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Multiplicative Neighborhood Indices

V.R. Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

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Abstract. We define the multiplicative total neighborhood index, multiplicative F-neighborhood index and generalized multiplicative version of these indices and determine exact formulas for line graphs of subdivision graphs of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$.

Keywords: Multiplicative neighborhood indices, line graph, subdivision graph, nanostructures.

AMS Mathematics Subject Classification (2010): 05C07, 05C35, 05C90

1. Introduction

Throughout this paper G is a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The line graph $L(G)$ of G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The subdivision graph $S(G)$ of G is the graph obtained from G by replacing each of its edges by a path of length two. Let $N_G(v) = \{u : uv \in E(G)\}$. Let

$$S_G(v) = \sum_{u \in N_G(v)} d_G(u)$$

be the degree sum of neighbor vertices. For undefined term and notation, we refer to [1].

We need the following results.

Lemma 1. Let G be a graph with p vertices and q edges. Then $S(G)$ has $p+q$ vertices and $2q$ edges.

Lemma 2. Let G be a graph with p vertices and q edges. Then $L(G)$ has q vertices and $\frac{1}{2}\sum_{i=1} d_G(u_i)^2 - q$ edges.

A molecular graph is a graph such that its vertices represent to the atoms and the edges to the bonds. Chemical Graph Theory has an important effect on the development of Chemical Sciences. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices have been considered in Chemistry and have found some useful applications, especially in $QSPR/QSAR$ research see [2, 3].

Recently, the first neighborhood Zagreb index was introduced and studied by Basavangoud et al. [4] and Mondal et al. [5], defined as

$$NM_1(G) = \sum_{u \in V(G)} S_G(u)^2.$$

The fifth M_1 and M_2 Zagreb indices were introduced by Graovac et al. in [6], defined as

$$M_1G_5(G) = \sum_{uv \in E(G)} \left[S_G(u) + S_G(v)\right], \qquad M_2G_5(G) = \sum_{uv \in E(G)} S_G(u) S_G(v).$$

The fifth multiplicative M_1 and M_2 Zagreb indices were proposed by Kulli in [7], defined as

$$M_1G_5II(G) = \prod_{uv \in E(G)} \left[S_G(u) + S_G(v)\right], \qquad M_2G_5II(G) = \prod_{uv \in E(G)} S_G(u) S_G(v).$$

Recently, the fifth arithmetic-geometric index [8], fourth multiplicative ABC index [9], fifth multiplicative arithmetic-geometric index [10], fifth multiplicative hyper Zagreb indices [11], multiplicative atom bond connectivity index [12] were introduced and studied.

In [7], Kulli introduced the multiplicative first neighborhood index, defined as

$$NM_1II(G) = \prod_{u \in V(G)} S_G(u)^2.$$

We introduce the multiplicative total neighborhood index of a graph G, defined as

$$T_nII(G) = \prod_{u \in V(G)} S_G(u).$$

We propose the multiplicative F_1-neighborhood index of a graph and it is defined as

$$F_1NII(G) = \prod_{u \in V(G)} S_G(u)^3.$$

We now continue the generalization and define the general first multiplicative neighborhood index of a graph G as

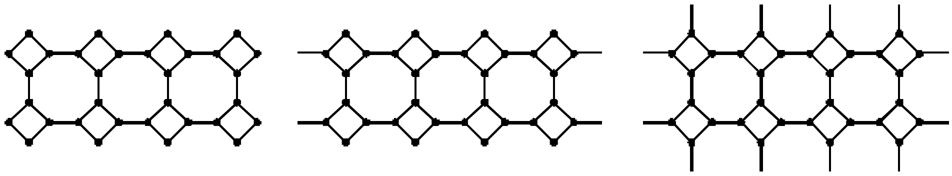
$$NM_1^a II(G) = \prod_{u \in V(G)} S_G(u)^a \tag{1}$$

where a is a real number.

In this paper, we compute the multiplicative first neighborhood index, multiplicative total neighborhood index, multiplicative F_1-neighborhood index and the general first multiplicative neighborhood index of line graphs of subdivision graphs of 2-*D* lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$.

2. 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$

In this section, the graphs of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[4,2]$ are shown in Figure 1.

