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Γ^*- Derivation Pair and Jordan Γ^*- Derivation Pair on Γ-ring M with Involution

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Abstract. Let M be a 6-torsion free Γ-ring M with involution and let (d, g): $M \to M$ be an additive mapping satisfying the condition that $a\alpha b\beta c = a\beta b\alpha c$ ($a,b,c \in M$ and $\alpha,\beta \in \Gamma$). In this paper we will give the relation between Γ*-derivation pair and Jordan Γ*-derivation pair. Also, we will prove that if (d, g) is a Jordan Γ*-derivation pair, then d is a Jordan Γ*-derivation.

Keywords: Γ-ring with involution, Jordan derivation pair, Γ*-derivation pair, Γ*-derivation

AMS Mathematics Subject Classification (2010): 16W10, 17C50

1. Introduction

The notion of Γ-ring was first introduced by Nobusawa [15] who also showed that Γ-rings are more general than rings. Barnes [3] slightly weakened the conditions in the definition of Γ-ring in the sense of Nobusawa. Barnes [3], Kyuno [11], Luh [13], Ceven [5], Hoque and Paul [6, 8, 9] and others had obtained a large numbers of important basic properties of Γ-rings in various ways and determined some more remarkable results of Γ-rings. We start with some definitions.

Let M and Γ be additive abelian groups. Define a mapping $M \times \Gamma \times M \to M$ by $(x, \alpha, y) \to (x\alpha y)$ which satisfies the conditions

(i) $x\alpha y \in M$.

(ii) $(x+y)\alpha z = x\alpha z + y\alpha z,\ x(\alpha+\beta)y = x\alpha y + x\beta y,\ x\alpha(y+z) = x\alpha y + x\alpha z$

(iii) $(x\alpha y)\beta z = x\alpha(y\beta z)$.

Then M is called a Γ-ring (see [13], [7]). Let M be a Γ-ring. Then an additive subgroup U of M is called a left (right) ideal of M if $M\Gamma U \subset U$ $(U\Gamma M \subset U)$. If U is both a left and a right ideal, then we say U is an ideal of M. Suppose again that M is a Γ-ring. Then M is said to be 2-torsion free if $2x = 0$ implies $x = 0$ for all $x \in M$. An ideal P_1 of a Γ-ring M is said to be prime if for some ideals A and B of M , $A\Gamma B \subseteq P_1$ implies $A \subseteq P_1$ or $B \subseteq P_1$.An ideal P_2 of a Γ-ring M is said to be semiprime if for any ideal U of M, $U\Gamma U \subseteq P_2$ implies $U \subseteq P_2$. A Γ-ring M is said to be prime if $a\Gamma M\Gamma b = (0)$ with a, $b \in M$, implies $a = 0$ or $b = 0$ and semiprime if $a\Gamma M\Gamma a = (0)$ with $a \in M$ implies $a = 0$. Furthermore, M is said to be a commutative Γ-ring if $x\alpha y = y\alpha x$ for all $x, y \in M$ and $\alpha \in \Gamma$. Moreover, the set Z (M) = $\{x \in M: x\alpha y = y\alpha x$ for all $\alpha \in \Gamma, y \in M\}$ is called the center of the Γ-ring M. If M is a Γ-ring, then $[x; y]_\alpha = x\alpha y - y\alpha x$ is known as the commutator of x and y with respect to α, where $x, y \in M$ and $\alpha \in \Gamma$. We make the basic commutator identities:

$$[x\alpha y, z]_\beta = [x, z]_\beta \alpha y + x[\alpha, \beta]_z y + x\alpha[y, z]_\beta \quad (1)$$

$$[x, y\alpha z]_\beta = [x, y]_\beta \alpha z + y[\alpha, \beta]_x z + y\alpha[x, z]_\beta \quad (2)$$

for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Now, we consider the following assumption:

(A) $\quad x\alpha y\beta z = x\beta y\alpha z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

According to assumption (A), the above commutator identities reduce to

$[x\alpha y, z]_\beta = [x, z]_\beta \alpha y + x\alpha[y, z]_\beta$ and $[x, y\alpha z]_\beta = [x, y]_\beta \alpha z + y\alpha[x, z]_\beta$

which we will extensively used.

