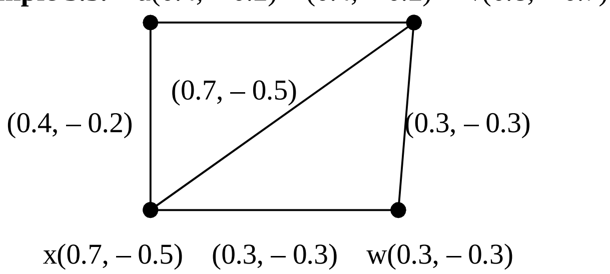

Figure 3.1:

In the above figure 3.1, G is (1.4, – 1.0) – edge regular bipolar fuzzy graph, but G is not a totally edge regular bipolar fuzzy graph.

Example 3.6. Consider the following bipolar fuzzy graph $G:(A,\ B)$.

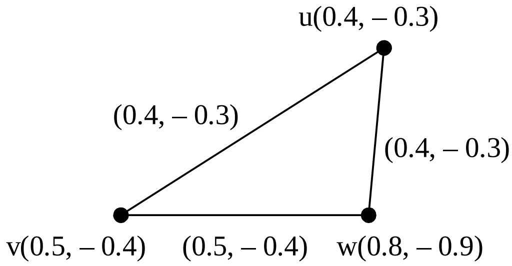

Figure 3.2:

Here, G is a (1.3, – 1.0) – totally edge regular bipolar fuzzy graph, but not an edge regular bipolar fuzzy graph.

Example 3.7. Consider the following fuzzy graph $G:(A,\ B)$.

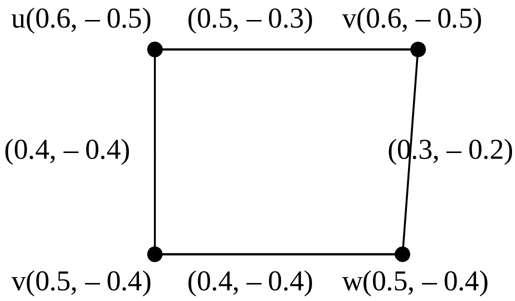

Figure 3.3:

Here, G is neither edge regular bipolar fuzzy graph nor totally edge regular bipolar fuzzy graph.

Example 3.8. Consider the following fuzzy graph $G:(A, B)$.

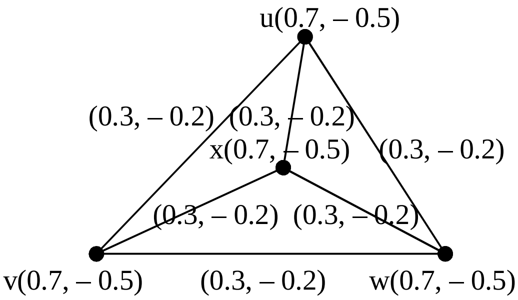

Figure 3.4:

In the figure 3.4, G is both edge regular bipolar fuzzy graph and totally edge regular bipolar fuzzy graph.

Example 3.9.

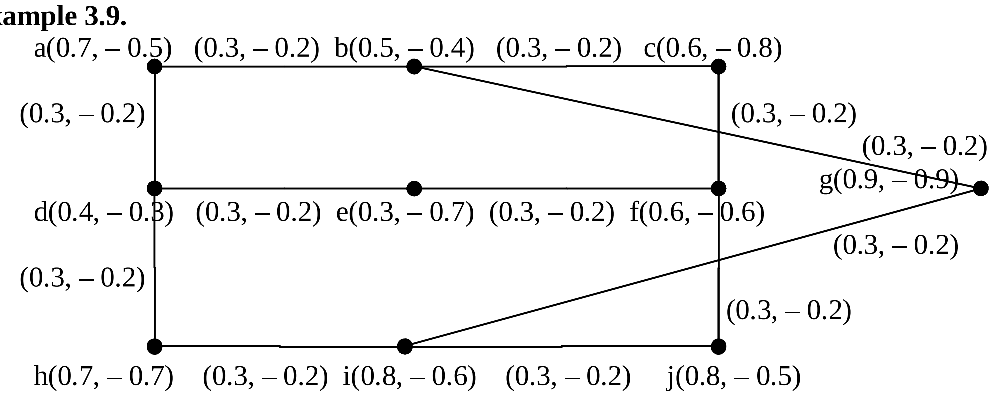

Figure 3.5:

Here, G is both (0.9, – 0.6) – edge regular bipolar fuzzy graph and (1.2, – 0.8) – totally edge regular bipolar fuzzy graph.

3.10. Equally edge regular bipolar fuzzy graph

Let $G:(A, B)$ be a bipolar fuzzy graph on $G^*:(V, E)$. If all the edges have the same open neighbourhood degree (k_1, k_2) with $k_1 = |k_2|$, then G is called an equally edge regular bipolar fuzzy graph. Otherwise it is unequally edge regular bipolar fuzzy graph.

Remark 3.11. From the above examples, it is clear that in general there does not exist any relationship between edge regular bipolar fuzzy graphs and totally edge regular bipolar fuzzy graphs. However, a necessary and sufficient condition under which two types of bipolar fuzzy graphs, edge regular bipolar fuzzy graphs and totally edge regular bipolar fuzzy graphs are equivalent in some particular case is provided in the following theorem.

Theorem 3.12. Let $G:(A,\, B)$ be a bipolar fuzzy graph on $G^* = (V,\, E)$. Then B is a constant function if and only if the following are equivalent:
(1). G is an edge regular bipolar fuzzy graph.
(2). G is a totally edge regular bipolar fuzzy graph.
Proof: Suppose that B is a constant function.

Let $B = (\mu^P(uv), \mu^N(uv)) = (c_1, c_2)$, for every $uv \in E$, where c_1 and c_2 are constant.
Assume that G is a (k_1, k_2) – edge regular bipolar fuzzy graph. Then
$d_G(uv) = (d_G^P(uv), d_G^N(uv)) = (k_1, k_2)$, for all $uv \in E$.

$\therefore \quad td_G(uv) = (td_G^P(uv), td_G^N(uv))$, for all $uv \in E$.

$= (d_G^P(uv) + \mu^P(uv), d_G^N(uv) + \mu^N(uv))$

$\Rightarrow \quad td_G(uv) = (k_1 + c_1, k_2 + c_2)$, for all $uv \in E$.

Hence G is a $(k_1 + c_1, k_2 + c_2)$ – totally edge regular bipolar fuzzy graph.
Thus (1) $\Rightarrow$ (2) is proved.
Now, suppose that G is a (m_1, m_2) – totally edge regular bipolar fuzzy graph.

Then $td_G(uv) = (m_1, m_2)$, for all $uv \in E$.

$\Rightarrow \quad td_G^P(uv) = m_1,\; td_G^N(uv) = m_2$, for all $uv \in E$.

$\Rightarrow \quad d_G^P(uv) + \mu^P(uv) = m_1,\; d_G^N(uv) + \mu^N(uv) = m_2$, for all $uv \in E$.

$\Rightarrow \quad d_G^P(uv) = m_1 - c_1,\; d_G^N(uv) = m_2 - c_2$, for all $uv \in E$.

Hence G is a $(m_1 - c_1, m_2 - c_2)$ – edge regular bipolar fuzzy graph.
Thus (2) $\Rightarrow$ (1) is proved.
Hence (1) and (2) are equivalent.
Conversely, assume that (1) and (2) are equivalent.
i.e., G is edge regular if and only if G is totally edge regular.
To prove that B is a constant function.
Suppose B is not a constant function.
Then $\mu^P(uv) \neq \mu^P(xy)$ and $\mu^N(uv) \neq \mu^N(xy)$ for atleast one pair of edges $uv, xy \in E$.