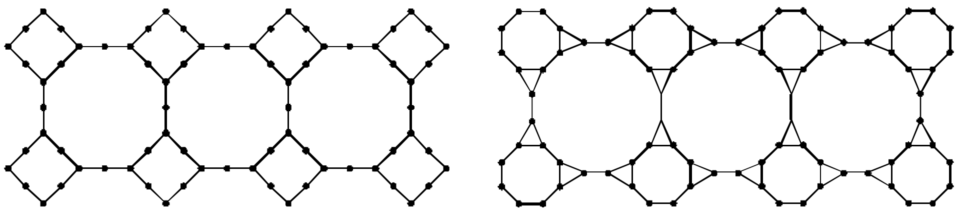


G_1 (a) H_1 (b) K_1 (c)

Figure 1:
(a) 2D-lattice of $TUC_4C_8[4, 2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8[4, 2]$ nanotorus

3. 2-D lattice of $TUC_4C_8[p, q]$

A subdivision graph of 2-D lattice and line graph of subdivision graph of 2-D lattice of $TUC_4C_8[4, 2]$ are shown in Figure 2(a) and Figure 2(b) respectively.



(a) (b)

(a) subdivision graph of 2D-lattice of $TUC_4C_8[4,2]$ (b) line graph of the subdivision graph of 2D-lattice of $TUC_4C_8[4, 2]$

Figure 2:

Let G be a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$, where p is the number of squares in a row and q is the number of rows of squares. The graph of 2-D lattice of $TUC_4C_8[p, q]$ has $4pq$ vertices and $6pq - p - q$ edges. By Lemma 1, a subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$ has $10pq - p - q$ vertices and $2(6pq - p - q)$ edges. Thus by Lemma 2, G has $2(6pq - p - q)$ vertices and $18pq - 5p - 5q$ edges. From Figure 2(b), we see that G has vertices of degree 2 or 3. The vertex partition based on the degree sum of neighbor vertices is obtained as given in Table 1 and Table 2.

$S_G(u)\backslash u\in V(G)$	4	5	8	9
Number of vertices	8	$4(p+q-2)$	$4(p+q-2)$	$2(6pq-5p-5q+4)$

Table 1: Vertex partition of G when $p > 1$, $q > 1$

$S_G(u)\backslash u\in V(G)$	4	5	8	9
Number of vertices	8	$4(p-1)$	$4(p-1)$	$2(p-1)$

Table 2: Vertex partition of G when $p > 1$, $q = 1$

Theorem 1. The general first multiplicative neighborhood index of a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$ is given by

$$NM_1^a II(G) = 4^{8a} \times 5^{4a(p+q-2)} \times 8^{4a(p+q-2)} 9^{2a(6pq-5p-5q+4)}, \text{ if } p>1,\ q>1, \qquad (2)$$

$$= 4^{8a} \times 5^{4a(p-1)} \times 8^{4a(p-1)} \times 9^{2a(p-1)}, \text{ if } p>1,\ q=1. \qquad (3)$$

Proof: Let G be a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$.
Case 1. Suppose $p > 1$, $q > 1$.
From equation (1) and by using Table 1, we deduce

$$NM_1^a II(G) = \prod_{u \in V(G)} S_G(u)^a$$
$$= (4^a)^8 \times (5^a)^{4(p+q-2)} \times (8^a)^{4(p+q-2)} \times (9^a)^{2(6pq-5p-5q+4)}$$
$$= 4^{8a} \times 5^{4a(p+q-2)} \times 8^{4a(p+q-2)} \times 9^{2a(6pq-5p-5q+4)}$$

Case 2. Suppose $p > 1$, $q = 1$.
By using equation (1) and Table 2, we derive

$$NM_1^a II(G) = \prod_{u \in V(G)} S_G(u)^a$$
$$= (4^a)^8 \times (5^a)^{4(p-1)} \times (8^a)^{4(p-1)} \times (9^a)^{2(p-1)}$$
$$= 4^{8a} \times 5^{4a(p-1)} \times 8^{4a(p-1)} \times 9^{2a(p-1)}.$$

We obtain the following results by Theorem 1.

