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Ranking of Generalized Dodecagonal Fuzzy Numbers Using Centroid of Centroids

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Abstract. The fuzzy set theory has been applied in many fields such as management, engineering etc. In modern management applications ranking using fuzzy numbers is the most important aspect in decision making process. In this paper, we proposed the ranking of generalized dodecagonal fuzzy numbers (DoFN). The proposed approach is based on rank, mode, divergence and spread.

Keywords: Ranking function; centroid points; generalized dodecagonal fuzzy numbers.

AMS Mathematics Subject Classification (2010): 03E72, 62F07.

1. Introduction

Ranking of fuzzy number play an important role in decision making. Zadeh [23] introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. The method for ranking was first proposed by Jain [9]. Yager [21] first used horizontal coordinate of the centroid point in ranking fuzzy numbers. Murakami et al. [14] have used both the horizontal and vertical coordinates of the centroid point as the ranking index. In Kaufmann and Gupta [12] proposed an approach for the ranking of fuzzy numbers.

Campos and Gonzalez [1] proposed a subjective approach for ranking fuzzy numbers. Cheng [6] presented a method for ranking fuzzy numbers by using the distance method. Chu and Tsao [7] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [8] presented a centroid-index method for ranking fuzzy numbers. Chen and Chen [3] presented a method for ranking generalized trapezoidal fuzzy numbers. Wang and Lee [20] used the centroid concept in developing their ranking index.

Chen and Tang [5] proposed a method for ranking p-norm trapezoidal fuzzy numbers. Since then several methods have been proposed by various researchers which includes ranking fuzzy numbers using maximizing and minimizing set [2] decomposition principle and signed distance [22], different heights and spreads[4], rank, mode, divergence and spread [13], area compensation distance method [14], Ordering of trapezoidal fuzzy numbers[19]. Gani and Mohamed [10] used a new ranking method for ranking the fuzzy numbers. Rajarajeswari and Sudha [13] proposed a ranking method for ordering fuzzy numbers based on Area, Mode, divergence, Spreads and Weights of generalized (non-normal) hexagonal fuzzy numbers. Rajarajeswari and Sudha [15] proposed a new method on the incentre of centroids and uses of Euclidean distance to ranking generalized hexagonal fuzzy numbers.

2. Preliminaries

2.1. Fuzzy set [9]

A fuzzy set $\tilde{A}$ is defined by $\tilde{A}$= {(x, μ_A(x)) : x ϵ A, μ_A(x) ϵ [0,1] }. In the pair (x, μ_A(x)), the first element x belong to the classical set A, the second element μ_A(x), belong to the interval [0, 1] called Membership function.

2.2. Fuzzy number [9]

A fuzzy set $\tilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number,
(i) $\tilde{A}$ must be a normal fuzzy set; (ii) $^{\alpha}\tilde{A}$ must be closed interval for every α ϵ [0,1]
(iii) the support of $\tilde{A}$, $^{0+}\tilde{A}$, must be bounded.

2.3. Dodecagonal fuzzy numbers [18]

A fuzzy number $\tilde{A}$ is a DoFN denoted by $\tilde{A}$ = ($a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12}$) where $a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12}$ are real numbers and its membership function is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \le a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k_1 & a_2 \le x \le a_3 \\ k_1 + (k_2 - k_1)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ k_2 & a_4 \le x \le a_5 \\ k_2 + (1 - k_2)\left(\frac{x-a_5}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ 1 & a_6 \le x \le a_7 \\ k_2 + (1 - k_2)\left(\frac{a_8-x}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ k_2 & a_8 \le x \le a_9 \\ k_1 + (k_2 - k_1)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ k_1 & a_{10} \le x \le a_{11} \\ k_1\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11} \le x \le a_{12} \\ 0 & a_{12} \le x \end{cases} \qquad \text{where } 0 < k_1 < k_2 < 1$$

