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Lie Ideals on Prime Γ-Rings with Jordan Right Derivations

***Omar Faruk*[1] and *Md. Mizanor Rahman*[2]**

Department of Mathematics, Jagannath University
Dhaka-1100, Bangladesh
[2]Email: mizanorrahman@gmail.com
[1]Corresponding author. Email: omar.fr92@yahoo.com

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Abstract. In this paper, we analyzed the basic properties and related theorem of Lie ideals on prime Γ-rings with Jordan right derivations. We mainly focused on the characterizations of 2-torsion free prime Γ-rings by using Lie ideals and Jordan right derivations. Our main objective is to prove the theorem that if M be a 2-torsion free prime Γ-ring and U be a Lie ideal of M such that $u\alpha u \in U$ for all $u, v \in U$ and $\alpha \in \Gamma$ and $d: M \to M$ is an additive mapping such that $d(u\alpha u) = 2\, d(u)\alpha u$ for all $u \in U$ and $\alpha \in \Gamma$, then $d(u\alpha v) = d(u)\alpha v + d(v)\alpha u$ for all $u, v \in U$ and $\alpha \in \Gamma$.

Keywords: Gamma Rings, Prime Gamma Rings, Lie ideals, Jordan Derivation, Jordan Right Derivation.

AMS Mathematics Subject Classification (2010): 05C25, 06F25

1. Introduction

The notion of a Γ-ring was first introduced as an extensive generalization of the concept of a classical ring. From its first appearance, the extensions and generalizations of various important results in the theory of classical rings to the theory of Γ-rings have been attracted a wider attentions as an emerging area of research to the modern algebraists to enrich the world of algebra. All over the world, many prominent mathematicians have worked out on this interesting area of research to determine many basic properties of Γ-rings and have executed more productive and creative results of Γ-rings in the last few decades.

Nobusawa [1] introduced Γ-ring as a generalization of ternary rings. Barnes [2] generalized the concept of Nobusawa Γ-ring and gave a concrete definition of a Γ-ring. Barnes, Luh and Kyuno studied the structure of Γ-rings and obtained various generalizations analogous to corresponding parts in ring theory. Since then some papers have been published on the topic of Γ-rings. Asci and Ceran [9] studied on a nonzero left derivation d on a prime Γ-ring M for which M is commutative with the conditions $d(U) \subseteq U$ and $d^2(U) \subseteq Z$, where U is an ideal of M and Z is the centre of M. Sapanci and Nakajima [10] defined a derivation and a Jordan derivation on Γ-rings and investigated a Jordan derivationon a certain type of completely prime Γ-ring which is a derivation. They proved that every Jordan derivation on a 2-torsion free completely prime Γ-rings is a derivation. They also gave examples of a derivation and a Jordan derivation of Γ-rings. Rahman and Paul [3,4] worked on Jordan derivations and Jordan left derivations of 2 and 3-torsion free semiprime Γ-rings. Rahman and Paul [5,6] have also worked derivations and generalized derivations on Lie ideals of completely semiprime Γ-rings. Reddy, Nagesh and Kumar [7] and Reddy, Kumar and Reddy [8] worked on left generalized derivations on prime γ-rings and results of symmetric reverse bi-derivations on prime rings respectively Kadhim et al. [11] studied on higher characterization on Γ-ring called Gamma * derivation pair and jordan gamma*− derivation pair on gamma-ring M with involution.

2. Preliminaries and notations

In this section some definitions have been discussed which are important for representing our main objective in the later sections.

Γ-ring 2.1. Let M and Γ be two abelian groups. If there is a mapping $M \times \Gamma \times M \to M$ such that the conditions

1. $x\alpha y \in M$
2. $(x+y)\alpha z = x\alpha z + y\alpha z, x(\alpha+\beta)y = x\alpha y + x\beta y,\ x\alpha(y+z) = x\alpha y + x\alpha z$
3. $(x\alpha y)\beta z = x\alpha(y\beta z)$

are satisfied for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, then M is called a Γ-ring.