Let G be a (k_1, k_2) – edge regular bipolar fuzzy graph.

Then $d_G(uv) = d_G(xy) = (k_1, k_2)$. Therefore,

$td_G(uv) = (d_G^P(uv) + \mu^P(uv), d_G^N(uv) + \mu^N(uv)) = (k_1 + \mu^P(uv), k_2 + \mu^N(uv))$ and
$td_G(xy) = (d_G^P(xy) + \mu^P(xy), d_G^N(xy) + \mu^N(xy)) = (k_1 + \mu^P(xy), k_2 + \mu^N(xy))$.

Since $\mu^P(uv) \neq \mu^P(xy)$ and $\mu^N(uv) \neq \mu^N(xy)$, we have

$td_G(uv) \neq td_G(xy)$.

Hence G is not a totally edge regular, which is a contradiction to our assumption.
Now, let G be a totally edge regular bipolar fuzzy graph.

Then $td_G(uv) = td_G(xy)$.

$$(d_G^P(uv)+\mu^P(uv), d_G^N(uv)+\mu^N(uv)) = (d_G^P(xy)+\mu^P(xy), d_G^N(xy)+\mu^N(xy)).$$

$\Rightarrow d_G^P(uv)+\mu^P(uv) = d_G^P(xy)+\mu^P(xy)$ and $d_G^N(uv)+\mu^N(uv) = d_G^N(xy)+\mu^N(xy)$.

$\Rightarrow d_G^P(uv)-d_G^P(xy) = \mu^P(xy)-\mu^P(uv)$ and $d_G^N(uv)-d_G^N(xy) = \mu^N(xy)-\mu^N(uv)$.

$\Rightarrow d_G^P(uv)-d_G^P(xy) \neq 0$ and $d_G^N(uv)-d_G^N(xy) \neq 0$, since $\mu^P(uv) \neq \mu^P(xy)$ and $\mu^N(uv) \neq \mu^N(xy)$.

$\Rightarrow \quad d_G^P(uv) \neq d_G^P(xy)$ and $d_G^N(uv) \neq d_G^N(xy)$.

$\Rightarrow \quad d_G(uv) \neq d_G(xy)$.

Thus G is not an edge regular bipolar fuzzy graph.
This is, a contradiction to our assumption.
Hence, B is a constant function.

Theorem 3.13. If a bipolar fuzzy graph G is both edge regular and totally edge regular, then B is a constant function.

Proof: Let G be a (k_1, k_2) – edge regular and (m_1, m_2) – totally edge regular bipolar fuzzy graph.

Then $d_G(uv) = (k_1, k_2)$, for all $uv \in E$ and $td_G(uv) = (m_1, m_2)$, for all $uv \in E$.

Now, $\quad td_G(uv) = (m_1, m_2)$, for all $uv \in E$.

$\Rightarrow \quad (d_G^P(uv)+\mu^P(uv), d_G^N(uv)+\mu^N(uv)) = (m_1, m_2)$, for all $uv \in E$.

Since $d_G(uv) = (d_G^P(uv), d_G^N(uv)) = (k_1, k_2)$, for all $uv \in E$,

$(k_1+\mu^P(uv), k_2+\mu^N(uv)) = (m_1, m_2)$, for all $uv \in E$.

$\Rightarrow \quad k_1+\mu^P(uv) = m_1, k_2+\mu^N(uv) = m_2$, for all $uv \in E$.

$\Rightarrow \quad \mu^P(uv) = m_1-k_1, \mu^N(uv) = m_2-k_2$, for all $uv \in E$.

$\Rightarrow \quad B = (\mu^P(uv), \mu^N(uv)) = (m_1-k_1, m_2-k_2)$, for all $uv \in E$.

Hence B is a constant function.

Remark 3.14. The converse of theorem 3.13 need not be true. It can be seen from the following example.