2.4. Generalized dodecagonal fuzzy number

A fuzzy number $\tilde{A}$ = ($a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12}$;u,v,w) is said to be generalized dodecagonal fuzzy number if its membership function is given be

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \le a_1 \\ u\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ u & a_2 \le x \le a_3 \\ u + (v - u)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ v & a_4 \le x \le a_5 \\ v + (w - v)\left(\frac{x-a_5}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ w & a_6 \le x \le a_7 \\ v + (w - v)\left(\frac{a_8-x}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ k_2 & a_8 \le x \le a_9 \\ u + (v - u)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ u & a_{10} \le x \le a_{11} \\ u\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11} \le x \le a_{12} \\ 0 & a_{12} \le x \end{cases} \quad \text{where } 0 < u < v < w \le 1$$

3. Proposed ranking method of dodecagonal fuzzy number

The centroid of a DoFN is considered to be the balancing point of the dodecagon (Fig. 1). Divide the dodecagon into eight triangles and one hexagon ABM, BCN, CDO, DEP, HIS, IJT, JKU, KWV and EFGHRQ respectively. Let the centroids of nine trapezoids be G_1, G_2, G_3, G_4, G_6, G_7, G_8, G_9 and G_5 respectively.

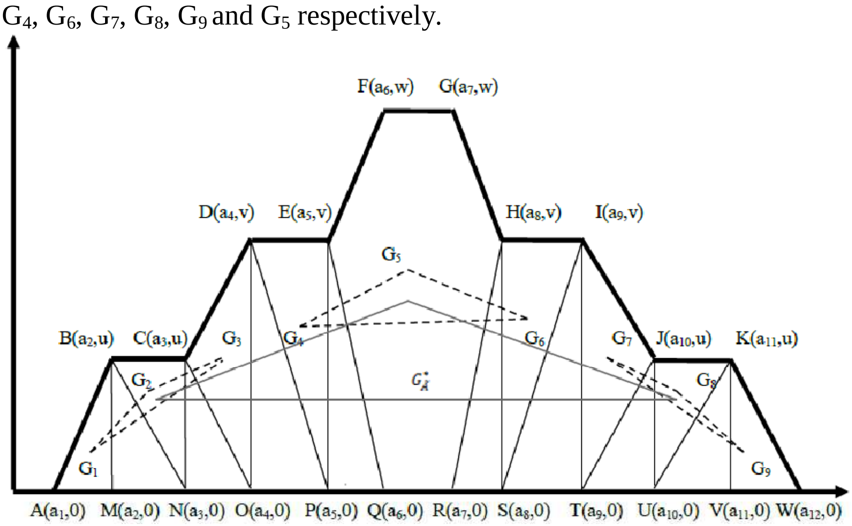

Figure 1: Generalized dodecagonal fuzzy number

The centroid of the nine plane figure is

$G_1=\left(\frac{a_1+2a_2}{3}, \frac{u}{3}\right)$; $G_2=\left(\frac{a_2+2a_3}{3}, \frac{2u}{3}\right)$; $G_3=\left(\frac{a_3+2a_4}{3}, \frac{u+v}{3}\right)$;

$G_4=\left(\frac{a_4+2a_5}{3}, \frac{2v}{3}\right)$; $G_5=\left(\frac{a_5+2(a_6+a_7)+a_8}{6}, \frac{v+w}{3}\right)$; $G_6=\left(\frac{2a_8+a_9}{3}, \frac{2v}{3}\right)$;

$G_7=\left(\frac{2a_9+a_{10}}{3}, \frac{u+v}{3}\right)$; $G_8=\left(\frac{2a_{10}+a_{11}}{3}, \frac{2u}{3}\right)$; $G_9=\left(\frac{2a_{11}+a_{12}}{3}, \frac{u}{3}\right)$

(a) G_1, G_2 and G_3 are non-collinear and they form triangle. We define the centroid G_1^* of the triangle with vertices G_1, G_2 and G_3 as $G_1^* = \left(\frac{a_1+3(a_2+a_3)+2a_4}{9}, \frac{4u+v}{9}\right)$.