Example 2.1. Let R be a ring of characteristic 2 having a unity element 1. Let $M = M_{\ 1,2}(R)$ and $\Gamma = \left\{\begin{pmatrix} n.1 \\ 0 \end{pmatrix} : n \in Z\right\}$, then M is a Γ-ring. If we assume $N = \{(x,x): x \in R\} \subseteq M$, then N is also a Γ-ring of M.

Prime Γ-ring 2.2. A Γ-ring M is said to be a prime Γ-ring if $x\Gamma M\Gamma y = 0$ (with $x, y \in M$) implies $x = 0$ or $y = 0$. In similar manner, M is said to be prime if the zero ideal is prime.

Semiprime Γ-ring 2.3. A Γ-ring M is said to be a prime Γ-ring if $x\Gamma M\Gamma x = 0$ (with $x \in M$) implies $x = 0$.

Commutative Γ-ring 2.4. A Γ-ring M is said to be a commutative Γ-ring if $x\alpha y = y\alpha x$ for all $x, y \in M$ and $\alpha \in \Gamma$. Again, $[x,y]_\alpha = x\alpha y - y\alpha x$ is called a commutator of x and y with respect to α, where $x, y \in M$ and $\alpha \in \Gamma$.

Nilpotent ideal 2.5. An ideal A of aΓ-ring M is called nilpotent if $(A\Gamma)^n A = (A\Gamma A\Gamma \ldots \ldots \ldots \Gamma A\Gamma)A = 0$, where n is the least positive integer.

n-torsion free or characteristic not equal to n 2.6. A Γ-ring M is said to be n-torsion free or characteristic not equal to n, denoted as char. $M \neq n$, if $nx = 0$ implies $x = 0$ for all $x \in M$. M is said to be 2-torsion free if $2x = 0$ implies $x = 0$ for all $x \in M$.

Derivation 2.7. Let M be a Γ-ring and $d: M \to M$ an additive map. Then d is called a derivation if

$$d(x\alpha y) = d(x)\alpha y + x\alpha d(y) \qquad where \;\; x, y \in M, \alpha \in \Gamma.$$

Example 2.2. Let M be a Γ-ring. If we define the map $D: M \to M$ by $D\big((x,y)\big) = \big(d(x), d(y)\big)$ then D is a derivation on M. Let $d: M \to M$ defined by $d(A) = d\left(\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & y \\ 0 & 0 \end{pmatrix}$ then d is a derivation.

Right derivation 2.8. Let M be a Γ-ring and $d: M \to M$ an additive map. Then d is called a right derivation if

$$d(a\alpha b) = d(a)\alpha b + d(b)\alpha a \qquad where \;\; a, b \in M, \alpha \in \Gamma$$

Jordan derivation 2.9. Let M be a Γ-ring and $d: M \to M$ an additive map. Then d is called a Jordan derivation if

$$d(x\alpha x) = d(x)\alpha x + x\alpha d(x) \quad where \;\; x \in M, \alpha \in \Gamma.$$

Example 2.3. Let M be a Γ-ring. If we define the map $D: M \to M$ by $D\big((x,x)\big) = \big(d(x), d(x)\big)$ then D is a Jordan derivation on M.

Jordan right derivation 2.10. Let M be a Γ- ring and $d: M \to M$ an additive map. Then d is called a Jordan right derivation if

$$d(a\alpha a) = 2d(a)\alpha a \quad where \;\; a \in M, \alpha \in \Gamma.$$

3 Main result

In this section for the sake of clarity, we prefer to split our presentation into two parts. The first part concerns with lemmas and the second part deals with our main objective.

Lemma 3.1. Let $U \nsubseteq Z(M)$ be a Lie ideal of a 2-torsion free prime Γ-ring M and $a, b \in M$ such that $a\alpha U\beta b = 0$. Then $a = 0$ or $b = 0$.