Corollary 1.1. The multiplicative first neighborhood index of a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$ is given by

$$NM_1 II(G) = 4^{16} \times 5^{8(p+q-2)} \times 8^{8(p+q-2)} \times 9^{4(6pq-5p-5q+4)}, \text{ if } p>1, q>1,$$
$$= 4^{16} \times 5^{8(p-1)} \times 8^{8(p-1)} \times 9^{4(p-1)}, \qquad \text{if } p>1, q=1.$$

Proof: Put $a = 2$ in equations (2) and (3), we get the desired results.

Corollary 1.2. The multiplicative total neighborhood index of a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$ is given by

$$T_n II(G) = 4^8 \times 5^{4(p+q-2)} \times 8^{4(p+q-2)} \times 9^{2(6pq-5p-5q+4)}, \text{ if } p>1, q>1,$$
$$= 4^8 \times 5^{4(p-1)} \times 8^{4(p-1)} \times 9^{2(p-1)}, \qquad \text{if } p>1, q = 1.$$

Proof: Put $a = 1$ in equations (2) and (3), we get the desired results.

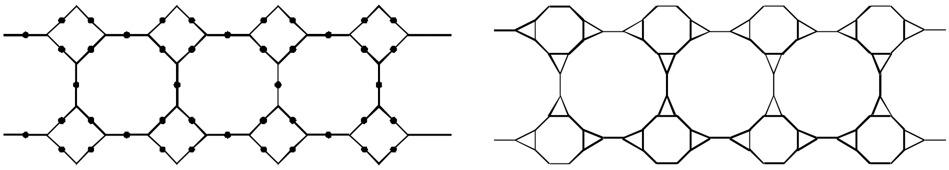
Corollary 1.3. The multiplicative F_1-neighborhood index of a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$ is given by

$$F_1 NII(G) = 4^{24} \times 5^{12(p+q-2)} \times 8^{12(p+q-2)} \times 9^{6(6pq-5p-5q+4)}, \qquad \text{if } p>1, q>1,$$
$$= 4^{24} \times 5^{12(p-1)} \times 8^{12(p-1)} \times 9^{6(p-1)}, \qquad \text{if } p>1, q = 1.$$

Proof: Put $a = 3$ in equations (2) and (3), we obtain the desired results.

4. $TUC_4C_8[p, q]$ nanotubes

In this section, a subdivision graph of $TUC_4C_8[4, 2]$ nanotube and a line graph of subdivision graph of $TUC_4C_8[4, 2]$ nanotube are presented in Figure 3(a) and Figure 3(b) respectively.



(a) Subdivision graph of $TUC_4C_8[4, 2]$ nanotube

(b) Line graph of subdivision graph of $TUC_4C_8[4, 2]$ nanotube

Figure 3:

Let H be a line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. A graph of $TUC_4C_8[p, q]$ nanotube has $4pq$ vertices and $6pq – p$ edges. By Lemma 1, a subdivision graph of $TUC_4C_8[p, q]$ nanotube has $10\ pq – p$ vertices and $12\ pq – 2p$ edges. Hence by Lemma 2, H has $12pq – p$ vertices and $12pq – 2p$ vertices and $18pq – 5p$ edges. From Figure 3(b), we see that H has vertices of degree 2 or 3. The vertex partition based on the degree sum of neighbor vertices is obtained as given in Table 3 and Table 4.

$S_H(u)\backslash u \in V(H)$	5	8	9
Number of vertices	$4p$	$4p$	$12pq – 10p$

Table 3: Vertex partition of H if $p > 1$, $q > 1$

$S_H(u)\backslash u \in V(H)$	5	8	9
Number of vertices	$4p$	$4p$	$2p$

Table 4: Vertex partition of H if $p > 1$, $q = 1$

Theorem 2. Let H be a line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then the general first multiplicative neighborhood index of H is

$$NM_1^a II(H) = 5^{4ap} \times 8^{4ap} \times 9^{a(12pq-10p)}, \text{ if } p>1,\ q>1, \tag{4}$$

$$= 5^{4ap} \times 8^{4ap} \times 9^{2ap}, \text{ if } p>1,\ q=1. \tag{5}$$

Proof: Case 1. Suppose $p >1$, $q >1$.
By using equation (1) and Table 3, we deduce

$$NM_1^a II(H) = \prod_{u \in V(H)} S_H(u)^a$$
$$= (5^a)^{4p} \times (8^a)^{4p} \times (9^a)^{12pq-10p}$$
$$= 5^{4ap} \times 8^{4ap} \times 9^{a(12pq-10p)}.$$