(b) G_4, G_5 and G_6 are non-collinear and they form triangle. We define the centroid G_2^* of the triangle with vertices G_4, G_5 and G_6 as $G_2^* = \left(\frac{2(a_4+a_6+a_7+a_9)+5(a_5+a_8)}{18}, \frac{5v+w}{9}\right)$

(c) G_7, G_8 and G_9 are non-collinear and they form triangle. We define the centroid G_3^* of the triangle with vertices G_7, G_8 and G_9 as $G_3^* = \left(\frac{2a_9+3(a_{10}+a_{11})+a_{12}}{9}, \frac{4u+v}{9}\right)$

Also, G_1^*, G_2^* and G_3^* are non-collinear and they form triangle. We define the centroid $G_{\tilde{A}}^*$ of the triangle with vertices G_1^*, G_2^* and G_3^* as

$$G_{\tilde{A}}^* = \left(\frac{2(a_1+a_6+a_7+a_{12})+6(a_2+a_3+a_4+a_9+a_{10}+a_{11})+5(a_5+a_8)}{54}, \frac{8u+7v+w}{27}\right)$$

If we take $u = \frac{w}{3}$ and $v = \frac{2w}{3}$ then

$$G_{\tilde{A}}^* = \left(\frac{2(a_1+a_6+a_7+a_{12})+6(a_2+a_3+a_4+a_9+a_{10}+a_{11})+5(a_5+a_8)}{54}, \frac{25w}{81}\right) \quad (1)$$

The ranking function of the generalized DoFN $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};w)$, which maps the set of all fuzzy number to a set of real numbers is defined as

$$R(\tilde{A}) = \left(\frac{2(a_1+a_6+a_7+a_{12})+6(a_2+a_3+a_4+a_9+a_{10}+a_{11})+5(a_5+a_8)}{54}\right)\left(\frac{25w}{81}\right) \quad (2)$$

This is the area between the centroid of the centroids $G_{\tilde{A}}^*$ as defined in (1) and (2) the original point.

The mode of the generalized DoFN $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};w)$ is

$$\text{Mode} = \frac{w}{2}(a_6 + a_7) \quad (3)$$

The divergence of the generalized DoFN $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};w)$ is

$$\text{Divergence} = w(a_{12} - a_1) \quad (4)$$

The left spread of the generalized DoFN $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};w)$ is

$$\text{Left spread} = \frac{w}{3}(a_6 - a_1) \quad (5)$$

The right spread of the generalized DoFN $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};w)$ is

$$\text{Right spread} = \frac{w}{3}(a_{12} - a_7) \quad (6)$$

4. Some important results

In this section some important results, that are useful for the proposed approach, are proved.

Proposition 4.1. Let $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};w_1)$ and $\tilde{B}= (b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},b_{12};w_2)$ be two generalized dodecagonal fuzzy numbers such that

(i) $R(\tilde{A}) = R(\tilde{B})$, (ii) mode $(\tilde{A})$ = mode $(\tilde{B})$ and (iii) divergence $(\tilde{A})$ = divergence $(\tilde{B})$ then

(a) Left spread $(\tilde{A})$ > Left spread $(\tilde{B})$ iff $a_6w_1 > b_6w_2$

(b) Left spread $(\tilde{A})$ < Left spread $(\tilde{B})$ iff $a_6w_1 < b_6w_2$

(c) Left spread $(\tilde{A})$ = Left spread $(\tilde{B})$ iff $a_6w_1 = b_6w_2$

Proof: We have

(i) $R(\tilde{A}) = R(\tilde{B})$

i.e., $\left(\frac{2(a_1+a_6+a_7+a_{12})+6(a_2+a_3+a_4+a_9+a_{10}+a_{11})+5(a_5+a_8)}{54}\right)\left(\frac{25w_1}{81}\right) =$
$\left(\frac{2(b_1+b_6+b_7+b_{12})+6(b_2+b_3+b_4+b_9+b_{10}+b_{11})+5(b_5+b_8)}{162}\right)\left(\frac{25w_2}{81}\right)$ (7)