Barnes [3], Luh [13], Kyuno [11], Hoque and Paul [8] as well as Uddin and Islam [16,17] studied the structure of Γ-rings and obtained various generalizations of corresponding parts in ring theory. Note that during the last few decades, many authors have studied derivations in the context of prime and semiprime rings and Γ-rings with involution [1,2,4,10,18].The notion of derivation pair and Jordan derivation pair on a *-ring R were defined by [12, 14, 19,20].

Definition 1.1. [5] An additive mapping $D: M \to M$ is called a derivation if $D(x\alpha y) = D(x)\alpha y + x\alpha D(y)$ which holds for all $x, y \in M$ and $\alpha \in \Gamma$.

Definition 1.2. [5] An additive mapping $D: M \to M$ is called a Jordan derivation if $D(x\alpha x) = D(x)\alpha x + x\alpha D(x)$ which holds for all $x \in M$ and $\alpha \in \Gamma$.

Definition 1.3. An additive mapping $(x\alpha x) \to (x\alpha x)^*$ on a Γ-ring M is called an involution if $(x\alpha y)^* = y^*\alpha x^*$ and $(x\alpha x)^{**} = x\alpha x$ for all $x, y \in M$ and $\alpha \in \Gamma$. A Γ-ring M equipped with an involution is called a Γ-ring M with involution (also known as Γ*-ring).

Definition 1.4. An additive mapping $(d,\ g)\ :M\to M$ is called a Γ^*-derivation pair if the following system of equations

$$d(x\alpha y\beta x) = d(x)\alpha y^*\beta x^* + x\alpha g(y)\beta x^* + x\alpha y\beta d(x) \tag{3}$$

which holds for all $x,\ y\in M$ and $\alpha,\beta\in\Gamma$ and

$$g(x\alpha y\beta x) = g(x)\alpha y^*\beta x^* + x\alpha d(y)\beta x^* + x\alpha y\beta g(x) \tag{4}$$

which holds for all $x,\ y\in M$ and $\alpha,\beta\in\Gamma$ are satisfied.

Definition 1.5. An additive mapping $(d,\ g)\ :M\to M$ is called a Jordan Γ^*-derivation pair if the following system of equations

$$d(x\alpha x\beta x) = d(x)\alpha x^*\beta x^* + x\alpha g(x)\beta x^* + x\alpha x\beta d(x) \tag{5}$$

which holds for all $x\in M$ and $\alpha,\beta\in\Gamma$, and

$$g(x\alpha x\beta x) = g(x)\alpha x^*\beta x^* + x\alpha d(x)\beta x^* + x\alpha x\beta g(x) \tag{6}$$

which holds for all $x\in M$ and $\alpha,\beta\in\Gamma$ are satisfied

Example 1. Let R be a commutative ring with ChR=2.

Define $M=\left\{\begin{bmatrix} a & b\\ 0 & a\end{bmatrix}: a,b\in R\right\}$, and $\Gamma=\left\{\begin{bmatrix} \alpha & 0\\ 0 & \alpha\end{bmatrix}: \alpha\in\Gamma\right\}$, then M is a Γ-ring under addition and multiplication of matrices.

Define a mapping $d:M\to M$ by $d\left(\begin{bmatrix} a & b\\ 0 & a\end{bmatrix}\right)=\begin{bmatrix} 0 & b\\ 0 & 0\end{bmatrix}$ and $g\left(\begin{bmatrix} c & d\\ 0 & c\end{bmatrix}\right)=\begin{bmatrix} 0 & d\\ 0 & 0\end{bmatrix}$

To show that $(d,\ g)$ is a Γ^*-derivation pair, let $x=\begin{bmatrix} a & b\\ 0 & a\end{bmatrix}$, $y=\begin{bmatrix} c & d\\ 0 & c\end{bmatrix}$,

$y^*=\begin{bmatrix} -c & d\\ 0 & -c\end{bmatrix}$, $\beta=\begin{bmatrix} \beta & 0\\ 0 & \beta\end{bmatrix}$,

then $d(x\alpha y\beta x) = d(x)\alpha y^*\beta x^* + x\alpha g(y)\beta x^* + x\alpha y\beta d(x)$. Hence $(d,\ g)$ is a Γ^*-derivation pair.

