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ij-Generalized Delta Semi Closed Sets

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Abstract. The aim of this communication is to introduce the concepts of $ij -$ generelized δ - semi closed sets and $ij -$ generelized δ - semi open sets and study their fundamental basic properties. Also we define $ij - g\delta$s continuous functions, $ij - g\delta$s irresolute functions in bitopological spaces and investigate some of their properties. Furthermore by using this set, we introduce and define $ij\text{–}T_{g\delta s}$, $ij - g\delta$s $T_{1/2}$ bitopological spaces. Also we study some properties of $ij - g\delta$s closure and interior operators.

Keywords: $ij - \delta$ open set, $ij - \delta$ semi open set, $ij - g\delta$s closed set, $ij - g\delta$s continuous function, $ij - g\delta$s irresolute function, $ij - g\delta$s closure, $ij - g\delta$s interior, $ij\text{–}T_{g\delta s}$ space, $ij - g\delta$s $T_{1/2}$ space.

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1. Introduction

In 1987, Banerjee [2] introduced the notion δ – open sets in bitopological space. Khedr[9], introduced and study more about $ij - \delta$ open sets. Using $ij - \delta$ open, Edward Samuel and Balan introduced one kind of $ij - \delta$ semi open sets in bitopological space and investigated some of their properties in [6]. In 1990, Arya and Nour [1] defined generalized semi closed sets. Generalized closed and generalized semi closed sets are independent notions. In this paper, we introduce the notion of $ij -$ Generalized δ - semi closed sets in bitopological spaces. Moreover the fundamental properties of this new concept will be studied. As application of $ij -$ Generalized δ - semi closed sets, we introduce and study some notions like $ij - g\delta$s closure, $ij - g\delta$s interior and $ij\text{–}T_{g\delta s}$, $ij - g\delta$s $T_{1/2}$ spaces.

2. Preliminaries

Throughout the present paper, (X, τ_1, τ_2) (or briefly X) always mean a bitopological space on which no separation axioms are assumed unless explicitly stated. Also $i, j = 1, 2$ and $i \neq j$. Let A be a subset of (X, τ_1, τ_2). By $i - Int(A)$ and $i - Cl(A)$, we mean respectively the interior and the closure of A in the topological space (X, τ_i) for $i = 1, 2$.

A subset A of X is called $ij-$ semi open[8,9] (respectively $ij-\alpha$ open, $ij-$ regular open) if $A \subseteq j-cl[i-\ int(A)]$ (respectively $A \subseteq i-\ int[j-cl[i-\ int(A)]]$, $A \subseteq i-\ int[j-cl(A)]$). A point x of X is called an $ij-\ \delta-$ cluster point of A if $i-Int(j-Cl(U))\ \cap\ A\ \neq \emptyset$ for every τ_i-open set U containing x. The set of all $ij-\ \delta-$ cluster points of A is called the $ij-\ \delta-$ closure of A and is denoted by $ij-\delta Cl(A)$.

Definition 2.1. [9] A subset A is said to be $ij-\ \delta$ closed if $ij-\delta Cl(A)=A$. The complement of an $ij-\ \delta$ closed set is said to be $ij-\ \delta$ open. The set of all $ij-\ \delta$ open (respectively $ij-\ \delta$ closed) sets of X will be denoted by $ij-\ \delta$O(X)(respectively $ij-\ \delta$C(X)).

Definition 2.2. [6] A subset A of a bitopological space (X,τ_1,τ_2) is called $ij-\ \delta$ semi open if there exists an $ij-\ \delta$ open set U such that $U \subseteq A \subseteq j-Cl(U)$. Complement of $ij-\ \delta$ semi open is called $ij-\ \delta$ semi closed. The family of $ij-\ \delta$ semi open (respectively $ij-\ \delta$ semi closed) set of X is denoted by $ij-\ \delta SO(X)$ (respectively $ij-\ \delta SC(X)$).

Recall that, arbitrary union of $cl(A_i), i\ \in\ I$ is contained in closure of arbitrary union of subsets A_i in any topological space. The equality holds if the collection $\{A_i, i\ \in\ I\}$ is locally finite.

3. ij – Generalized δ - semi closed sets

Definition 3.1. A subset A of a bitopological space (X,τ_1,τ_2) is called $ij-$generalized δ - semi closed (ij– $g\delta s$ closed) if $ji-scl(A) \subseteq U$ whenever $A\ \subseteq U$ and U is $ij-\delta$ open in X.

Example 3.1. Let $X=\{a,b,c,d\}$, $\tau_1=\{\emptyset,X,\{a\},\{a,b\},\{a,c,d\}\}$, $\tau_2=\{\emptyset,X,\{a\},\{d\}, \{a,d\}\}$. Then 12 – $g\delta$s closed sets and 21 – $g\delta$s closed sets are P(X).

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Theorem 3.1. Let (X,τ_1,τ_2) be a bitopological space and $A \subset X$. Then the following are true,

(a) If A is τ_j- closed set, then A is ij– $g\delta s$ closed.

(b) If A is $ji-$ semi closed set, then A is ij– $g\delta s$ closed.

Proof. (a) Suppose that A is τ_j- closed set in a bitopological space (X,τ_1,τ_2). Let $A \subseteq U$ and U is an $ij-\delta$ open in X. Since A is τ_j- closed, we have $j-\mathrm{cl(A)}=\mathrm{A} \subseteq U$. Then $ji-\mathrm{scl(A)} \subseteq j-\mathrm{cl(A)}=\mathrm{A} \subseteq U$. Therefore A is ij– $g\delta s$ closed.

(b) Suppose that A is $ji-$ semi closed, then $ji-s\mathrm{cl(A)}=\mathrm{A} \subseteq U$ and U is an $ij-\delta$ open in X. Therefore A is ij– $g\delta s$ semi closed.

Theorem 3.2. If A is ij– $g\delta s$ closed set in a bitopological space (X,τ_1,τ_2) and $A \subseteq B \subseteq ji-\mathrm{scl(A)}$, then B is ij– $g\delta s$ closed.