(ii) mode ($\tilde{A}$) = mode ($\tilde{B}$)

i.e., $\frac{w_1}{2}(a_6+a_7) = \frac{w_2}{2}(b_6+b_7)$ (8)

(iii) divergence ($\tilde{A}$) = divergence ($\tilde{B}$)

i.e., $(a_{12}-a_1)w_1=(b_{12}-b_1)w_2$ (9)

Solving(7), (8) and (9) we get

$a_1w_1= b_1w_2$

$(a_2+a_3+ a_4+a_9+a_{10}+a_{11})w_1=(b_2+b_3+b_4+b_9+b_{10}+b_{11})w_2$

$(a_5+a_8)w_1=(b_5+b_8)w_2$

(a) Left spread ($\tilde{A}$) > Left spread ($\tilde{B}$)

iff $(a_6-a_1)\frac{w_1}{3} > (b_6-b_1)\frac{w_2}{3}$

iff $(a_6-a_1)w_1> (b_6-b_1)w_2$

iff $a_6w_1 > b_6w_2$ ($\because a_1w_1= b_1w_2$)

Hence, Left spread ($\tilde{A}$) > Left spread ($\tilde{B}$) iff $a_6w_1 > b_6w_2$

(b) Left spread ($\tilde{A}$) < Left spread ($\tilde{B}$)

iff $(a_6-a_1)\frac{w_1}{3} < (b_6-b_1)\frac{w_2}{3}$

iff $(a_6-a_1)w_1< (b_6-b_1)w_2$

iff $a_6w_1 < b_6w_2$ ($\because a_1w_1= b_1w_2$)

Hence, Left spread ($\tilde{A}$) < Left spread ($\tilde{B}$) iff $a_6w_1 < b_6w_2$

(b) Left spread ($\tilde{A}$) = Left spread ($\tilde{B}$)

iff $(a_6-a_1)\frac{w_1}{3} = (b_6-b_1)\frac{w_2}{3}$

iff $(a_6-a_1)w_1= (b_6-b_1)w_2$

iff $a_6w_1 = b_6w_2$ ($\because a_1w_1= b_1w_2$)

Hence, Left spread ($\tilde{A}$) = Left spread ($\tilde{B}$) iff $a_6w_1 = b_6w_2$

Corollary 4.1. All the results of proposition 4.1 also hold for right spread.

Proposition 4.2. Let $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};w_1)$ and $\tilde{B}=(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},b_{12};w_2)$ be two generalized dodecagonal fuzzy numbers such that

(i) R($\tilde{A}$) = R($\tilde{B}$), (ii) mode ($\tilde{A}$) = mode ($\tilde{B}$) and (iii) divergence ($\tilde{A}$) = divergence ($\tilde{B}$) then

(a) Left spread ($\tilde{A}$) > Left spread ($\tilde{B}$) iff Right spread ($\tilde{A}$) > Right spread ($\tilde{B}$)

(b) Left spread ($\tilde{A}$) < Left spread ($\tilde{B}$) iff Right spread ($\tilde{A}$) < Right spread ($\tilde{B}$)

(c) Left spread ($\tilde{A}$) = Left spread ($\tilde{B}$) iff Right spread ($\tilde{A}$) = Right spread ($\tilde{B}$)

Proof: From proposition 4.1, we have

$a_1w_1= b_1w_2$

$(a_2+a_{11})w_1=(b_2+b_{11})w_2$

$(a_3+a_{10})w_1=(b_3+b_{10})w_2$

$(a_4+a_9)w_1=(b_4+b_9)w_2$

$(a_5+a_8)w_1=(b_5+b_8)w_2$

(a) Left spread ($\tilde{A}$) > Left spread ($\tilde{B}$)

iff $a_6w_1 > b_6w_2$ (from proposition 4.1)
iff $a_7w_1 < b_7w_2$ ($\because \frac{w_1}{2}(a_6+a_7) = \frac{w_2}{2}(b_6+b_7)$ or $w_1(a_6+a_7) = w_2(b_6+b_7)$)
iff $-a_7w_1 > -b_7w_2$
iff $(a_{12}-a_7)w_1 > (b_{12}w_2-b_7)w_2$ ($\because a_{12}w_1 = b_{12}w_2$)
iff Right spread $(\tilde{A})$ > Right spread $(\tilde{B})$
Similarly (b) and (c) can be proved.