It is clear that every Γ^*-derivation pair is a Jordan Γ^*-derivation pair, but the converse in general is not true. In the following example we give an additive mapping which is a Jordan Γ^*-derivation pair, but not a Γ^*-derivation pair.

Example 2. Let *M* be a 2-torsion free Γ-ring with involution with $x\alpha x=0$ for all $x\in M$ and $\alpha\in\Gamma$, and $x\alpha y\beta x^*\neq x\alpha y^*\beta x^*$ for some $x,\ y\in M$ and $\alpha,\beta\in\Gamma$. If we define a map $d: M \to M$ by $d(x) = x - x^*$ for all $x \in M$ and $g: M \to M$ by $g(x) = x^* - x$ for all $x \in M$, then (d, g) is a Jordan Γ*-derivation pair but not a Γ*-derivation pair.

In this paper, we will give the relation between a Γ*-derivation pair and a Jordan Γ* derivation pair. Also we will prove that if (d, g) is a Jordan Γ*-derivation pair, then *d* is a Jordan Γ*-derivation.

2. Γ*-derivation pair and Jordan Γ*-derivation pair

To prove our main results we need the following lemmas.

Lemma 2.1. Let *M* be a 6-torsion free Γ-ring with involution with an identity element satisfying assumption (A), and let (*d*, *g*) be a Jordan Γ*-derivation pair. Then *d* + *g* is a Jordan Γ*-derivation.

Proof. Define an additive mapping $k: M \to M$ by

$$k(x) = d(x) + g(x) \tag{7}$$

for all $x \in M$. Then by using (5) and (6) and the above relation, we get

$$k(x\alpha x\beta x) = k(x)\alpha x^*\beta x^* + x\alpha k(x)\beta x^* + x\alpha x\beta k(x) \tag{8}$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$. Linearization of the relation (8), we get

$$\begin{aligned} k(x\alpha y\beta x + y\alpha x\beta x + y\alpha y\beta x + x\alpha x\beta y + x\alpha y\beta y + y\alpha x\beta y) &= k(x)\alpha y^*\beta x^* + x\alpha \\ k(y)\beta x^* + x\alpha y\beta k(x) + k(y)\alpha x^*\beta x^* + y\alpha k(x)\beta x^* + y\alpha x\beta k(x) + k(y)\alpha y^*\beta x^* \\ + y\alpha k(y)\beta x^* + y\alpha y\beta k(x) + k(x)\alpha x^*\beta y^* + x\alpha k(x)\beta y^* + x\alpha x\beta k(y) + k(x)\alpha y^* \\ \beta y^* + x\alpha k(y)\beta y^* + x\alpha y\beta k(y) + k(y)\alpha x^*\beta y^* + y\alpha k(x)\beta y^* + y\alpha x\beta k(y) \end{aligned} \tag{9}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Replace *x* by −*x* in relation (9), we get

$$\begin{aligned} k(x\alpha y\beta x + y\alpha x\beta x - y\alpha y\beta x + x\alpha x\beta y - x\alpha y\beta y - y\alpha x\beta y) &= k(x)\alpha y^*\beta x^* + x\alpha \\ k(y)\beta x^* + x\alpha y\beta k(x) + k(y)\alpha x^*\beta x^* + y\alpha k(x)\beta x^* + y\alpha x\beta k(x) - k(y)\alpha y^*\beta x^* \\ - y\alpha k(y)\beta x^* - y\alpha y\beta k(x) + k(x)\alpha x^*\beta y^* + x\alpha k(x)\beta y^* + x\alpha x\beta k(y) - k(x)\alpha y^* \\ \beta y^* - x\alpha k(y)\beta y^* - x\alpha y\beta k(y) - k(y)\alpha x^*\beta y^* - y\alpha k(x)\beta y^* - y\alpha x\beta k(y) \end{aligned} \tag{10}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. According to relation (9) and (10) we obtain