Proof. Suppose that A is ij– $g\delta s$ closed set in a bitopological space (X,τ_1,τ_2) and $A \subseteq B \subseteq ji-$ scl(A). Let $B \subseteq U$ and U is an $ij-\delta$ open in X. Since $A \subseteq B$ and $B \subseteq U$, we have $\mathrm{A} \subseteq U$. Since A is ij– $g\delta s$ closed set, then $ji-$ scl(A) $\subseteq U$. Also since

$B \subseteq ji -$ scl(A), then ji $-$ scl(B) $\subseteq$ ji $-$ scl[ji $-$ scl(A)] $\subseteq$ ji $-$ scl(A) $\subseteq$ U. Therefore B is $ij\text{–}\, g\delta s$ closed.

Preposition 3.1. If A and B are $ij\text{–}\, g\delta s$ closed sets in a bitopological space (X, τ_1, τ_2), then $A \cup B$ is also a $ij\text{–}\, g\delta s$ closed set.

Proof. Suppose A and B are $ij\text{–}\, g\delta s$ closed sets in a bitopological space (X, τ_1, τ_2). Let U be a $ij - \delta$ open in X and $A \cup B \subseteq U$. Since $A \cup B \subseteq U$, we have $A \subseteq U$ and $B \subseteq U$. Since U is $ij - \delta$ open in X and A and B are $ij\text{–}\, g\delta s$ closed sets, we have $ji -$ scl(A) $\subseteq U$ and $ji -$ scl(B) $\subseteq U$. Therefore $[\, ji -$ scl(A)$] \cup [ji -$ scl(B)$] \subseteq U \cup U$. This implies $ji -$ scl($A \cup$ B) $\subseteq U$. Hence $A \cup B$ is also a $ij\text{–}\, g\delta s$ closed set.

Preposition 3.2. If A and B are $ij\text{–}\, g\delta s$ closed sets in a bitopological space (X, τ_1, τ_2), then $A \cap B$ is also a $ij\text{–}\, g\delta s$ closed set.

Proof. Suppose A and B are $ij\text{–}\, g\delta s$ closed sets in a bitopological space (X, τ_1, τ_2). Let U be a $ij - \delta$ open in X and $A \cap B \subseteq U$. Since $A \cap B \subseteq U$, we have $A \subseteq U$ and $B \subseteq U$. Since U is $ij - \delta$ open in X and A and B are $ij\text{–}\, g\delta s$ closed sets, we have $ji -$ scl(A) $\subseteq U$ and $ji -$ scl(B) $\subseteq U$. Therefore $[ji -$ scl(A)$] \cap [ji -$ scl(B)$] \subseteq U \cap U$. This implies $ji -$ scl($A \cap$ B) $\subseteq U$. Hence $A \cap B$ is also a $ij\text{–}\, g\delta s$ closed set.

Theorem 3.3. The arbitrary union of $ij\text{–}\, g\delta s$ closed sets $\{A_i,\ i \in I\}$ in a bitopological space (X, τ_1, τ_2) is $ij\text{–}\, g\delta s$ closed if the family $\{A_i, i \in I\}$ is τ_j– locally finite.

Proof. Let $\{A_i, i \in I\}$ be τ_j– locally finite and A_i is $ij\text{–}\, g\delta s$ closed in X for each $i \in I$. Let $\bigcup A_i \subseteq U$ and U is $ij - \delta$ open in X. Then $A_i \subseteq U$ and U is $ij - \delta$ open in X for each i. Since A_i is $ij\text{–}\, g\delta s$ closed in X for each $i \in I$, we have $ji - scl(A_i) \subseteq$ U. Consequently, $\bigcup\big[\, ji - scl(A_i)\big] \subseteq$ U. Since the family $\{A_i, i \in I\}$ is τ_j–locally finite, $ji - scl[\bigcup(A_i)] = \bigcup\big[\, ji - scl(A_i)\big] \subseteq U$. Therefore, $\bigcup A_i$ is $ij\text{–}\, g\delta s$ closed in X.

Theorem 3.4. Let $B \subset A \subset X$ where A is $ij - \delta$ open and $ij\text{–}\, g\delta s$ closed in X. Then B is $ij\text{–}\, g\delta s$ closed relative to A if and only if B is $ij\text{–}\, g\delta s$ closed relative to X.

Proof. Suppose that $B \subset A \subset X$ where A is $ij - \delta$ open and $ij\text{–}\, g\delta s$ closed in X. Suppose that B is $ij\text{–}\, g\delta s$ closed relative to A. Let $B \subseteq U$, U is $ij - \delta$ open in X. Since $A \subset X$, A is $ij - \delta$ open we have $A \cap U$ is $ij - \delta$ open in X. Then $A \cap U$ is $ij - \delta$ open in A. Since $\subset A$, $B \subset U$, we have $B \subset A \cap U$. Then $ji - scl(B_A) \subseteq A \cap U$, Since B is $ij\text{–}\, g\delta s$ closed relative to A. This implies that $ji - scl(B_A) \subseteq U$. Since A is $ij - \delta$ open, A is $ij\text{–}\, \delta$sg closed in X. This implies that $ji - scl(B_A) = ji - scl(B) \cap A = ji - scl(B) \subseteq U$, since $ji - sc(B) \subseteq A$. Therefore B is $ij\text{–}\, g\delta s$ closed relative to X.

Conversely, Suppose that B is $ij\text{–}\, g\delta s$ closed relative to X. Let $B \subset U$ and U is $ij\text{–}\, \delta$ open in A. Since $A \subset X$, we have U is $ij\text{–}\, \delta$ open in X. This implies that $ji - scl(B) \subseteq U$. Now $ji - scl(B_A) = ji - scl(B) \cap A = ji - scl(B) \subseteq U$. Therefore B is $ij\text{–}\, g\delta s$ closed relative to A.

Theorem 3.5. A set A be $ij-g\delta s$ closed in X if and only if $ji-scl(A)-A$ contains no non-empty $ij-\delta$ closed set.