Case 2. Suppose $p > 1$, $q = 1$.
From equation (1) and by using Table 4, we derive

$$NM_1^a II(H) = \prod_{u \in V(H)} S_H(u)^a$$
$$= (5^a)^{4p} \times (8^a)^{4p} \times (9^a)^{2p}$$
$$= 5^{4ap} \times 8^{4ap} \times 9^{2ap}.$$

We establish the following results by Theorem 2.

Corollary 2.1. The multiplicative first neighborhood index of H is given by

$$NM_1II(H) = 5^{8p} \times 8^{8p} \times 9^{2(12pq-10p)}, \qquad \text{if } p>1,\ q>1,$$
$$= 5^{8p} \times 8^{8p} \times 9^{4p}, \qquad \text{if } p>1,\ q=1.$$

Proof: Put a = 2 in equations (4) and (5), we get the desired results.

Corollary 2.2. The multiplicative total neighborhood index of H is given by

$$T_nII(H) = 5^{4p} \times 8^{4p} \times 9^{12pq-10p}, \ \text{if } p>1,\ q>1,$$
$$= 5^{4p} \times 8^{4p} \times 9^{2p}, \qquad \text{if } p>1,\ q=1.$$

Proof: Put a = 1 in equations (4) and (5), we get the desired results

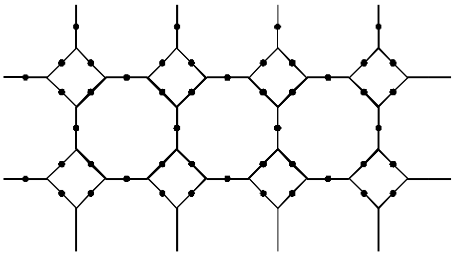
Corollary 2.3. The multiplicative F_1-neighborhood index of H is given by

$$F_1NII(H) = 5^{12p} \times 8^{12p} \times 9^{3(12pq-10p)}, \qquad \text{if } p>1,\ q>1,$$
$$= 5^{12p} \times 8^{12p} \times 9^{6p}, \qquad \text{if } p>1,\ q=1.$$

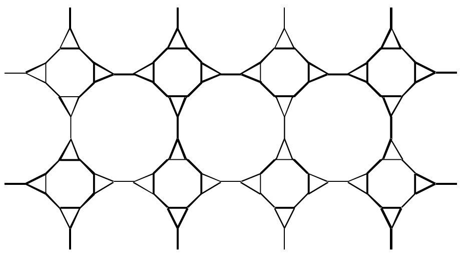
Proof: Put a = 3 in equations (4) and (5), we obtain the desired results.

5. $TUC_4C_8[p, q]$ nanoturus

In this section, a subdivision graph of TUC_4C_8[4, 2] nanotorus and line graph of subdivision graph of TUC_4C_8[4, 2] nanotorus are shown in Figure 4(a) and Figure 4(b).



(a) Subdivision graph of $TUC_4C_8[p, q]$ nanotorus



(b) Line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotorus

Figure 4:

Let K be a line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotorus. A graph of $TUC_4C_8[p, q]$ nanotorus has $4pq$ vertices and $6pq$ edges. By Lemma 1, a subdivision graph of $TUC_4C_8[p, q]$ nanotorus has $10pq$ vertices and $12pq$ edges. Therefore by Lemma 2, K has $12pq$ vertices and $18pq$ edges. Clearly the degree of each vertex is 3. The vertex partition based on the degree sum of neighbor vertices of each vertex is as given in Table 5.

$S_K(u) \setminus u \in V(K)$	9
Number of vertices	$12pq$

Table 5: Vertex partition of K

Theorem 3. Let K be a line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotorus. Then

(i) $NM_1^aII(K) = (9^a)^{12pq}$.

(ii) $NM_1II(K) = 9^{24pq}$.

(iii) $T_n II(K) = 9^{12pq}$.

(iv) $F_1 NII(K) = 9^{36pq}$.

Proof: By using definitions and Table 5, we get the desired results.

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