$$\begin{aligned} k(x\alpha y\beta x + y\alpha x\beta x + x\alpha x\beta y) &= k(x)\alpha y^*\beta x^* + x\alpha k(y)\beta x^* + x\alpha y\beta k(x) \\ + k(y)\alpha x^*\beta x^* + y\alpha k(x)\beta x^* + y\alpha x\beta k(x) + k(x)\alpha x^*\beta y^* + x\alpha k(x)\beta y^* \\ + x\alpha x\beta k(y) \end{aligned} \tag{11}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Setting $x = y = 1$ in relation (11), and since *M* is 6-torsion free, we get $k(1) = 0$. Now replace *y* by 1 in relation (11), we get

$$k(x\alpha x) = k(x)\alpha x^* + x\alpha k(x) \tag{12}$$

for all $x \in M$ and $\alpha \in \Gamma$. Hence $d + g$ *is* a Jordan Γ^*-derivation.

Lemma 2.2. Let *M* be a 6-torsion free Γ−ring with involution with an identity element satisfying assumption (A), and (d, g) is a Jordan Γ*-derivation pair, then

$$(d-g)_x = a\alpha x^* + x\beta a \tag{13}$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$. And

$$(g-d)_y = b\alpha y^* + y\beta b \tag{14}$$

for all $y \in M$ and $\alpha, \beta \in \Gamma$, where $a = d(1)$ and $b = g(1)$.

Proof. From relation (5), we obtain (see how the relation (11) was obtained from (9))

$$d(x\alpha y\beta x + y\alpha x\beta x + x\alpha x\beta y) = d(x)\alpha y^*\beta x^* + x\alpha g(y)\beta x^* + x\alpha y\beta d(x) + d(y)\alpha x^*\beta x^* + y\alpha g(x)\beta x^* + y\alpha x\beta d(x) + d(x)\alpha x^*\beta y^* + x\alpha g(x)\beta y^* + x\alpha x\beta d(y) \tag{15}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Setting $x = 1$ in the relation (15), we get

$$d(y) = 2(a\alpha y^* + y\beta a) + (b\alpha y^* + y\alpha b) + g(y) \tag{16}$$

for all $y \in M$ and $\alpha, \beta \in \Gamma$. Similarly, we can show that

$$g(y) = 2(b\alpha y^* + y\alpha b) + (a\alpha y^* + y\beta a) + d(y) \tag{17}$$

for all $y \in M$ and $\alpha, \beta \in \Gamma$. Comparing the relation (16) and (17), we arrive at (13) and (14).

Theorem 2.3. Let *M* be a 6-torsion free Γ-ring *M* with involution with an identity element satisfying assumption (A) and let *(d, g)* be a Jordan Γ*-derivation pair. Then (d, g) is a Γ*-derivation pair.

Proof. Putting 1 for *y* in relation (15), we get

$$3d(x\alpha x) = 2A(x) + B(x) + a\alpha x^*\beta x^* + x\alpha x\beta a + x\alpha b\beta x^* \tag{18}$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$, where $A(x) = d(x)\alpha x^* + x\alpha d(x)$ and $B(x) = g(x)\alpha x^* + x\alpha g(x)$. By using Lemma (2.2) and relation (18), we get

$$3d(x\alpha x) = 2A(x) + B(x) + (d-g)_{(x\alpha x)} + x\alpha b\beta x^* \tag{19}$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$. Hence,

$$d(x\alpha x) + (d+g)_{(x\alpha x)} = 2A(x) + B(x) + x\alpha b\beta x^* \tag{20}$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$. By using Lemma (2.1), we get

$$d(x\alpha x) + (d+g)_{(x)}\alpha x^* + x\alpha(d+g)_{(x)} = 2A(x) + B(x) + x\alpha b\beta x^* \tag{21}$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$. Then from relation (21), we obtain

$$d(x\alpha x) = A(x) + x\alpha b\beta x^* \tag{22}$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$. By linearization of relation (22), we obtain

$$d(x\alpha y + y\alpha x) = d(x)\alpha y^* + d(y)\alpha x^* + x\alpha d(y) + y\alpha d(x) + x\alpha b\beta y^* + y\alpha b\beta x^* \tag{23}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Replace y by $x\delta y + y\beta x$ in relation (23), we get