Proof. Suppose that A is $ij-g\delta s$ closed in X. Let F be $ij-\delta$ closed and $F \subseteq ji-scl(A)-A$. Since F be $ij-\delta$ closed, we have $F^C$is $ij-\delta$ open. Since $F \subseteq ji-scl(A)-A$, we have $\{ji-scl(A)-A\}^C \subseteq F^C$. Then $A \subseteq F^C$. Also since A is $ij-g\delta s$ closed in X, we have $ji-scl(A) \subseteq F^C$. This implies that $\{F^C\}^C \subseteq \{ji-scl(A)\}^C = ji-scl(A^C)$. Then $F \subseteq ji-scl(A^C)$. Also since $F \subseteq ji-scl(A)-A$, we have $F \subseteq ji-scl(A)$. This implies that $F \cap F \subseteq \{ji-scl(A^C)\} \cap \{ji-scl(A)\} = ji-scl(A^C \cap A) = ji-scl(\phi)$. This implies $F \subseteq \phi$. Hence $ji-scl(A)-A$ contains no non-empty $ij-\delta$ closed set.

Conversely, Suppose that $ji-scl(A)-A$ contains no non-empty $ij-\delta$ closed set. Let $A \subseteq U$ and U is $ij-\delta$ open in X. Suppose that $ji-scl(A) \nsubseteq U$. Then $ji-scl(A) \cap U^C \neq \phi$. Since $A \subseteq U$, we have $U^C \subseteq A^C$. Then $ji-scl(A) \cap U^C \subseteq ji-scl(A) \cap A^C = ji-scl(A)-A$. Since is $ij-\delta$ open in X, we have U^C is $ij-\delta$ closed in X. Then $ji-scl(A) \cap U^C$is $ij-\delta$ closed in X. Which is contradiction, therefore $ji-scl(A) \subseteq U$. Hence A is $ij-g\delta s$ closed in X.

Theorem 3.6. A set A be $ij-g\delta s$ closed in X. Then A is $ji-$semi closed if and only if $ji-scl(A)-A$ is $ij-\delta$ closed set.

Proof. Suppose that A is $ij-g\delta s$ closed in X and τ_j-semi closed. Since A is $ji-$semi closed, we have $ji-scl(A)=A$. Then $ji-scl(A)-A=\phi$ is $ij-\delta$ closed.

Conversely, Suppose that A is $ij-g\delta s$ closed and $ji-scl(A)-A$ is $ij-\delta$ closed. Since A is $ij-g\delta s$ closed, we have by theorem 3.5, $ji-scl(A)-A$ contains no non-empty $ij-\delta$ closed set. Since $ji-scl(A)-A$ is itself $ij-\delta$ closed, we have $ji-scl(A)-A=\phi$. Then $ji-scl(A)=A$. Hence A is $ji-$semi closed.

Theorem 3.7. If A is $ij-g\delta s$ closed in X and $A \subset B \subset ji-scl(A)$, then $ji-scl(B)-B$ contains no non-empty $ij-\delta$ closed set.

Proof. Let A be $ij-g\delta s$ closed in X and $A \subset B \subset ji-scl(A)$, then by theorem 3.2, B is $ij-g\delta s$ closed. By theorem 3.6, $ji-scl(B)-B$ contains no non-empty $ij-\delta$ closed set.

Definition 3.2. A subset A of a bitopological space (X,τ_1,τ_2) is called $ij-$generalized δ - semi open set ($ij-g\delta s$ open set) if X − A is $ij-g\delta s$ closed.

Theorem 3.8. A set A is $ij-g\delta s$ open if and only if $F \subseteq ji-\text{sint}(A)$, whenever $F \subseteq A$ and F is $ij-\delta$ closed.

Proof. Suppose that A is $ij-g\delta s$ open. Then A^C is $ij-g\delta s$ closed. Suppose that F is $ij-\delta$ closed and $F \subseteq A$. Then $F^C$is $ij-\delta$ open and $A^C \subseteq F^C$. Since A^C is $ij-g\delta s$ closed. Therefore $ji-scl(A^C) \subseteq F^C$. Since $ji-scl(A^C) = [ji-\text{sint}(A)]^C$, we have $[ji-\text{sint}(A)]^C \subseteq F^C$. Hence $F \subseteq ji-\text{sint}(A)$.

Conversely, Suppose that $F \subseteq ji-\text{sint}(A)$ whenever $F \subseteq A$ and F is $ij-\delta$ closed.

Then $A^C \subseteq F^C$ and F^C is $ij-\delta$ open. Take $U = F^C$, since $F \subseteq ji\text{-}\operatorname{sint}(A)$, $[ji\text{-}\operatorname{sint}(A)]^C \subseteq F^C = U$. Since $ji\text{-}scl(A^C) = [ji\text{-}\operatorname{sint}(A)]^C$, we have $ji\text{-}scl(A^C) \subseteq U$. Then A^C is $ij\text{-}g\delta s$ closed. Therefore A is $ij\text{-}g\delta s$ open.

Preposition 3.3. If A and B are $ij\text{-}g\delta s$ open sets in a bitopological space (X, τ_1, τ_2), then $A \cup B$ is also a $ij\text{-}g\delta s$ open set.

Proof. Suppose A and B are $ij\text{-}g\delta s$ open sets in a bitopological space (X, τ_1, τ_2). Let U be a $ij-\delta$ closed in X and $U \subseteq A \cup B$. Since $U \subseteq A \cup B$, we have U$\subseteq A$ and $U \subseteq B$. Since U is $ij-\delta$ closed in X and A and B are $ij\text{-}g\delta s$ open sets, we have $U \subseteq ji-$ sint(A) and $U \subseteq ji-$ sint(B). Therefore $U \cup U \subseteq ji-$ sint(A)] $\cup$ $[ji-$ sint(B)]. This implies $U \subseteq ji-$ sint($A \cup$ B). Hence $A \cup B$ is also a $ij\text{-}g\delta s$ open set.

Preposition 3.4. If A and B are $ij\text{-}g\delta s$ open sets in a bitopological space (X, τ_1, τ_2), then $A \cap B$ is also a $ij\text{-}g\delta s$ open set.