Proof: Since M is 2-torsion free prime Γ-ring, there exists an ideal I of M such that $[I, M]_\alpha \subseteq U$ but $[I, M]_\alpha \nsubseteq Z(M)$. Now taking $u \in U$, $e \in I$ and $m \in M$, we have $[e\alpha a\alpha u, m]_\alpha \in [I, M]_\alpha \subseteq U$ and so

$$0 = a\alpha[e\alpha a\alpha u, m]_\beta \beta b$$

$= a\alpha[e\alpha a, m]_\alpha \beta u\beta b + a\alpha e\beta a\alpha[u,m]_\alpha \beta b$, $(since\ a\alpha b\beta c = a\beta b\alpha c$, for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma)$

$= a\alpha[e\alpha a, m]_\alpha \beta u\beta b$, $\quad since\ a\alpha[u,m]_\alpha \in a\alpha U\beta b$

$= a\alpha e\alpha a\alpha m\beta u\beta b - a\alpha m\alpha e\alpha a\beta u\beta b$

$= a\alpha e\alpha a\alpha m\beta u\beta b - a\alpha m\alpha e\beta a\alpha u\beta b$

$= a\alpha e\alpha a\alpha m\beta u\beta b$, $\quad$ By assumption.

Thus, $a\alpha I\alpha a\alpha M\beta U\beta b = 0$.

If $a \neq 0$ then $U\beta b = 0$, by the primeness of M. Now, if $u \in U$ and $m \in M$ then $u\alpha m - m\alpha u \in U$ and hence $(u\alpha m - m\alpha u)\beta b = 0$ implies $u\alpha m\beta b = 0$, that is $u\alpha M\beta b = 0$. Since $U \neq 0$, we must have$b = 0$. In the similar fashion, it can be shown that if $b \neq 0$ then $a = 0$.

Lemma 3.2. Let M be a 2-torsion free prime Γ-ring and let U be a Lie ideal of M such that $u\alpha u \in U$, for all $u \in U$ and $\alpha \in \Gamma$. If $d: M \to M$ is an additive mapping satisfying $d(u\alpha u) = 2d(u)\alpha u$for all $u \in U$ and $\alpha \in \Gamma$, then

(i) $d(u\alpha v + v\alpha u) = 2d(u)\alpha v + 2d(v)\alpha u$

(ii) $d(u\alpha v\beta u) = d(v)\beta u\alpha u + 3d(u)\beta v\alpha u - d(u)\beta u\alpha v$

(iii) $d(u\alpha v\beta w + w\alpha v\beta u) = d(v)\beta(u\alpha w + w\alpha u) + 3d(u)\beta v\alpha w$
$+3d(w)\beta v\alpha u - d(u)\beta w\alpha v - d(w)\beta u\alpha v$

(iv) $d(u)\beta u\alpha[u,v]_\alpha = d(u)\beta[u,v]_\alpha \alpha u$

(v) $(d(u\alpha v) - d(v)\alpha u - d(u)\alpha v)\beta[u,v]_\alpha = 0$

for all $u, v, w \in U$ and $\alpha, \beta \in \Gamma$.

Proof: $(\boldsymbol{i})$Since $u\alpha v + v\alpha u = (u+v)\alpha(u+v) - u\alpha u - v\alpha v$, we have $u\alpha v + v\alpha u \in U$ for all $u, v \in U$ and $\alpha \in \Gamma$. Then $d(u\alpha v + v\alpha u) = d((u+v)\alpha(u+v)) - d(u\alpha u) - d(v\alpha v)$ with our hypothesis yields the required result.

$(\boldsymbol{ii})$Replacing v by $u\beta v + v\beta u$ in (i), we have

$$d(u\alpha(u\beta v + v\beta u) + (u\beta v + v\beta u)\alpha u) = 2d(u)\alpha(u\beta v + v\beta u) + 2d(u\beta v + v\beta u)\alpha u$$

Since $u\alpha v + v\alpha u \in U$ and $a\alpha b\beta c = a\beta b\alpha c$, we get

$$d(u\alpha(u\beta v + v\beta u) + (u\beta v + v\beta u)\alpha u)$$
$$= 4d(v)\beta u\alpha u + 6d(u)\beta v\alpha u + 2d(u)\beta u\alpha v \qquad (1)$$

On the other hand

$$d(u\alpha(u\beta v + v\beta u) + (u\beta v + v\beta u)\alpha u) = d(u\alpha u\beta v + v\beta u\alpha u) + 2d(u\alpha v\beta u)$$
$$= 2d(v)\beta u\alpha u + 4d(u)\beta u\alpha v + 2d(u\alpha v\beta u)$$