$$d(x\alpha x\delta y + x\alpha y\beta x + x\delta y\alpha x + y\beta x\alpha x) = d(x)\alpha y^*\delta x^* + d(x)\alpha x^*\beta y^* + x\alpha d(x)\alpha y^* + x\alpha d(y)\alpha x^* + x\alpha x\alpha d(y) + x\alpha y\alpha d(x) + x\alpha x\beta by^* + x\alpha y\beta b\delta x^* + d(x)\alpha y^*\beta x^* + d(y)\alpha x^*\beta x^* + x\alpha d(y)\beta x^* + y\alpha d(x)\alpha x^* + x\alpha b\beta y^*\alpha x^* + y\alpha b \beta x^*\alpha x^* + x\delta y\alpha d(x) + y\beta x\alpha d(x) + x\alpha b\beta y^*\delta x^* + x\alpha b\beta x^*\beta y^* + x\delta y\alpha b\beta x^* + y\beta x\alpha b\beta x^*$$

for all $x, y \in M$ and $\alpha, \beta, \delta \in \Gamma$. Hence and by using assumption (A), we get

$$d(x\alpha x\beta y + y\beta x\alpha x + 2d(x\alpha y\beta x) = 2(d(x)\alpha y^*\beta x^* + x\alpha d(y)\beta x^* + x\alpha y\beta d(x) + x\alpha y\beta b\delta x^* + x\alpha b\beta y^*\delta x^*) + d(x)\alpha x^*\beta y^* + x\alpha d(x)\alpha y^* + x\alpha x\alpha d(y) + x\alpha x\beta by^* + d(y)\alpha x^*\beta x^* + y\alpha d(x)\alpha x^* + y\alpha b\beta x^*\alpha x^* + y\beta x\alpha d(x) + x\alpha b\beta x^*\beta y^* + y\beta x\alpha b\beta x^* \tag{24}$$

for all $x,y \in M$ and $\alpha, \beta \in \Gamma$. Replace x by $x\alpha x$ in the relation (23) and using (22), we get

$$d(x\alpha x\beta y + y\beta x\alpha x) = d(x)\alpha x^*\beta y^* + x\alpha d(x)\beta y^* + x\alpha b\beta x^*\alpha y^* + d(y)\alpha x^*\beta x^* + x\alpha x\beta d(y) + y\alpha d(x)\alpha x^* + y\alpha x\beta d(x) + y\alpha x\alpha b\beta x^* + x\alpha x\alpha b\beta y^* + y\alpha b\beta x^*\alpha x^* \tag{25}$$

for all $x,y \in M$ and $\alpha, \beta \in \Gamma$. Comparing the relation (24) and (25) and using assumption (A), we get

$$d(x\alpha y\beta x) = d(x)\alpha y^*\beta x^* + x\alpha d(y)\beta x^* + x\alpha y\beta d(x) + x\alpha y\beta b\delta x^* + x\alpha b\beta y^*\alpha x^* \tag{26}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Using Lemma (2.2), then we get

$$d(x\alpha y\beta x) = d(x)\alpha y^*\beta x^* + x\alpha g(y)\beta x^* + x\alpha y\beta d(x) + x\alpha y\beta b\alpha x^* + x\alpha b\beta y^*\alpha x^* + x\alpha a\beta y^*\alpha x^* + x\alpha y\beta a\alpha x^*.$$

Hence,

$$d(x\alpha y\beta x) = d(x)\alpha y^*\beta x^* + x\alpha g(y)\beta x^* + x\alpha y\beta d(x) + x\alpha(a\alpha y^* + y\alpha a) + (b\beta y^* + y\beta b)\alpha x^* \tag{27}$$

for all $x,y \in M$ and $\alpha,\beta \in \Gamma$. Since $(a\alpha y^* + y\alpha a) + (b\beta y^* + y\beta b) = 0$ (see the relation (16) and (17)), then from relation (27) we obtain

$$d(x\alpha y\beta x) = d(x)\alpha y^*\beta x^* + x\alpha g(y)\beta x^* + x\alpha y\beta d(x)$$

for all $x,y \in M$ and $\alpha,\beta \in \Gamma$. And

$$g(x\alpha y\beta x) = g(x)\alpha y^*\beta x^* + x\alpha d(y)\beta x^* + x\alpha y\beta g(x)$$

for all $x,y \in M$ and $\alpha,\beta \in \Gamma$. Then we get (d, g) is a Γ*- derivation pair.