Consider the bipolar fuzzy graph $G:(A, B)$ given in figure 3.6.

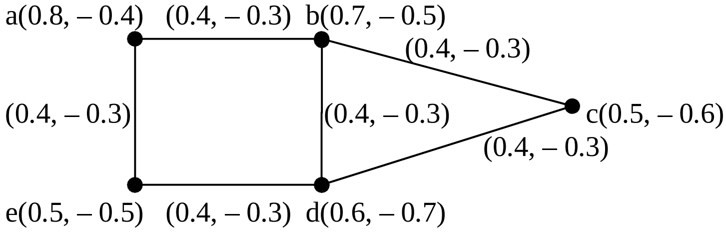

Figure 3.6:

Here, B is a constant function, but G is not an edge regular and also not a totally edge regular bipolar fuzzy graph.

Theorem 3.15. Let $G:(A, B)$ be a fuzzy graph on $G^* = (V, E)$. If B is a constant function, then G is edge regular if and only if G^* is edge regular.

Proof: Given that B is a constant function.

Let $B = (\mu^P(xy), \mu^N(xy)) = (c_1, c_2)$, for all $xy \in E$, where c_1 and c_2 are constants.

Assume that G is edge regular.

Let $d_G(xy) = (d_G^P(xy), d_G^N(xy)) = (k_1, k_2)$, for all $xy \in E$.

Then $d_G^P(xy) = k_1$, for all $xy \in E$.

$$\Rightarrow \sum_{zx\in E} \mu^P(zx) + \sum_{yz\in E} \mu^P(yz) - 2\mu^P(xy) = k_1, \text{ for all } xy \in E.$$

$$\Rightarrow \sum_{zx\in E} c_1 + \sum_{yz\in E} c_1 - 2c_1 = k_1, \text{ for all } xy \in E.$$

$$\Rightarrow c_1 d_{G^*}(x) + c_1 d_{G^*}(y) - 2c_1 = k_1, \text{ for all } xy \in E.$$

$$\Rightarrow c_1(d_{G^*}(x) + d_{G^*}(y) - 2) = k_1, \text{ for all } xy \in E.$$

$$\Rightarrow c_1 d_{G^*}(xy) = k_1, \text{ for all } xy \in E.$$

$$\Rightarrow d_{G^*}(xy) = \frac{k_1}{c_1}, \text{ for all } xy \in E.$$

Hence, G^* is edge regular.

Conversely, assume that G^* is edge regular.

Let $d_{G^*}(xy) = m$, for all $xy \in E$, where m is a constant.

Consider $d_G^P(xy) = \sum_{zx\in E} \mu^P(zx) + \sum_{yz\in E} \mu^P(yz) - 2\mu^P(xy)$, for all $xy \in E$.

$$\begin{aligned} d_G^P(xy) &= \sum_{zx\in E} c_1 + \sum_{yz\in E} c_1 - 2c_1 \\ &= c_1 d_{G^*}(x) + c_1 d_{G^*}(y) - 2c_1 \\ &= c_1(d_{G^*}(x) + d_{G^*}(y) - 2) \\ &= c_1 d_{G^*}(xy) \end{aligned}$$

$$\Rightarrow d_G^P(xy) = mc_1, \text{ for all } xy \in E.$$

Now, $d_G^N(xy) = \sum_{zx\in E} \mu^N(zx) + \sum_{yz\in E} \mu^N(yz) - 2\mu^N(xy)$, for all $xy \in E$.

$$\begin{aligned} d_G^N(xy) &= \sum_{zx\in E} c_2 + \sum_{yz\in E} c_2 - 2c_2 \\ &= c_2 d_{G^*}(x) + c_2 d_{G^*}(y) - 2c_2 \\ &= c_2(d_{G^*}(x) + d_{G^*}(y) - 2) \\ &= c_2 d_{G^*}(xy) \end{aligned}$$

$$\Rightarrow d_G^N(xy) = mc_2, \text{ for all } xy \in E.$$

Thus $d_G(xy) = (d_G^P(xy), d_G^N(xy)) = (mc_1, mc_2)$, for all $xy \in E$.

