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Product Connectivity Leap Index and ABC Leap Index of Helm Graphs

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

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Abstract. Recently, some leap Zagreb indices of a graph based on the second degrees of vertices were introduced. In this paper, we propose the product connectivity leap index and *ABC* leap index of a graph. We compute the sum connectivity leap index, product connectivity leap index, ABC leap index and geometric-arithmetic leap index of helm graphs.

Keywords: connectivity leap indices, helm graph

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*35

1. Introduction

We consider only finite, simple connected graphs. The degree $d_G(v)$ of a vertex v is the number of edges incident to v. The number of edges in a shortest path connecting any two vertices u and v of G is the distance between these two vertices u and v, and denoted by $d(u,v)$. For a positive integer k and $v \in V(G)$, the open neighborhood of v in G is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$. The k-distance degree of v in G is the number of k neighbors of v in G and denoted by $d_k(v)$, see [1]. Any undefined terminologies and notations may be found in [2].

In [3], Kulli proposed the sum connectivity leap index and geometric-arithmetic leap index, defined as

$$SL(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_2(u) + d_2(v)}}. \tag{1}$$

$$GAL(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_2(u) d_2(v)}}{d_2(u) + d_2(v)}. \tag{2}$$

Motivated by the above two definitions of connectivity leap indices, we introduce the product connectivity leap index and atom bond connectivity (ABC) leap index as follows:

$$PL(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_2(u)d_2(v)}}. \tag{3}$$

$$ABCL(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_2(u)+d_2(v)-2}{d_2(u)d_2(v)}}. \tag{4}$$

Recently, some novel variants of leap indices were introduced such as leap hyper-Zagreb indices [4], F-leap indices [5], minus leap and square leap indices [6], augmented leap index [7]. In recent years, some new connectivity indices have been introduced and studied such as sum connectivity index [8], product connectivity index [9], sum connectivity Revan index [10], geometric-arithmetic reverse and sum connectivity reverse indices [11], sum connectivity Gourava index [12], connectivity Banhatti indices [13]. Also some other connectivity indices were studied, for example, in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

In this paper, the connectivity leap indices of helm graphs are determined. For helm graphs see [277].

2. Helm graphs

A wheel W_{n+1}, $n\geq 3$ is the join of C_n and K_1. Clearly $|V(W_{n+1})|=n+1$ and $|E(W_{n+1})|=2n$. A helm graph, denoted by H_n, is a graph obtained from W_{n+1} by attaching an end edge to each rim vertex of W_{n+1}, where the vertices corresponding to C_n are known as rim vertices. A graph H_n is presented in Figure 1.

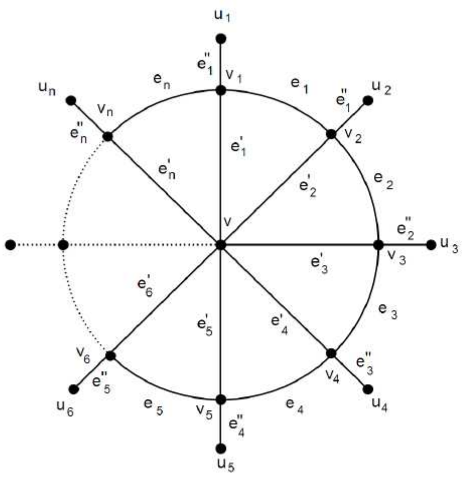

Figure 1: A graph H_n.

It is easy to see that $|V(H_n)|=2n+1$ and $|E(H_n)|=3n$. Then H_n has 3 types of the 2-distance degrees of edges as given in Table 1.

$d_2(u), d_2(v)\backslash uv \in E(H_n)$	$(n, n-1)$	$(3, n-1)$	$(n-1, n-1)$
Number of edges	N	n	N

Table 1

Theorem 1. The sum connectivity leap index of a helm graph H_n is

$$SL(H_n)=n\left(\frac{1}{\sqrt{2n-1}}+\frac{1}{\sqrt{n+2}}+\frac{1}{\sqrt{2n-2}}\right).$$