Combining the above equation with (1)and using the condition that M is 2-torsion free, we obtain

$$d(u\alpha v\beta u) = d(v)\beta u\alpha u + 3d(u)\beta v\alpha u - d(u)\beta u\alpha v.$$

(***iii***)Replacing $u + w$ for u in (ii) and using $a\alpha b\beta c = a\beta b\alpha c$, we get

$$\begin{aligned} &d\big((u+w)\alpha v\beta(u+w)\big) \\ &= d(v)\beta u\alpha u + d(v)\beta w\alpha w + d(v)\beta(u\alpha w + w\alpha u) + 3d(u)\beta v\alpha u \\ &\quad +3d(u)\beta v\alpha w + 3d(w)\beta v\alpha u + 3d(w)\beta v\alpha w - d(u)\beta u\alpha v - d(u)\beta w\alpha v \\ &\quad -d(w)\beta u\alpha v - d(w)\beta w\alpha v \end{aligned} \tag{2}$$

On the other hand with $a\alpha b\beta c = a\beta b\alpha c$, we have

$$\begin{aligned} d((u+w)\alpha v\beta(u+w)) &= d(u\alpha v\beta u) + d(w\alpha v\beta w) + d(u\alpha v\beta w + w\alpha v\beta u) \\ &= d(v)\beta u\alpha u + 3d(u)\beta v\alpha u - d(u)\beta u\alpha v + d(v)\beta w\alpha w \\ &\quad +3d(w)\beta v\alpha w - d(w)\beta w\alpha v + d(u\alpha v\beta w + w\alpha v\beta u) \end{aligned} \tag{3}$$

Combining (2)and (3), we obtain

$$\begin{aligned} &d(u\alpha v\beta w + w\alpha v\beta u) \\ &\qquad = d(v)\beta(u\alpha w + w\alpha u) + 3d(u)\beta v\alpha w + 3d(w)\beta v\alpha u - d(u)\beta w\alpha v \\ &\qquad\quad - d(w)\beta u\alpha v. \end{aligned}$$

(***iv***)Since $u\alpha v + v\alpha u$ and $u\alpha v - v\alpha u$ are in U, we find that $2u\alpha v \in U$, for all $u, v \in U$ and $\alpha \in \Gamma$. By hypothesis, we have $d((u\alpha v)\beta(u\alpha v)) = 2d(u\alpha v)\beta u\alpha v$. Replacing w by $2u\beta v$ in (iii) with $a\alpha b\beta c = a\beta b\alpha c$ and the condition that M is 2-torsion free, we get

$$\begin{aligned} &d(u\alpha v\beta(u\beta v) + (u\beta v)\alpha v\beta u) \\ &\qquad = d(v)\beta(u\alpha u\beta v + u\alpha v\beta u) + 3d(u)\beta v\beta u\alpha v + 3d(u\beta v)\beta v\alpha u - d(u) \\ &\qquad\quad \beta u\beta v\alpha v - d(u\beta v)\beta u\alpha v \end{aligned} \tag{4}$$

On the other hand with $a\alpha b\beta c = a\beta b\alpha c$, we have

$$\begin{aligned} d(u\alpha v\beta(u\beta v) + (u\beta v)\alpha v\beta u) &= d\big((u\beta v)\alpha(u\beta v) + u\alpha v\beta v\beta u\big) \\ &= 2d(u\beta v)\beta u\alpha v + 2d(v)\beta v\beta u\alpha u + 3d(u)\beta v\beta u\alpha v - d(u)\beta u\beta v\alpha v) \end{aligned} \tag{5}$$

Combining (4) and (5), we have

$$d(u\beta v)\beta[u,v]_\alpha = d(v)\beta[u,v]_\beta \alpha u + d(u)\beta[u,v]_\beta \alpha v \tag{6}$$