5. Proposed approach for ranking of generalized dodecagonal fuzzy numbers

Let $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};w_1)$ & $\tilde{B}=(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},b_{12};w_2)$ be two generalized dodecagonal fuzzy numbers then use the following steps to compare $\tilde{A}$ and $\tilde{B}$

Step 1: Find R $(\tilde{A})$ and R $(\tilde{B})$
Case (i) If R $(\tilde{A})$ > R $(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If R $(\tilde{A})$ < R $(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (ii) If R $(\tilde{A})$ = R $(\tilde{B})$ then go to step 2.

Step 2: Find mode $(\tilde{A})$ and mode $(\tilde{B})$
Case (i) If mode $(\tilde{A})$ > mode $(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If mode $(\tilde{A})$ < mode $(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (ii) If mode $(\tilde{A})$ = mode $(\tilde{B})$ then go to step 3.

Step 3: Find divergence $(\tilde{A})$ and divergence $(\tilde{B})$
Case (i) If divergence $(\tilde{A})$ > divergence $(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If divergence $(\tilde{A})$ < divergence $(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (ii) If divergence $(\tilde{A})$ = divergence $(\tilde{B})$ then go to step 4.

Step 4: Find Left Spread $(\tilde{A})$ and Left Spread $(\tilde{B})$
Case (i) If Left Spread $(\tilde{A})$ > Left Spread $(\tilde{B})$
i.e. $a_6w_1 > b_6w_2$ then $\tilde{A} \succ \tilde{B}$ (from proposition 4.1)
Case (ii) If Left Spread $(\tilde{A})$ < Left Spread $(\tilde{B})$
i.e. $a_6w_1 < b_6w_2$ then $\tilde{A} \prec \tilde{B}$ (from proposition 4.1)
Case (ii) If Left Spread $(\tilde{A})$ = Left Spread $(\tilde{B})$
i.e. $a_6w_1 < b_6w_2$ then go to step 5. (from proposition 4.1)

Step 5: Find w_1 and w_2
Case (i) If $w_1 > w_2$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If $w_1 < w_2$ then $\tilde{A} \prec \tilde{B}$
Case (ii) If $w_1 = w_2$ then $\tilde{A} \sim \tilde{B}$.

6. Numerical problems

Example 6.1. Let $\tilde{A}$ = (0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1,1.1,1.2;0.7) and $\tilde{B}$ = (0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2,2.2,2.4;0.35)

Step 1: R $(\tilde{A})$ = 0.14 and R $(\tilde{B})$ =0.14. Since R $(\tilde{A})$ = R $(\tilde{B})$ go to step 2
Step 2: Mode $(\tilde{A})$ = 0.455 and Mode $(\tilde{B})$ =0.455. Since Mode $(\tilde{A})$ = Mode $(\tilde{B})$ go to step 3
Step 3: Divergence $(\tilde{A})$ = 0.77 and Divergence $(\tilde{B})$ = 0.77.
Since Divergence $(\tilde{A})$ = Divergence $(\tilde{B})$ go to step 4
Step 4: Left Spread $(\tilde{A})$ = 0.117 and Left Spread $(\tilde{B})$ = 0.117.
Since Left Spread $(\tilde{A})$ = Left Spread $(\tilde{B})$ go to step 5

Step 5: $w_1 = 0.7$ and $w_2 = 0.35$. Since $w_1 > w_2$ then $\widetilde{A} \succ \widetilde{B}$