Proof. Suppose A and B are $ij\text{-}g\delta s$ open sets in a bitopological space (X, τ_1, τ_2). Let U be a $ij-\delta$ closed in X and $U \subseteq A \cap B$. Since $U \subseteq A \cap B$, we have $U \subseteq A$ and $U \subseteq B$. Since U is $ij-\delta$ closed in X and A and B are $ij\text{-}g\delta s$ open sets, we have $U \subseteq ji-$ sint(A) and $U \subseteq ji-$ sint(B). Therefore $U \cup U \subseteq ji-$ sint(A)] $\cap$ $[ji-$ sint(B)]. This implies $U \subseteq ji-$ sint($A \cap$ B). Hence $A \cap B$ is also a $ij\text{-}g\delta s$ open set.

Theorem 3.9. The arbitrary intersection of $ij\text{-}g\delta s$ open sets $\{A_i,\ i \in I\}$ in a bitopological space (X, τ_1, τ_2) is $ij\text{-}g\delta s$ open if the family $\{A_i^C, i \in I\}$ is $j-$ locally finite.

Proof. Let $\{A_i^C, i \in I\}$ be $j-$ locally finite and A_i is $ij\text{-}g\delta s$ open in X for each $i \in I$. Then A_i^C is $ij\text{-}g\delta s$ closed in X for each $i \in I$. Then by theorem we have $\bigcup A_i^C$ is $ij\text{-}g\delta s$ closed. Consequently, $\{\bigcap(A_i)\}^C$ is $ij\text{-}g\delta s$ closed in X. Therefore $\bigcap A_i$ is $ij\text{-}g\delta s$ open in X.

Theorem 3.10. If A is $ij\text{-}g\delta s$ open and $ji\text{-}\operatorname{sint}(A) \subseteq B \subseteq A$, then B is $ij\text{-}g\delta s$ open

Proof. Suppose that A is $ij\text{-}g\delta s$ open and $ji\text{-}\operatorname{sint}(A) \subseteq B \subseteq A$. Let F be a $ij-\delta$ closed and $F \subseteq B$. Since $F \subseteq B$, $B \subseteq A$, we have F $\subseteq A$. Since A is $ij\text{-}g\delta s$ open, we have $F \subseteq ji\text{-}\operatorname{sint}(A)$. Since $ji\text{-}\operatorname{sint}(A) \subseteq B$, we have $ji\text{-}\operatorname{sint}[ji\text{-}\operatorname{sint}(A)] \subseteq ji\text{-}\operatorname{sint}(B)$. Then $ji\text{-}\operatorname{sint}(A) \subseteq ji\text{-}\operatorname{sint}(B)$. Since $F \subseteq ji\text{-}\operatorname{sint}(A)$, then $ji\text{-}\operatorname{sint}(F) \subseteq ji\text{-}\operatorname{sint}(B)$ Therefore B is is $ij\text{-}g\delta s$ open.

Theorem 3.11. A set A is $ij\text{-}g\delta s$ closed in X if and only if $ji\text{-}$scl(A) $-$ A is $ij\text{-}g\delta s$ open.

Proof. Suppose that A is $ij\text{-}g\delta s$ closed in X. Let F be a $ij-\delta$ closed and $F \subseteq ji\text{-}\operatorname{scl}(A) - A$. Since A is is $ij\text{-}g\delta s$ closed in X, $ji\text{-}\operatorname{scl}(A) - A$ contains no non-empty $ij-\delta$ closed set. Since $F \subseteq ji\text{-}\operatorname{scl}(A) - A$, $F = \phi \subseteq ji\text{-}\operatorname{sint}[ji\text{-}\operatorname{scl}(A) - A]$. Then $ji\text{-}\operatorname{scl}(A) - A$ is $ij\text{-}g\delta s$ open.

Conversely, Suppose that $ji\text{–}\operatorname{scl}(A) - A$ is $ij\text{–}\, g\delta s$ open and suppose that U is $ij - \delta$ open, $A \subseteq U$. Since $A \subseteq U$, we have $U^C \subseteq A^C$. Therefore $ji\text{–}\operatorname{scl}(A) \cap U^C = ji\text{–}\operatorname{scl}(A) - A$. Since U is $ij - \delta$ open in X, we have U^C is $ij - \delta$ closed in X. Also since $ji\text{–}\operatorname{scl}(A)$ is $ij\text{–}\,\delta$ closed in X and U^C is $ij - \delta$ closed in X. Then $[ji\text{–}\operatorname{scl}(A)] \cap U^C$ is $ij\text{–}\,\delta$ closed in X. Since $ji\text{–}\operatorname{scl}(A) - A$ is $ij\text{–}\, g\delta s$ open. Then $\left[ji\text{–}\operatorname{scl}(A)\right] \cap U^C \subseteq ji\text{–}\operatorname{sint}\left[ji\text{–}\operatorname{scl}(A) - A\right] = ji\text{–}\operatorname{sint}\left[ji\text{–}\operatorname{scl}(A) \cap A^C\right] = \phi$. That is $ji\text{–}\operatorname{scl}(A) \subseteq U$. Therefore A is $ij\text{–}\, g\delta s$ closed.

Theorem 3.12. The intersection of a $ij\text{–}\, g\delta s$ open set and a $ij - \delta$ open set is always $ij\text{–}\, g\delta s$ open.

Proof. Suppose that A is $ij\text{–}\, g\delta s$ open and B is $ij\text{–}\,\delta$ open. Since B is $ij\text{–}\,\delta$ open, then B^C is $ij\text{–}\,\delta$ closed. Since every $ij\text{–}\,\delta$ closed set is $ij\text{–}\, g\delta s$ closed. Therefore B^C is $ij\text{–}\, g\delta s$ closed. This implies that B is $ij\text{–}\, g\delta s$ open. By Preposition 3.2, we have $A \cap B$ is $ij\text{–}\, g\delta s$ open.

Theorem 3.13. If a set A is $ij\text{–}\, g\delta s$ open in a bitopological space (X, τ_1, τ_2), then $G = X$ whenever G is $ij - \delta$ open and $[ji\text{–}\operatorname{sint}(A)] \cup A^C \subseteq G$.