Replacing $u + v$ for v in (6), we have

$$d\big(u\beta(u+v)\big)\beta[u,u+v]_\alpha = d(u+v)\beta[u,u+v]_\beta \alpha u + d(u)\beta[u,u+v]_\beta \alpha(u+v)$$

$$\Longrightarrow 2d(u)\beta u\beta u\alpha u + 2d(u)\beta u\beta u\alpha v - 2d(u)\beta u\beta u\alpha u - 2d(u)\beta u\beta v\alpha u$$
$$+ d(u\beta v)\beta u\alpha u + d(u\beta v)\beta u\alpha v - d(u\beta v)\beta u\alpha u - d(u\beta v)\beta v\alpha u$$
$$= d(u)\beta u\beta u\alpha u + d(u)\beta u\beta v\alpha u - d(u)\beta u\beta u\alpha u - d(u)\beta v\beta u\alpha u$$
$$+ d(v)\beta u\beta u\alpha u + d(v)\beta u\beta v\alpha u - d(v)\beta u\beta u\alpha u - d(v)\beta v\beta u\alpha u$$
$$+ d(u)\beta u\beta u\alpha u + d(u)\beta u\beta u\alpha v - d(u)\beta u\beta u\alpha u - d(u)\beta u\beta u\alpha v$$
$$+ d(u)\beta u\beta v\alpha u + d(u)\beta u\beta v\alpha v - d(u)\beta v\beta u\alpha u - d(u)\beta v\beta u\alpha v$$

$$\therefore 2d(u)\beta u\beta[u,v]_\alpha + d(u\beta v)\beta[u,v]_\alpha$$
$$= 2d(u)\beta[u,v]_\beta\alpha u + d(v)\beta[u,v]_\beta\alpha u + d(u)\beta[u,v]_\beta\alpha v \tag{7}$$

From (6) and (7), we get

$$d(u)\beta u\beta[u,v]_\alpha = d(u)\beta[u,v]_\beta\alpha u$$
$$\therefore d(u)\beta u\alpha[u,v]_\alpha = d(u)\beta[u,v]_\alpha\alpha u.$$

$(\boldsymbol{v})$ Linearizing (iv) on u, we have

$$d(u)\beta u\alpha[u,v]_\alpha + d(v)\beta v\alpha[u,v]_\alpha + d(v)\beta u\alpha[u,v]_\alpha + d(u)\beta v\alpha[u,v]_\alpha$$
$$= d(u)\beta[u,v]_\beta\alpha u + d(v)\beta[u,v]_\beta\alpha u + d(u)\beta[u,v]_\beta\alpha v + d(v)\beta[u,v]_\beta\alpha v,$$
$$for\ all\ u,v \in U and \alpha,\beta \in \Gamma.$$

Application of (iv) and (6) yield that

$$d(v)\beta u\alpha[u,v]_\alpha + d(u)\beta v\alpha[u,v]_\alpha = d(u\beta v)\beta[u,v]_\alpha$$

and hence

$$(d(u\alpha v) - d(v)\alpha u - d(u)\alpha v)\beta[u,v]_\alpha = 0, for\ all u,v \in U and \alpha,\beta \in \Gamma.$$

Lemma 3.3. Let M be a 2-torsion free Γ-ring and U a Lie ideal of M such that $u\alpha u \in U$ for all $u \in U$ and $\alpha \in \Gamma$. If $d: M \to M$ is an additive mapping satisfying $d(u\alpha u) = 2d(u)\alpha u$for all $u \in U$ and $\alpha \in \Gamma$ then

(i) $d([u,v]_\alpha)\beta[u,v]_\alpha = 0$

(ii) $d(v)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u + v\alpha u\alpha u) = 0,$
$for\ all\ u,v \in U\ and\ \alpha,\beta \in \Gamma.$

Proof: $(\boldsymbol{i})$ By Lemma 3.2(i) and Lemma 3.2(v), we get

$$d(u\alpha v + v\alpha u) = 2(d(u)\alpha v + d(v)\alpha u) \tag{8}$$

$$\text{and } (d(u\alpha v) - d(v)\alpha u - d(u)\alpha v)\beta[u,v]_\alpha = 0 \tag{9}$$

Combining (8) and (9), we have

$$(d(v\alpha u) - d(u)\alpha v - d(v)\alpha u)\beta[u,v]_\alpha = 0 \tag{10}$$

Using $(9) - (10)$, we get

$$d([u,v]_\alpha)\beta[u,v]_\alpha = 0\ for\ all\ u,v \in U\ and\ \alpha,\beta \in \Gamma.$$