Proof: Let H_n be a helm graph with $2n$+1 vertices and $3n$ edges. From definition (1), we have

$$SL(H_n)=\sum_{uv\in E(H_n)}\frac{1}{\sqrt{d_2(u)+d_2(v)}}.$$

Then by using Table 1, we obtain

$$SL(H_n)=\left(\frac{1}{\sqrt{n+n-1}}\right)n+\left(\frac{1}{\sqrt{3+n-1}}\right)n+\left(\frac{1}{\sqrt{n-1+n-1}}\right)n$$

$$=n\left(\frac{1}{\sqrt{2n-1}}+\frac{1}{\sqrt{n+2}}+\frac{1}{\sqrt{2n-2}}\right).$$

Theorem 2. The product connectivity leap index of a helm graph H_n is

$$PL(H_n)=\frac{n}{\sqrt{n-1}}\left(\frac{1}{\sqrt{n}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{n-1}}\right).$$

Proof: Let H_n be a helm graph with $2n$+1 vertices and $3n$ edges. From definition (3), we have

$$PL(H_n)=\sum_{uv\in E(H_n)}\frac{1}{\sqrt{d_2(u)d_2(v)}}.$$

Then by using Table 1, we obtain

$$PL(H_n)=\left(\frac{1}{\sqrt{n+(n-1)}}\right)n+\left(\frac{1}{\sqrt{3+(n-1)}}\right)n+\left(\frac{1}{\sqrt{(n-1)(n-1)}}\right)n$$

$$=\frac{n}{\sqrt{n-1}}\left(\frac{1}{\sqrt{n}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{n-1}}\right).$$

Theorem 3. The geometric-arithmetic leap index of a helm graph H_n is

$$GAL(H_n)=2n\sqrt{n-1}\left(\frac{\sqrt{n}}{2n-1}+\frac{\sqrt{3}}{n+2}+\frac{\sqrt{n-1}}{2n-2}\right).$$

Proof: Let H_n be a helm graph with $2n$+1 vertices and $3n$ edges. From definition (2), we obtain

$$GAL(H_n)=\sum_{uv\in E(H_n)}\frac{2\sqrt{d_2(u)d_2(v)}}{d_2(u)+d_2(v)}.$$

Then by using Table 1, we deduce

$$GAL(H_n)=\left(\frac{2\sqrt{(n-1)}}{n+(n-1)}\right)n+\left(\frac{2\sqrt{3(n-1)}}{n+(n-1)}\right)n+\left(\frac{2\sqrt{(n-1)(n-1)}}{n-1+n-1}\right)n$$

$$= 2n\sqrt{n-1}\left(\frac{\sqrt{n}}{2n-1} + \frac{\sqrt{3}}{n+2} + \frac{\sqrt{n-1}}{2n-2}\right).$$

Theorem 4. The atom bond connectivity leap index of a helm graph H_n is

$$ABCL(H_n) = \frac{n}{\sqrt{n-1}}\left(\sqrt{\frac{2n-3}{n}} + \sqrt{\frac{n}{3}} + \sqrt{\frac{2n-4}{n-1}}\right).$$

Proof: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. From definition (4), we have

$$ABCL(H_n) = \sum_{uv \in E(H_n)} \sqrt{\frac{d_2(u) + d_2(v) - 2}{d_2(u)d_2(v)}}.$$

Then by using Table 1, we derive

$$ABCL(H_n) = \left(\sqrt{\frac{n+n-1-2}{n(n-1)}}\right)n + \left(\sqrt{\frac{3+n-1-2}{3(n-1)}}\right)n + \left(\sqrt{\frac{n-1+n-1-2}{(n-1)(n-1)}}\right)n$$

$$= \frac{n}{\sqrt{n-1}}\left(\sqrt{\frac{2n-3}{n}} + \sqrt{\frac{n}{3}} + \sqrt{\frac{2n-4}{n-1}}\right).$$