Proof. Suppose that A is $ij\text{–}\, g\delta s$ open in a bitopological space (X, τ_1, τ_2) and G is $ij\text{–}\,\delta$ open and $[ji\text{–}\operatorname{sint}(A)] \cup A^C \subseteq G$. Then $G^C \subseteq \{[ji\text{–}\operatorname{sint}(A)] \cup A^C\}^C = [ji\text{–}\operatorname{sint}(A)]^C \cap (A^C)^C = ji\text{–}\operatorname{sint}(A^C) \cap A = ji\text{–}\operatorname{sint}(A^C) - A^C$. Since G is $ij\text{–}\,\delta$ open, G^C is $ij\text{–}\,\delta$ closed and A is $ij\text{–}\, g\delta s$ open , A^C is $ij\text{–}\, g\delta s$ closed. This implies that $ji\text{–}\operatorname{scl}(A^C) - A^C$ contains no non-empty $ij\text{–}\,\delta$ closed set in X. Then $G^C = \phi$. Therefore $G = X$.

Remark 3.1. The converse of the above theorem is not true in general.

Definition 3.3. The $ij -$ generalized $\delta -$ semi closure of a subset A of a bitopological space (X, τ_1, τ_2) is the intersection of all $ij\text{–}\, g\delta s$ closed sets containing A and is denoted by $ij\text{–}\, g\delta scl(A)$.

Theorem 3.14. Let A be a subset of a bitopological space (X, τ_1, τ_2). Then $A \subseteq ij - g\delta scl(A) \subseteq ji - scl(A) \subseteq j - cl(A)$.

Proof. It follows from the facts that every $\tau_j -$ closed set is $ji -$ semi closed and every $ji -$ semi-closed set is $ij - g\delta s$ closed.

Theorem 3.15. If A is $ij - g\delta s$ closed set, then $A = ij - g\delta scl(A)$.

Proof. By above theorem, $A \subseteq ij - g\delta scl(A)$. Now, we show that $ij - g\delta scl(A) \subseteq A$. Since $ij - g\delta scl(A) = \cap \{ F : A \subseteq F$ and F is $ij - g\delta s$ closed in X$\}$ and A is $ij - g\delta s$ closed set, then $ij - g\delta scl(A) \subseteq A$. Thus $A = ij - g\delta scl(A)$.

Definition 3.4. A point *x* of a bitopological space (X, τ_1, τ_2) is called an ij-generalized $\delta -$ semi limit point (briefly $ij - g\delta s$ limit point) of a subset *A* of *X*, if for each $ij - g\delta s$ open set *U* containing *x*, $A \cap U \backslash \{x\} \neq \phi$. The set of all $ij - g\delta s$ limit points of *A* will be denoted by $ij - g\delta sd(A)$ and is called the $ij -$ generalized $\delta -$ semi derived set of A.

Theorem 3.16. Let A and B be subsets of a bitopological space (X, τ_1, τ_2). If $A \subset B$, then $ij - g\delta sd(A) \subset ij - g\delta sd(B)$.
Proof. Obvious.

Theorem 3.17. If A is a subset of a bitopological space (X, τ_1, τ_2), then $ij - g\delta scl(A) = A \cup ij - g\delta sd(A)$.
Proof. First we prove that $A \cup ij - g\delta sd(A) \subseteq ij - g\delta scl(A)$. By Definition 3.25, $ij - g\delta sd(A) \subseteq ij - g\delta scl(A)$. Since $A \subset ij - g\delta scl(A)$, then $A \cup ij - g\delta sd(A) \subset ij - g\delta scl(A)$.
Conversely, suppose that $x \notin (A \cup ij - g\delta sd(A))$. Then $x \notin A$ and $x \notin ij - g\delta sd(A)$. Since $x \notin ij - g\delta sd(A)$, then there exists an $ij - g\delta s$ open set U such that $x \in U$ and $A \cap U \backslash \{x\} = \phi$. Since $x \notin A$, then $U \cap A = \phi$. Since $x \notin X \backslash U$ where $X \backslash U$ is $ij - g\delta s$ closed and $A \subset X \backslash U$. Then $x \notin ij - g\delta scl(A)$. Hence $ij - g\delta scl(A) \subset A \cup ij - g\delta sd(A)$ and consequently $ij - g\delta scl(A) = A \cup ij - g\delta sd(A)$.

Theorem 3.18. A point $x \in ij - g\delta scl(A)$ if and only if every $ij - g\delta s$ open set U containing x, $U \cap A \neq \phi$.
Proof. Let $x \in ij - g\delta scl(A)$ and U be an $ij - g\delta s$ open set containing x. Suppose that $U \cap A = \phi$. Then $A \subset X \backslash U$ where $X \backslash U$ is $ij - g\delta s$ closed set. Thus $x \in X \backslash U$ which is a contradiction. Therefore $U \cap A \neq \phi$.
Conversely, suppose that for every $ij - g\delta s$ open set U containing x, $U \cup A \neq \phi$. Let $x \notin ij - g\delta scl(A)$, then there exists $ij - g\delta s$ closed F in X such that $A \subset F$ and $x \notin F$. Hence $x \in X \backslash F$ where $X \backslash F$ is $ij - g\delta s$ open set and $X \backslash F \cap A = \phi$, which is a contradiction. Therefore $x \in ij - g\delta scl(A)$.