$(\boldsymbol{ii})$ For any $u,v \in U$ and $\alpha,\beta \in \Gamma$, we have

$$d([u,v]_\alpha\beta[u,v]_\alpha) = 2d([u,v]_\alpha)\beta[u,v]_\alpha$$

By (i) we have

$$d([u,v]_\alpha \beta [u,v]_\alpha) = 0 \; for\ all\ u,v \in U\ and\ \alpha,\beta \in \Gamma \tag{11}$$

Since $2u\alpha v \in U$ for all $u,v \in U$ and $\alpha \in \Gamma$replacing u by $2u\beta v$ in $u\alpha v + v\alpha u \in U$ and $u\alpha v - v\alpha u \in U$ and adding the results with $a\alpha b\beta c = a\beta b\alpha c$ we find that $4v\alpha u\beta v \in U$ for all $u,v \in U$ and $\alpha,\beta \in \Gamma$. Replacing $4v\alpha u\beta v$ for v in Lemma 3.2(i) and since M is 2-torsoin free, we have

$$d(u\alpha(v\alpha u\beta v) + (v\alpha u\beta v)\alpha u) = 2(d(u)\alpha v\alpha u\beta v + d(v\alpha u\beta v)\alpha u) \tag{12}$$

Using (12) in (11) with $a\alpha b\beta c = a\beta b\alpha c$, we have

$$\begin{aligned} 0 &= d(u\alpha(v\alpha u\beta v) + (v\alpha u\beta v)\alpha u) - d(u\alpha(v\alpha v)\beta u) - d(v\alpha(u\alpha u)\beta v) \\ &= 2(d(u)\alpha v\alpha u\beta v + d(v\alpha u\beta v)\alpha\, u) - d(v\alpha v)\beta u\alpha u - 3d(u)\beta v\alpha v\alpha u \\ &\quad + d(u)\beta u\alpha v\alpha v - d(u\alpha u)\beta v\alpha v - 3d(v)\beta u\alpha u\alpha v + d(v)\beta v\alpha u\alpha u \\ &= -3d(v)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u + v\alpha u\alpha u) - d(u)\beta(u\alpha v\alpha v - 2v\alpha u\alpha v + v\alpha v\alpha u) \end{aligned}$$

and hence

$$\begin{aligned} 3d(v)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u + v\alpha u\alpha u) + d(u)\beta(u\alpha v\alpha v - 2v\alpha u\alpha v\ + v\alpha v\alpha u) \\ = 0, for\ all\ u,v \in U\ and\ \alpha,\beta \in \Gamma \end{aligned} \tag{13}$$

In view of Lemma 3.2(iv), we get

$$d(u)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u\ + v\alpha u\alpha u) = 0 \tag{14}$$

Replacing u by $u+v$ in (14), we get

$$\big(d(u) + d(v)\big)\beta\big((u\alpha u\alpha v - 2u\alpha v\alpha u\ + v\alpha u\alpha u) - (v\alpha v\alpha u - 2v\alpha u\alpha v + u\alpha v\alpha v)\big) = 0$$

Now using (14) in the above expression, we have

$$\begin{aligned} d(v)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u + v\alpha u\alpha u) - d(u)\beta(v\alpha v\alpha u - 2v\alpha u\alpha v + u\alpha v\alpha v) = 0, \\ for\ all\ u,v \in U\ and\ \alpha,\beta \in \Gamma \end{aligned} \tag{15}$$

Combining (13) and (15), we obtain

$$d(v)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u + v\alpha u\alpha u) = 0 \tag{16}$$

By (15) and (16), we arrive at

$$d(v)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u + v\alpha u\alpha u) = 0, for\ all\ u,v \in U\ and\ \alpha,\beta \in \Gamma.$$

Theorem 3.4. Let M be a 2-torsion free prime Γ-ring and U be a Lie ideal of M such that $u\alpha u \in U$ for all $u,v \in U$ and $\alpha \in \Gamma$. If $d: M \to M$ is an additive mapping such that $d(u\alpha u) = 2\ d(u)\alpha u$ for all $u \in U$ and $\alpha \in \Gamma$, then $d(u\alpha v) = d(u)\alpha v + d(v)\alpha u$ for all $u,v \in U$ and $\alpha \in \Gamma$.