Hence, G is edge regular.

Remark 3.16. The above theorem 3.15 need not be true, when B is not a constant function.

In the following figure 3.7, G^* is edge regular, but G is not an edge regular and also B is not a constant function.

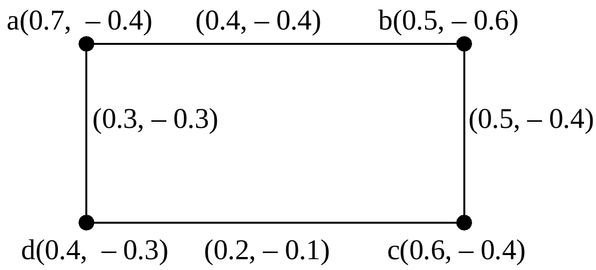

Figure 3.7:

4. Properties of edge regular fuzzy graphs

Theorem 4.1. Let $G = (A, B)$ be a bipolar fuzzy graph on G*: (V, E). Then $\sum_{uv \in E} d_G^P(uv) = \sum_{uv \in E} d_{G^*}(uv)\mu^P(uv)$ and $\sum_{uv \in E} d_G^N(uv) = \sum_{uv \in E} d_{G^*}(uv)\mu^N(uv)$ where $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$, for all u, v $\in$ V.

Proof: By the definition of edge degree in a fuzzy graph, edge degree is sum of the membership values of its adjacent edges.

Therefore, in $\sum_{uv \in E} d_G^P(uv)$, every edge contributes its positive membership value exactly the number of edges adjacent to that edge times.

Thus, in $\sum_{uv \in E} d_G^P(uv)$, each $\mu^P(uv)$ appears $d_{G^*}(uv)$ times and these are the only values in that sum.

Hence $\sum_{uv \in E} d_G^P(uv) = \sum_{uv \in E} d_{G^*}(uv)\mu^P(uv)$.

Similarly, $\sum_{uv \in E} d_G^N(uv) = \sum_{uv \in E} d_{G^*}(uv)\mu^N(uv)$.

Theorem 4.2. Let $G = (A, B)$ be a bipolar fuzzy graph on G*: (V, E). Then $\sum_{uv \in E} td_G^P(uv) = \sum_{uv \in E} d_{G^*}(uv)\mu^P(uv) + S^P(G)$ and

$\sum_{uv \in E} td_G^N(uv) = \sum_{uv \in E} d_{G^*}(uv)\mu^N(uv) + S^N(G)$.

Proof: We know that $td_G(uv) = d_G(uv) + \mu(uv)$.

Therefore $\sum_{uv\in E} td_G^P(uv) = \sum_{uv\in E}\left(d_G^P(uv)+\mu^P(uv)\right)$

$$= \sum_{uv\in E} d_G^P(uv) + \sum_{uv\in E}\mu^P(uv)$$

$\sum_{uv\in E} td_G^P(uv) = \sum_{uv\in E} d_{G^*}(uv)\mu^P(uv) + S^P(G)$. (by theorem 4.1)

Similarly, $\sum_{uv\in E} td_G^N(uv) = \sum_{uv\in E} d_{G^*}(uv)\mu^N(uv) + S^N(G)$.

Theorem 4.3. The size of a (k_1, k_2) – edge regular bipolar fuzzy graph $G:(A,\ B)$ on a k – edge regular crisp graph $G^*:(V,\ E)$ is $\left(\frac{qk_1}{k}, \frac{qk_2}{k}\right)$, where $q = |E|$.

Proof: Since G is (k_1, k_2) – edge regular bipolar fuzzy graph and $G^*:(V,\ E)$ is k – edge regular, $d_G(xy) = (d_G^P(xy), d_G^N(xy)) = (k_1, k_2)$ and $d_{G^*}(xy) = k$, for all $xy\in E$.