Theorem 3.19. If A and B are subsets of a bitopological space (X, τ_1, τ_2), then the following are true:
(i) $ij - g\delta sd(A \cup B) = ij - g\delta sd(A) \cup ij - g\delta sd(B)$
(ii) $ij - g\delta scl(A \cup B) = ij - g\delta scl(A) \cup ij - g\delta scl(B)$
(iii) $ij - g\delta scl(A) = ij - g\delta scl(ij - g\delta scl(A))$.
Proof. (i) Let A and B be subsets of X. Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By Theorem 3.16, and $ij - g\delta sd(A) \subseteq ij - g\delta sd(A \cup B)$ and $ij - g\delta sd(B) \subseteq ij - g\delta sd(A \cup B)$. Hence $ij - g\delta sd(A) \cup ij - g\delta sd(B) \subseteq ij - g\delta sd(A \cup B)$.
Conversely, let $x \notin ij - g\delta sd(A) \cup ij - g\delta sd(B)$. Then $x \notin ij - g\delta sd(A)$, $x \notin ij - g\delta sd(B)$ and there exist two $ij - g\delta s$ open sets U, V such that $x \in U$, $x \in V$, $A \cap U \backslash \{x\} = \phi$ and $B \cap V \backslash \{x\} = \phi$. Hence $x \in U \cap V$, where $U \cap V$ is an $ij - g\delta s$ open set of X by Preposition 3.2. This implies $(U \cap V) \backslash \{x\} \cap (A \cup B) = \phi$ and $x \notin ij - g\delta sd(A \cup B)$. Thus $ij - g\delta sd(A \cup B) \subseteq ij - g\delta sd(A) \cup ij - g\delta sd(B)$ and $ij - g\delta sd(A \cup B) = ij - g\delta sd(A) \cup ij - g\delta sd(B)$.
(ii) the proof is similar to (i).
(iii) By Theorem 3.19(iii), $ij - g\delta scl(A) \subseteq ij - g\delta scl(ij - g\delta scl(A))$. Now, let $x \notin ij - g\delta scl(A)$. This means that by Theorem 3.31, there exists an $ij - g\delta s$ open set U of X containing x and $U \cap A = \phi$. Suppose that $U \cap ij - g\delta scl(A) \neq \phi$. Then there is $y \in U \cap ij - g\delta scl(A)$, so $y \in ij - g\delta scl(A)$. This implies for every $ij - g\delta s$ open set V containing y we have $V \cap A \neq \phi$. But U is an $ij - g\delta s$ open set containing y. Hence

$U \cap A \neq \phi$, which is a contradiction. Thus $U \cap ij - g\delta scl(A) = \phi$ and $x \notin ij - g\delta scl(ij - g\delta scl(A))$. Hence $ij - g\delta scl(A) = ij - g\delta scl(ij - g\delta scl(A))$.

Definition 3.5. The $ij -$ generalized $\delta -$ semi interior of a subset A of a bitopological space (X, τ_1, τ_2) is the union of all $ij - g\delta s$ open sets contained in A and is denoted by $ij - g\delta sint(A)$.

Theorem 3.20. For any subset A of a bitopological space (X, τ_1, τ_2), we have $j - int(A) \subseteq ji - sint(A) \subseteq ij - g\delta sint(A)$.
Proof. The proof follows from the facts that every $\tau_j -$ open set is $ji -$ semi open and every $ji -$ semi open set is $ij - g\delta s$ open.

Theorem 3.21. For any subset A of a bitopological space (X, τ_1, τ_2), we have:
(i) $ij - g\delta scl(X\backslash A) = X\backslash ij - g\delta sint(A)$
(ii) $ij - g\delta sint(X\backslash A) = X\backslash ij - g\delta scl(A)$.
Proof. (i) Let $x \notin ij - g\delta scl(X\backslash A)$, there exists an $ij - g\delta s$ open set U of X containing x such that $U \cap (X\backslash A) = \phi$. Hence $x \in U \subset A$ and $x \in ij - g\delta sint(A)$. Thus $x \notin X\backslash ij - g\delta sint(A)$.
Conversely, let $x \notin X\backslash ij - g\delta sint(A)$. Thus $x \in ij - g\delta sint(A)$ and there exists an $ij - g\delta s$ open set U of X such that $x \in U \subset A$. Hence $U \cap (X\backslash A) = \phi$ and $x \notin ij - g\delta scl(X\backslash A)$.
(ii) The proof is similar to that of (i).

4. $ij\text{–} g\delta s$ continuous functions

Definition 4.1. A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be $ij\text{–} g\delta s$ continuous, if $f^{-1}(V)$ is $ij\text{–} g\delta s$ closed set in X for every $\sigma_j -$ closed set V in Y.

Example 4.1. Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}, \{a, c, d\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{d\}, \{a, d\}$, $\sigma_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_2 = \{\phi, X, \{c\}, \{d\}, \{c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Let $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ defined by $f(\{a\}) = f(\{d\}) = \{c\}$, $f(\{b\}) = \{d\}$, $f(\{c\}) = \{a\}$. Then f is $ij\text{–} g\delta s$ continuous.

Theorem 4.1. Every pairwise continuous function is $ij\text{–} g\delta s$ continuous.
Proof. Let $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be pairwise continuous function. Let U be a $j -$ closed set in Y. Then $f^{-1}(U)$ is $j -$ closed set in X. Since every $j -$ closed set is $ij\text{–} g\delta s$ closed, $i \neq j$ and $i, j = 1,2$, we have f is $ij\text{–} g\delta s$ continuous.

Theorem 4.2. The following are equivalent for a function is $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$,

(a) f is $ij\text{–} g\delta s$ continuous.

(b) $f^{-1}(U)$ is $ij\text{–} g\delta s$ open for each $\sigma_j -$ open set U in Y, $i \neq j$ and $i, j = 1,2$.

Proof. (a) $\Rightarrow$ (b) Suppose that f is $ij\text{–} g\delta s$ continuous. Let A be a $\sigma_j -$ open in Y. Then A^C is $\sigma_j -$ closed in Y. Since f is $ij\text{–} g\delta s$ continuous, we have $f^{-1}(A^c)$ is $ij\text{–} g\delta s$ closed in X, $i \neq j$ and $i, j = 1,2$. Consequently, $f^{-1}(A)$ is $ij\text{–} g\delta s$ open in X.

(b) $\Rightarrow$ (a) Suppose that $f^{-1}(U)$ is $ij\text{–}\ g\delta s$ open for each $\sigma_j -$ open set U in Y, $i \neq j$ and $i, j = 1{,}2$. Let V be $\sigma_j -$ closed set in Y. Then V^c is $\sigma_j -$ open in Y. Therefore by our assumption, $f^{-1}(V^c)$ is $ij\text{–}\ g\delta s$ open in X, $i \neq j$ and $i, j = 1{,}2$. Hence $f^{-1}(V)$ is $ij\text{–}\ g\delta s$ closed in X. Therefore f is $ij\text{–}\ g\delta s$ continuous.

Definition 4.2. A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is $ij\text{–}\ g\delta s$ irresolute if $f^{-1}(U)$ is $ij\text{–}\ g\delta s$ closed for each $ij\text{–}\ g\delta s$ closed set U in Y , $i \neq j$ and $i, j = 1{,}2$.