Proof: Suppose U is a commutative Lie ideal of M. Let $a \in U$ and $x \in M$, then $[a,x]_\alpha \in U$ and so commutes with a. Now for $x, y \in M$, we have $a\beta[a, x\alpha y]_\alpha = [a, x\alpha y]_\alpha \beta a$ for all $\alpha, \beta \in \Gamma$. Expanding $[a, x\alpha y]_\alpha$ as $[a,x]_\alpha \alpha y + x\alpha[a,y]_\alpha$ and using that a commutes with this, with $[a,x]_\alpha$ and with $[a,y]_\alpha$ we have $2[a,x]_\alpha \alpha [a,y]_\alpha = 0$. Since M is 2-torsion free so $[a,x]_\alpha \alpha [a,y]_\alpha = 0$. Replacing y by $a\beta x$ in $[a,x]_\alpha \alpha [a,y]_\alpha = 0$with $a\alpha b\beta c = a\beta b\alpha c$, we have $[a,x]_\alpha \alpha M \beta [a,x]_\alpha = 0$ for all $x \in M$ and $\alpha, \beta \in \Gamma$. Since M is prime so $[a,x]_\alpha = 0$ and $U \subset Z(M)$. Hence by Lemma 2.2 (i) we have $2d(u\alpha v) = 2(d(u)\alpha v + d(v)\alpha u)$. Since M is 2-torsion free hence $d(u\alpha v) = d(u)\alpha v + d(v)\alpha u$. Now we assume that U is a noncommutative Lie ideal of M i.e. $U \not\subset Z(M)$. Then by Lemma 3.2(iv) we have

$$d(u)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u + v\alpha u\alpha u) = 0, for\ all\ u, v \in U\ and\ \alpha, \beta \in \Gamma \tag{17}$$

Replacing u by $[u_1, w]_\alpha$ in (17), we get

$$d([u_1,w]_\alpha)\beta([u_1,w]_\alpha \alpha [u_1,w]_\alpha \alpha v) - d([u_1,w]_\alpha)\beta 2([u_1,w]_\alpha \alpha v \alpha [u_1,w]_\alpha) + d([u_1,w]_\alpha)\beta(v\alpha[u_1,w]_\alpha \alpha [u_1,w]_\alpha) = 0, for\ u, v, u_1, w \in U\ and\ \alpha, \beta \in \Gamma.$$

Using Lemma 3.3(i), we get $d([u_1,w]_\alpha)\beta v \alpha [u_1,w]_\alpha \alpha [u_1,w]_\alpha = 0$ and so

$$d([u_1,w]_\alpha)\beta U \alpha [u_1,w]_\alpha \alpha [u_1,w]_\alpha = 0.$$

Hence by Lemma 3.1 either $d([u_1,w]_\alpha) = 0$ or $[u_1,w]_\alpha \alpha [u_1,w]_\alpha = 0$.If $d([u_1,w]_\alpha) = 0$i.e. $d(u_1\alpha w) = d(w\alpha u_1)$ for all $u_1, w \in U$ and $\alpha \in \Gamma$, then by Lemma 3.2(i) and the fact that M is 2-torsion free, we get $d(u_1\alpha w) = d(u_1)\alpha w + d(w)\alpha u_1$. On the other hand, let $[u_1,w]_\alpha \alpha [u_1,w]_\alpha = 0$ for some $u_1, w \in U$ and $\alpha \in \Gamma$. By Lemma 3.3(ii) we get

$$d(v)\beta(u\alpha u\alpha v - 2u\alpha v\alpha u + v\alpha u\alpha u) = 0, \text{ for all } u, v \in U \text{ and } \alpha, \beta \in \Gamma \tag{18}$$

Replacing v by $[u_1,w]_\alpha$ in (18), we get

$$d([u_1,w]_\alpha)\beta(u\alpha u\alpha[u_1,w]_\alpha) - d([u_1,w]_\alpha)\beta 2(u\alpha[u_1,w]_\alpha \alpha u) + d([u_1,w]_\alpha)\beta([u_1,w]_\alpha \alpha u\alpha u) = 0, for\ u, v, u_1, w \in U\ and\ \alpha, \beta \in \Gamma.$$