In the following proposition, we will add a condition on the above theorem to obtain a Jordan Γ*-derivation from Jordan Γ*-derivation pair.

To prove our main result we need the following lemmas.

Lemma 2.4. Let M be a 2-torsion free Γ-ring with involution with an identity element satisfies assumption (A) and let (d, g) be a Jordan Γ*-derivation pair such that $d(1) = g(1)$, then $d(x) = g(x)$ for all $x \in M$.

Proof. Define the mapping $f: M \to M$ by $f(x) = d(x) - g(x)$ for all $x \in M$. Then by using (5) and (6), we get

$$f(x\alpha x\beta x) = f(x)\alpha x^*\beta x^* - x\alpha f(x)\beta x^* + x\alpha x\beta f(x) \tag{28}$$

for all $x \in M$ and $\alpha,\beta \in \Gamma$. By linearization the relation (28), we get

$$\begin{aligned} &f(x\alpha x\beta y + y\alpha x\beta x + x\alpha y\beta y + y\alpha y\beta x + x\alpha y\beta x + y\alpha x\beta y) = f(x)\alpha x^*\beta y^* \\ &+ f(x)\alpha y^*\beta x^* + f(x)\alpha y^*\beta y^* + f(y)\alpha x^*\beta y^* + f(y)\alpha y^*\beta x^* + f(y)\alpha x^* \\ &\beta x^* - x\alpha f(x)\beta y^* - x\alpha f(y)\beta x^* - y\alpha f(x)\beta x^* - y\alpha f(y)\beta x^* - y\alpha f(x)\beta y^* \\ &- x\alpha f(y)\beta y^* + x\alpha y\beta f(x) + y\alpha x\beta f(x) + y\alpha y\beta f(x) + x\alpha y\beta f(y) + \\ &y\alpha x\beta f(y) + x\alpha x\beta f(y) \end{aligned} \tag{29}$$

for all $x,y \in M$ and $\alpha,\beta \in \Gamma$. From relation (29), one obtains (see how the relation (11) was obtained from (9))

$$\begin{aligned} &f(x\alpha x\beta y + y\alpha x\beta x + x\alpha y\beta x) = f(x)\alpha x^*\beta y^* + f(x)\alpha y^*\beta x^* + f(y)\alpha x^*\beta x^* \\ &- x\alpha f(x)\beta y^* - x\alpha f(y)\beta x^* - y\alpha f(x)\beta x^* + x\alpha y\beta f(x) + y\alpha x\beta f(x) + x\alpha x\beta f(y) \end{aligned} \tag{30}$$

for all $x,y \in M$ and $\alpha,\beta \in \Gamma$. Replace x by 1 in relation (30), we get

$$2f(y) = f(1)y^* + yf(1) \tag{31}$$

for all $y \in M$. Since $f(1) = 0$, then from relation (31), we get $d(x) = g(x)$.

Lemma 2.5. Let M be a 2-torsion free Γ-ring with an identity element satisfies assumption (A), and let $d : M \to M$ be an additive mapping satisfies

$$d\left(x\alpha y\beta x\right) = d\left(x\right)\alpha y^{*}\beta x^{*} + x\alpha d\left(y\right)\beta x^{*} + x\alpha y\beta d\left(x\right) \tag{32}$$

Then d is a Jordan Γ*-derivation.

Proof. Setting $x = y = 1$ in relation (32), we get $d(1) = 0$. Replace y by 1 in relation (32), therefore we obtain

$$d\left(x\alpha x\right) = d\left(x\right)\alpha x^{*} + x\alpha d\left(x\right)$$

for all $x \in M$ and $\alpha \in \Gamma$. Then d is a Jordan Γ*-derivation.

Theorem 2.6. Let M be a 6-torsion free Γ-ring with involution with an identity element satisfying assumption (A), and let (d, g) be a Jordan Γ*-derivation pair such that $d\left(1\right) = g\left(1\right)$, then d is a Jordan Γ*-derivation.

Proof. By using Theorem 2.3, we get (d, g) is a Γ*-derivation pair, and by Lemma (2.4), d satisfies relation (32). Hence by using Lemma 2.5, we get d is a Jordan Γ*-derivation.

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