Example 4.2. Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}, \{a, c, d\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{d\}, \{a, d\}$, $\sigma_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_2 = \{\phi, X, \{c\}, \{d\}, \{c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Let $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ defined by $f(\{a\}) = f(\{d\}) = \{c\}$, $f(\{b\}) = \{d\}$, $f(\{c\}) = \{a\}$. Then f is $ij\text{–}\ g\delta s$ irresolute.

Theorem 4.3. Every $ij\text{–}\ g\delta s$ continuous function is $ij\text{–}\ g\delta s$ irresolute function.
Proof. Let A be $j\text{–}$ closed set in Y. Since every $j\text{–}$ closed set is $ij\text{–}\ g\delta s$ closed in Y, $i, j = 1{,}2$ and $i \neq j$. Since f is $ij\text{–}\ g\delta s$ continuous function. Then $f^{-1}(A)$ is $ij\text{–}\ g\delta s$ closed in X. Therefore f is $ij\text{–}\ g\delta s$ irresolute function.

Theorem 4.4. Let $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \longrightarrow (Z, \mu_1, \mu_2)$ be two functions. Then

(a) If f and g are $ij\text{–}\ g\delta s$ continuous, then $g \circ f$ is $ij\text{–}\ g\delta s$ continuous.
(b) If f and g are $ij\text{–}\ g\delta s$ irresolute, then $g \circ f$ is $ij\text{–}\ g\delta s$ irresolute.
(c) If f is $ij\text{–}\ g\delta s$ irresolute and g is $ij\text{–}\ g\delta s$ continuous, then $g \circ f$ is $ij\text{–}\ g\delta s$ continuous.

Proof. (a) Let $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \longrightarrow (Z, \mu_1, \mu_2)$ be $ij\text{–}\ g\delta s$ continuous. Let U be $j -$ closed set in Z, $i, j = 1{,}2$ and $i \neq j$. Since g is $ij\text{–}\ g\delta s$ continuous, $g^{-1}(U)$ is $ij\text{–}\ g\delta s$ closed in Y . Since f is $ij\text{–}\ g\delta s$ continuous, $(g \circ f)^{-1} = f^{-1}[g^{-1}(U)]$ is $ij\text{–}\ g\delta s$ closed in X. Therefore, $g \circ f$ is $ij\text{–}\ g\delta s$ continuous.
The proofs of (b) and (c) are similar.

Definition 4.3. A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is $ij -$ preδsg continuous if $f^{-1}(U)$ is $ij\text{–}\ g\delta s$ closed for each ij – δ semi closed set U in Y, $i, j = 1{,}2$ and $i \neq j$.

Definition 4.4. A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is $ij -$ pre gδs closed if $f(U)$ is $ij\text{–}\ g\delta s$ closed for each $ij\text{–}$ δ semi closed set U in X, $i, j = 1{,}2$ and $i \neq j$.

Theorem 4.5. Let $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is $ij\text{–}\ g\delta s$ continuous, then $f\big(ij - g\delta scl(A)\big) \subseteq j - cl(f(A))$ for every subset A of X.
Proof. Since $A \subseteq f^{-1}(f(A))$, we have $A \subseteq f^{-1}[j - cl\big(f(A)\big)]$. Now $j - cl(f(A))$ is a $j -$ closed set in Y and hence $f^{-1}[j - cl\big(f(A)\big)]$ is a $ij\text{–}\ g\delta s$ closed set containing A. Consequently $ij - g\delta scl(A) \subseteq f^{-1}[j - cl\big(f(A)\big)]$. Therefore $\big(ij - g\delta scl(A)\big) \subseteq f[f^{-1}\big[j - cl\big(f(A)\big)\big]] \subseteq j - cl(f(A))$.

Theorem 4.6. Let $f:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ be a function and let $g: X\text{x}X \longrightarrow Y$ be the bitopological graph function of f defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is ij– $g\delta s$ continuous, then f is ij– $g\delta s$ continuous.

Proof. Let U be an $j-$closed in Y. Then $X\text{x}U$ is an $ij-$closed in $X\text{x}X$. Since g is ij– $g\delta s$ continuous, then $f^{-1}(U) = g^{-1}(X\text{x}U)$ is ij– $g\delta s$ closed in X. Therefore f is ij– $g\delta s$ continuous.

Definition 4.5. A subset A of a bitopological space (X,τ_1,τ_2) is said to be ij– $g\delta s$ dense if $ij - g\delta scl(A) = X$.

Theorem 4.7. Assume that $ij - G\delta S$O(X) is closed under any intersection. If $f:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ and $g:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ are ij– $g\delta s$ continuous and Y is pairwise Urysohn, then $E = \{x \in X: f(x) = g(x)\}$ is ij– $g\delta s$ closed in X.

Proof Let $x \in X - E$, then $f(x) \neq g(x)$. Since Y is a pairwise Urysohn, there exists $i-$ open set V and $j-$ open set W such that $f(x) \in V, g(x) \in W$ and $j - cl(V) \cap i - cl(W) = \phi$. Since f and g are ij– $g\delta s$ continuous, $f^{-1}[j - cl(V)]$ and $g^{-1}[i - cl(W)]$ are ij– $g\delta s$ closed in X. Let $U = f^{-1}[j - cl(V)]$ and $G = g^{-1}[i - cl(W)]$.

Then U and G are ij– $g\delta s$ closed sets containing x. Set $A = U \cap G$, thus A is ij– $g\delta s$ closed in X. Hence $f(A) \cap g(A) = f(U \cap G) \cap g(U \cap G) \subseteq f(U) \cap g(G) = j - cl(V) \cap \sigma_1 - cl(W) = \phi$. Therefore $A \cap E = \phi$. This implies $x \notin ij - g\delta scl(E)$. Hence E is ij– $g\delta s$ closed in X.

Theorem 4.8. Assume that $ij - G\delta S$O(X) is closed under any intersection. If $f:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ and $g:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ are ij– $g\delta s$ continuous, Y is pairwise Urysohn and $f = g$ on ij– $g\delta s$ dense set $A \subset X$, then $f = g$ on X.