Applying Lemma 3.3(i), we have

$$d([u_1,w]_\alpha)\beta(u\alpha u\alpha[u_1,w]_\alpha) - d([u_1,w]_\alpha)\beta 2(u\alpha[u_1,w]_\alpha \alpha u) = 0, \quad for\ all\ u \in U\ and\ \alpha \in \Gamma. \tag{19}$$

Linearizing (19) on u and using (18), we have

$$d([u_1,w]_\alpha)\beta(v\alpha u\alpha[u_1,w]_\alpha) + d([u_1,w]_\alpha)\beta(u\alpha v\alpha[u_1,w]_\alpha) - d([u_1,w]_\alpha)\beta 2((u\alpha[u_1,w]_\alpha \alpha v) + (v\alpha[u_1,w]_\alpha \alpha u)) = 0, \quad for\ all\ u, v \in U\ and\ \alpha \in \Gamma \tag{20}$$

Replacing u by $2u\beta v_1$ in (20) and then using the fact the M is 2-torsion free with $a\alpha b\beta c = a\beta b\alpha c$, we have

$$d([u_1,w]_\alpha)\beta(v\alpha u\beta v_1\alpha[u_1,w]_\alpha) + d([u_1,w]_\alpha)\beta(u\alpha v_1\beta v\alpha[u_1,w]_\alpha) - d([u_1,w]_\alpha)\beta 2\left((u\alpha v_1\beta[u_1,w]_\alpha\alpha v) + (v\alpha[u_1,w]_\alpha\alpha u\beta v_1)\right) = 0$$

Further replacing v_1 by $[u_1,w]_\alpha$ in the above expression and then using Lemma $3.3(i)$ together with the fact that $[u_1,w]_\alpha\alpha[u_1,w]_\alpha = 0$, we find that

$$d([u_1,w]_\alpha)\beta v\alpha u\beta[u_1,w]_\alpha\alpha[u_1,w]_\alpha = 0$$

i.e. $d([u_1,w]_\alpha)\beta U\alpha u\beta[u_1,w]_\alpha\alpha[u_1,w]_\alpha = 0 \quad for\ all\ u \in U and \alpha,\beta \in \Gamma$.

Thus, by Lemma 3.1 either $d([u_1,w]_\alpha) = 0$ or $u\beta[u_1,w]_\alpha\alpha[u_1,w]_\alpha = 0$.

If $d([u_1,w]_\alpha) = 0$, then using the similar argument as above we get the required result. On the other hand, if $u\beta[u_1,w]_\alpha\alpha[u_1,w]_\alpha = 0$ for all $u \in U$and $\alpha,\beta \in \Gamma$ then by Lemma 3.1 we have $[u_1,w]_\alpha = 0$. Further by Lemma $3.2(i)$ and the fact that M is 2-torsion free we have $d(u_1\alpha w) = d(u_1)\alpha w + d(w)\alpha u_1$. Hence in both cases, we find that $d(u\alpha v) = d(u)\alpha v + d(v)\alpha u$, for all $u,v \in U$ and $\alpha \in \Gamma$. This completes the proof of the above theorem.

Corollary 3.4.1. Let M be a 2-torsion free prime Γ-ring and $d: M \to M$ be a Jordan right derivation. Then d is a right derivation on M.

Proof: If M is commutative, then $u\alpha v = v\alpha u$ for all $u,v \in M$ and $\alpha \in \Gamma$ and so by Lemma $3.2(i)$ we have $d(u\alpha v) = d(u)\alpha v + d(v)\alpha u$ for all $u,v \in M$ and $\alpha \in \Gamma$. If M is noncommutative then by the above Theorem, we have $d(u\alpha v) = d(u)\alpha v + d(v)\alpha u$ for all $u,v \in M$ and $\alpha \in \Gamma$. This completes the proof.

4. Conclusion

Γ-ring is one of the most important and modern branches of algebra in mathematics now a days. In this paper we tried to characterize some special properties of prime Γ-ring on Lie ideals with Jordan right derivations which may help future researchers to proceed further. In this paper, Γ-ring is supposed to be of 2-torsion free and prime and the properties of Jordan right derivations has been used to characterize prime Γ-ring on lie ideals.

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