Thus $\sum_{xy\in E} d_G^P(xy) = \sum_{xy\in E} d_{G^*}(xy)\mu^P(xy)$, (by theorem 4.1)

$\Rightarrow \quad \sum_{xy\in E} k_1 = k\sum_{xy\in E}\mu^P(xy)$.

$\Rightarrow \quad qk_1 = kS^P(G)$. (since $q = |E|$)

Therefore $S^P(G) = \frac{qk_1}{k}$.

Similarly, $S^N(G) = \frac{qk_2}{k}$.

Hence, $S(G) = (S^P(G), S^N(G)) = \left(\frac{qk_1}{k}, \frac{qk_2}{k}\right)$.

Theorem 4.4. The size of a (m_1, m_2) – totally edge regular bipolar fuzzy graph $G:(A,\ B)$ on a k – edge regular crisp graph $G^*:(V,\ E)$ is $\left(\frac{qm_1}{k+1}, \frac{qm_2}{k+1}\right)$, where $q = |E|$.

Proof: Since G is (m_1, m_2) – totally edge regular bipolar fuzzy graph and $G^*:(V,\ E)$ is k – edge regular, $td_G(xy) = (td_G^P(xy), td_G^N(xy)) = (m_1, m_2)$ and $d_{G^*}(xy) = k$, for all $xy\in E$.

Thus $\sum_{xy\in E} td_G^P(xy) = \sum_{xy\in E} d_{G^*}(xy)\mu^P(xy) + S^P(G)$, (by theorem 4.2)

$\Rightarrow \quad \sum_{xy\in E} m_1 = k\sum_{xy\in E}\mu^P(xy) + S^P(G)$.

$$\Rightarrow \qquad qm_1 = kS^P(G) + S^P(G). \text{ (since } q = |E| \text{)}$$

Therefore $S^P(G) = \dfrac{qm_1}{k+1}$.

Similarly, $S^N(G) = \dfrac{qm_2}{k+1}$.

Hence, $S(G) = \left(\dfrac{qm_1}{k+1}, \dfrac{qm_2}{k+1}\right)$.

REFERENCES

1. M.Akram and W.Dudek, Regular bipolar fuzzy graphs, *Neural Computing and Applications*, 21(1) (2011) 197 – 205.
2. F.Harary, *Graph Thoery,* Narosa /Addison Wesley, Indian Student Edition, 1988.
3. A.Nagoor Gani and J.Malarvizhi, Isomorphism on fuzzy graphs, *International Journal of Computational and Mathematical Sciences*, 2(4) (2008) 190–196.
4. A.Nagoorgani and J.Malarvizhi, Properties of μ-complement of a fuzzy graph, *International Journal of Algorithms, Computing and Mathematics*, 2(3) (2009) 73–83.
5. A.Nagoor Gani and K.Radha, On regular fuzzy graphs, *Journal of Physical Sciences*, 12 (2008) 33–40.
6. A.Nagoor Gani and K.Radha, Some sequences in fuzzy graphs, *Far East Journal of Applied Mathematics*, 31(3) (2008) 321–335.
7. K.Radha and N.Kumaravel, On edge regular fuzzy graphs, *International Journal of Mathematical Archive*, 5(9) (2014) 100–112.
8. K.Radha and N.Kumaravel, Some properties of edge regular fuzzy graphs, *Jamal Academic Research Journal* (JARJ), Special issue, 2014, 121 – 127.
9. S.Samanta and M.Pal, Irregular bipolar fuzzy graphs, *Inernational Journal of Applications of Fuzzy Sets*, 2(2012) 91–102.
10. S.Samanta and M.Pal, Some more results on bipolar fuzzy sets and bipolar fuzzy intersection graphs, *The Journal of Fuzzy Mathematics,* 22 (2) (2014) 253–262.