Example 6.2. Let $\widetilde{A}$ = (0.15,0.2,0.21,0.26,0.29,0.31,0.33,0.37,0.39,0.45,0.48,0.49;0.4) and $\widetilde{B}$ = (0.45,0.49,0.51,0.55,0.57,0.6,0.62,0.67,0.69,0.72,0.73,0.79;0.8)
Step 1: R ($\widetilde{A}$) = 0.041 and R ($\widetilde{B}$) = 0.152. Since R ($\widetilde{A}$) < R ($\widetilde{B}$), $\widetilde{A} \prec \widetilde{B}$

Example 6.3. Let $\widetilde{A}$ = (0.43,0.45,0.5,0.6,0.7,0.8,1,1.1,1.2,1.35,1.4,1.5;1) and $\widetilde{B}$ = (0.28, 0.45,0.5,0.6,0.75,0.8,0.9,1.15,1.2,1.3,1.45,1.5;1)
Step 1: R ($\widetilde{A}$) = 0.283 and R ($\widetilde{B}$) = 0.283. Since R ($\widetilde{A}$) = R ($\widetilde{B}$), go to step 2
Step 2: Mode ($\widetilde{A}$) = 0.9 and Mode ($\widetilde{B}$) = 0.85. Since Mode ($\widetilde{A}$) > Mode ($\widetilde{B}$), $\widetilde{A} \succ \widetilde{B}$

Example 6.4. Let $\widetilde{A}$ = (0.2,0.4,0.6,0.8,1.15,1.2,1.4,1.65,1.8,2,2.2,2.4;1) and $\widetilde{B}$ = (0.1,0.4, 0.7,0.8,1,1.2,1.4,1.6,1.8,2.1,2.2,2.4;1)
Step 1: R ($\widetilde{A}$) = 0.407 and R ($\widetilde{B}$) = 0. 407. Since R ($\widetilde{A}$) = R ($\widetilde{B}$) go to step 2
Step 2: Mode ($\widetilde{A}$) = 1.3 and Mode ($\widetilde{B}$) = 1.3. Since Mode ($\widetilde{A}$) = Mode ($\widetilde{B}$) go to step 3
Step 3: Divergence ($\widetilde{A}$) = 2.2 and Divergence ($\widetilde{B}$) =2.3.
Since Divergence ($\widetilde{A}$) < Divergence ($\widetilde{B}$), $\widetilde{A} \prec \widetilde{B}$

Example 6.5. Let $\widetilde{A}$ = (0.2,0.3,0.6,0.8,1,1.3,1.4,1.6,1.8,2,2.2,2.3;1) and $\widetilde{B}$ = (0.2,0.4,0.6,0.8,1,1.2,1.5,1.6,1.8,2,2.1,2.3;1)
Step 1: R ($\widetilde{A}$) = 0.403 and R ($\widetilde{B}$) = 0.403. Since R ($\widetilde{A}$) = R ($\widetilde{B}$) go to step 2
Step 2: Mode ($\widetilde{A}$) = 1.35 and Mode ($\widetilde{B}$) = 1.35. Since Mode ($\widetilde{A}$) = Mode ($\widetilde{B}$) go to step 3
Step 3: Divergence ($\widetilde{A}$) = 2.1 and Divergence ($\widetilde{B}$) = 2.1.
Since Divergence ($\widetilde{A}$) = Divergence ($\widetilde{B}$) go to step 4
Step 4: Left spread ($\widetilde{A}$) = 0.36 and Left Spread ($\widetilde{B}$) = 0.33.
Since Left Spread ($\widetilde{A}$) > Left Spread ($\widetilde{B}$), $\widetilde{A} \succ \widetilde{B}$

7. Conclusion

In this paper, we proposed a simpler and easier approach for ranking of generalized dodecagonal fuzzy number by using centroid of centroids. Also, proposed method is illustrated with the help of numerical example.

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