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REFERENCES

1. A.M.Naji, N.D.Soner and I.Gutman, On leap Zagreb indices of graphs, *Commun. Comb. Optim.,* 2 (2017) 99-117.
2. V.R.Kulli, *College Graph Theory,* Vishwa International Publications, Gulbarga, India (2012).
3. V.R.Kulli, Sum connectivity leap index and geometric-arithmetic leap indices of certain windmill graphs, submitted.
4. V.R.Kulli, Leap hyper-Zagreb indices and their polynomials of certain graphs, *International Journal of Current Research in Life Sciences,* 7(10) (2018) 2783-2791.
5. V.R.Kulli, On F-leap indices and F-leap polynomials of some graphs, submitted.
6. V.R.Kulli, Minus leap and square leap indices and their polynomials of some special graphs, *International Research Journal of Pure Algebra,* 8(11) (2018).
7. V.R.Kulli, On augmented leap index and its polynomial of some wheel graphs, submitted.
8. B.Zohu and N.Trinajstić, On a novel connectivity index, *J. Math. Chem*., 46 (2009) 1252-1270.
9. M. Randić, On characterization of molecular branching, *J. Am. Chem. Soc.,* 97 (1975) 6609-6615.
10. V.R.Kulli, The sum connectivity Revan index of silicate and hexagonal networks, *Annals of Pure and Applied Mathematics*, 14(3) (2017) 401-406.
11. V.R.Kulli, Geometric-arithmetic reverse and sum connectivity reverse indices of silicate and hexagonal networks, *International Journal of current Research in Science and Technology,* 3(10) (2017) 29-33.
12. V.R.Kulli, On the sum connectivity Gourava index, *International Journal of Mathematical Archive*, 8(7) (2017) 211-217.

13. V.R.Kulli, B.Chaluvaraju and H.S.Boregowda, Connectivity Banhatti indices for certain families of benzenoid systems, *Journal of Ultra Chemistry*, 13(4) (2017) 81-87.
14. V.R.Kulli, Atom bond connectivity reverse and product connectivity reverse indices of oxide and honeycomb networks, *International Journal of Fuzzy Mathematical Archive*, 15(1) (2018) 1-5.
15. V.R.Kulli, Multiplicative product connectivity and multiplicative sum connectivity indices of dendrimer nanostars, *International Journal of Engineering Sciences and Research Technology*, 7(2) (2018) 278-283.
16. V.R.Kulli, Multiplicative connectivity Revan indices of polycyclic aromatic hydrocarbons and benzenoid systems, *Annals of Pure and Applied Mathematics*,16(2) 2018) 337-343.
17. V.R.Kulli, Multiplicative connectivity Revan indices of certain families of benzenoid systems, *International Journal of Mathematical Archive*, 9(3) (2018) 235-241.
18. V.R.Kulli, Multiplicative connectivity Banhatti indices of benzenoid systems and polycyclic aromatic hydrocarbons, *Journal of Computer and Mathematical Sciences*, 9(3) (2018) 212-220.
19. V.R.Kulli, Multiplicative connectivity reverse indices of two families of dendrimer nanostars, *International Journal of Current Research in Life Sciences*, 7(2) (2018) 1102-1108.
20. V.R.Kulli, Degree based multiplicative connectivity indices of nanostructures, *International Journal of Current Advanced Research*, 7, 2(J) (2018).
21. V.R.Kulli, Edge version of multiplicative atom bond connectivity index of certain nanotubes and nanotorus, *International Journal of Mathematics and its Applications*, 6(1-E) (2018) 977-982.
22. V.R.Kulli, Multiplicative connectivity Banhatti indices of dendrimer nanostars, *Journal of Chemistry and Chemical Sciences*, 8(6) (2018) 964-973.
23. V.R.Kulli, Multiplicative atom bond connectivity and multiplicative geometric-arithmetic indices of chemical structures in drugs, *International Journal of Mathematical Archive*, 9(6) (2018) 155-163.
24. V.R.Kulli, Connectivity Revan indices of chemical structures in drugs, *International Journal of Engineering Sciences and Research Technology*, 7(5) (2018) 11-16.
25. V.R.Kulli, Computing reduced connectivity indices of certain nanotubes, *Journal of Chemistry and Chemical Sciences*, 8(11) (2018) 1174-1180.
26. V.R.Kulli and M.H.Akhbari, Multiplicative atom bond connectivity and multiplicative geometric-arithmetic indices of dendrimer nanostars, *Annals of Pure and Applied Mathematics*, 16(2) (2018) 429-436.
27. Shiladhar, A.M.Naji and N.D.Soner, Leap Zagreb indices of some wheel related graphs, *Journal of Computer and Mathematical Sciences*, 9(3) (2018) 221-231.