Proof Since f and g are ij– $g\delta s$ continuous, Y is pairwise Urysohn by theorem 4.14, $E = \{x \in X: f(x) = g(x)\}$ is ij– $g\delta s$ closed in X. By assumption, $f = g$ on ij– $g\delta s$ dense set $A \subset X$. Since $A \subset E$ and A is ij– $g\delta s$ dense set in X, then $X = ij - g\delta scl(A) \subseteq ij - g\delta scl(E) = E$. Hence $f = g$ on X.

Definition 4.6. A bitopological space (X,τ_1,τ_2) is called ij– $T_{g\delta s}$, if every ij– $g\delta s$ closed set is $\tau_j -$ closed, $i,j = 1,2$ and $i \neq j$.

Theorem 4.9. Let $f:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ be onto ij– $g\delta s$ irresolute and ij– pre$g\delta s$ closed map. If X is ij– $T_{g\delta s}$, then Y is also ij– $T_{g\delta s}$.

Proof. Let A be a ij– $g\delta s$ closed subset of Y, $i,j = 1,2$ and $i \neq j$. Since f is onto ij– $g\delta s$ irresolute, $f^{-1}(A)$ is ij– $g\delta s$ closed subset of X. Since X is ij– $T_{g\delta s}$ space, $f^{-1}(A)$ is $j-$ closed in X, $i,j = 1,2$ and $i \neq j$. Since f is ij– pre$g\delta s$ closed map, $f[f^{-1}(A)] = A$ is j– closed in Y. Therefore Y is ij– $T_{g\delta s}$.

Definition 4.7. A bitopological space (X,τ_1,τ_2) is called ij– $g\delta sT_{1/2}$, if every ij– $g\delta s$ closed set is $ji -$ semi closed, $i,j = 1,2$ and $i \neq j$.

Theorem 4.10. A bitopological space (X, τ_1, τ_2) is ij– $g\delta sT_{1/2}$ if and only if every singleton is $ji -$ semi open or $ij -$ semi closed.

Proof. Suppose $\{x\}$ is not $\tau_i -$ semi closed. Then $X\backslash\{x\}$ is $ij -$ $g\delta s$ closed. Since (X, τ_1, τ_2) is $ij -$ $g\delta sT_{1/2}$ space, $X\backslash\{x\}$ is $ji -$ semi closed and $\{x\}$ is $ji -$ semi open.

Conversely, let F be $ij -$ $g\delta s$ closed. For any $x \in ji - scl(F)$, {x} is $ji -$ semi open or $ij -$ semi closed by assumption.

Case 1. Suppose $\{x\}$ is $ji -$ semi open. Since $\{x\} \cap F \neq \phi$, then $x \in F$.

Case 2. Suppose $\{x\}$ is $ij -$ semi closed. If $x \notin F$, then this contradicts Theorem 3.9 since $\{x\} \subset ji - scl(F) \backslash F$. Thus $x \in F$.

From the above two cases we conclude that F is a $ji -$ semi-closed. Hence (X, τ_1, τ_2) is a $ij -$ $g\delta sT_{1/2}$ space.

Definition 4.8. A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be ij– $g\delta s$ closed, if for each $\tau_j -$ closed set U of X, $f(U)$ is ij– $g\delta s$ closed set in Y. If f is 12 – $g\delta s$ closed and 21 – $g\delta s$ closed, then f is called pairwise $g\delta s -$ closed.

Theorem 4.11. Every $ji -$ semi closed function is ij– $g\delta s$ closed function.

Proof. The proof follows from, every $ji -$ semi closed set is ij– $g\delta s$ closed set.

Theorem 4.12. For a function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$, the following are equivalent:

(i) f is ij– $g\delta s$ open.

(ii) $f[j - int(A)] \subset ij - g\delta sint[f(A)]$, for each subset A of X.

(iii) For each $x \in X$ and for $j -$ open set U containing x, there is an ij– $g\delta s$ open set V containing $f(x)$ such that $V \subset f(U)$.

(iv) If f is surjective, then $f^{-1}[ij - g\delta sint(B)] \subset j - cl[f^{-1}(B)]$, for each subset B of Y.

Proof. (i) $\Longrightarrow$ (ii) Let A be a subset of a bitopological space (X, τ_1, τ_2). Since $j - int(A) \subset A$, then $f[j - int(A)] \subset f(A)$. But $j - int(A)$ is $j -$ open set of X, then $f[j - int(A)]$ is ij– $g\delta s$ open set in Y, since f is ij– $g\delta s$ open. Hence $f[j - int(A)] \subset ij - g\delta sint[f[j - int(A)]] \subset ij - g\delta sint[f(A)]$. Thus $f[j - int(A)] \subset ij - g\delta sint[f(A)]$.

(ii) $\Longrightarrow$ (iii) Let $x \in X$ and U be a $j -$ open set containing x. Then by (ii), $f[j - int(U)] \subset ij - g\delta sint[f(U)]$ and this implies $f(U) \subset ij - g\delta sint[f(U)]$. Thus there exists an ij– $g\delta s$ open set V such that $f(x) \in V$ and $V \subset f(U)$.

(iii) $\Longrightarrow$ (iv) Let $B \subset Y$ and $x \in f^{-1}[ij - g\delta sint(B)]$. Then $f(x) \in ij - g\delta sint(B)$. If $x \notin j - cl[f^{-1}(B)]$, then $x \in U$, where $U = X \backslash j - cl[f^{-1}(B)]$, and hence by (iii), there is an ij– $g\delta s$ open set V such that $f(x) \in V \subset f(U)$. Now $V \subset f(U) \subset f[X \backslash f^{-1}(B)] \subset Y \backslash V$. Now $f(x) \in ij - g\delta sint(B)$. Hence $f(x) \notin V$ which is contradiction. Thus $f^{-1}[ij - g\delta sint(B)] \subset j - cl[f^{-1}(B)]$.

Theorem 4.13. Let $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \longrightarrow (Z, \eta_1, \eta_2)$ are two functions, then

(i) If f is j – closed and g is $ij-g\delta s$ closed, then $g \circ f$ is $ij-g\delta s$ closed.
(ii) If f is $ij-\delta$ continuous surjection and $g \circ f$ is $ij-g\delta s$ closed, then g is $ij-g\delta s$ closed.

Proof. Obvious.

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