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Applications of *b-open Sets and **b-open Sets in Topological Spaces

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Abstract. In this paper, we introduce D(c,*b)-set,D(c,**b)-set, **b-continuous, locally **b-closed continuous, D(c,*b)-continuous, D(c,**b)-continuous functions and discuss some properties of the above sets and continuous functions.

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Keywords : *b-open sets,**b-open sets, t-sets, t*-sets, B-sets, B*-sets.

1. Introduction

Levine [1963] introduced the notion of semi-open sets and semi-continuity in topological spaces. Andrijevic [1996] introduced a class of generalized open sets in topological spaces. Mashhour [1982] introduced pre open sets in topological spaces.The class of b-open sets is contained in the class of semi-open and pre-open sets.Tong [1989] introduced the concept of t-set and B-set in topological spaces. The class of *b-open set is both semi-open and pre open. Indira, Rekha [2012] introduced the concept of *b-open set, **b-open set, t*-set, B*-set, locally *b-closed set, locally **b-closed set, *b-continuous in topological spaces. In this paper we introduce the notion of **b-continuous, t*-continuous, B*-continuous, locally **b-closed continuous, D(c,*b)-continuous, D(c,**b)-continuous in Topological spaces. In this paper we discuss properties of the above sets and continuous functions. All through this paper (X,τ) and (Y,σ) stand for topological spaces with no separation assumed, unless otherwise stated. Let $A \subseteq X$,the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively.

2. Preliminaries

Definition 2.1. A subset A of a space X is said to be :
1. Semi-open [14] if $A \subseteq Cl(Int(A))$
2. Pre open[15] if $A \subseteq Int(Cl(A))$
3. α -open [16] if $A \subseteq Int(Cl(Int(A)))$
4. β -open [1,4] if $A \subseteq Cl(Int(Cl(A)))$
5. b-open [3] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$
6. *b-open [10] if $A \subseteq Cl(Int(A)) \cap Int(Cl(A))$
7. b**-open [6] if $A \subseteq Int(Cl(Int(A))) \cup Cl(Int(Cl(A)))$
8. **b-open [10] if $A \subseteq Int(Cl(Int(A))) \cap Cl(Int(Cl(A)))$

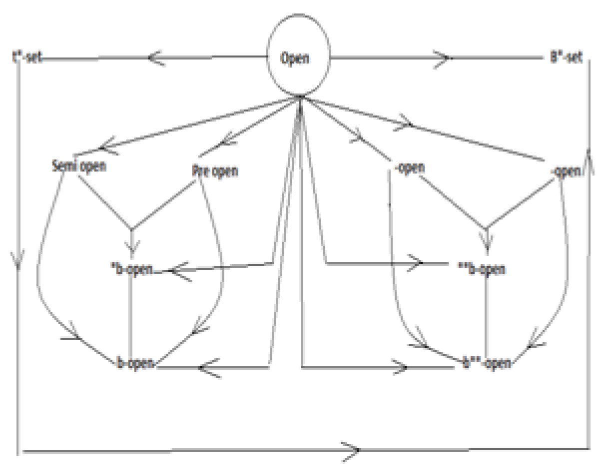
Definition 2.2. A subset A of a space X is called:
1. t-set [21] if $Int(A) = Int(Cl(A))$
2. t*-set [10] if $Cl(A) = Cl(Int(A))$
3. B-set [21] if $A = U \cap V$, where $U \in \tau$ and V is a t-set.
4. B*-set [10] if $A = U \cap V$, where $U \in \tau$ and V is a t*-set.
5. locally closed [5] if $A = U \cap V$, where $U \in \tau$ and V is a closed set.
6. locally b-closed [6] if $A = U \cap V$, where $U \in \tau$ and V is a b-closed set.
7. locally *b-closed [10,13] if $A = U \cap V$, where $U \in \tau$ and V is a *b-closed set.
8. locally b**-closed [6] if $A = U \cap V$, where $U \in \tau$ and V is a b**-closed set.
9. locally **b-closed [10] if $A = U \cap V$, where $U \in \tau$ and V is a **b-closed set.
10. D(c,b) –set [21] if $Int(A) = bInt(A)$
11. D(c,b**) –set [6] if $Int(A) = b^{**}Int(A)$

Definition 2.3. A function $f : X \to Y$ is called [1,2,12,13,14,15,16,22]:
1. semi continuous if $f^{-1}(V)$ is semi open in X for each open set V of Y .
2. pre continuous if $f^{-1}(V)$ is pre open in X for each open set V of Y .
3. α -continuous if $f^{-1}(V)$ is α - open in X for each open set V of Y .
4. β -continuous if $f^{-1}(V)$ is β -open in X for each open set V of Y .
5. b-continuous if $f^{-1}(V)$ is b-open in X for each open set V of Y .
6. *b-continuous if $f^{-1}(V)$ is *b-open in X for each open set V of Y .
7. b**-continuous if $f^{-1}(V)$ is b**-open in X for each open set V of Y .

8. t-continuous if $f^{-1}(V)$ is t-set in X for each open set V of Y .
9. B-continuous if $f^{-1}(V)$ is B-set in X for each open set V of Y .
10. locally closed continuous if $f^{-1}(V)$ is locally closed in X for each open set V of Y .
11. locally b-closed continuous if $f^{-1}(V)$ is locally b-closed in X for each open set V of Y .
12. locally *b-closed continuous if $f^{-1}(V)$ is locally *b-closed in X for each open set V of Y .
13. locally b**-closed continuous if $f^{-1}(V)$ is locally b**-closed in X for each open set V of Y .
14. completely continuous if $f^{-1}(V)$ is regular open in X for each open set V of Y .
15. D(c,b)-continuous if $f^{-1}(V)$ is D(c,b)-set in X for each open set V of Y .
16. D(c,*b)-continuous if $f^{-1}(V)$ is D(c,*b)-set in X for each open set V of Y .
17. D(c,b**)-continuous if $f^{-1}(V)$ is D(c,b**)-set in X for each open set V of Y .

The Figure 1 and Figure 2 give the relations between the above sets.

Figure 1



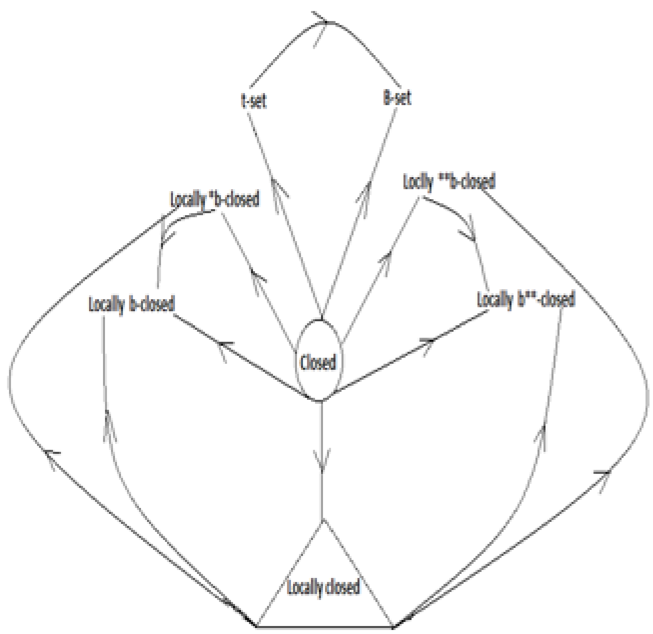
3. Properties of *b-open sets and **b-open sets

Definition 3.1. A subset A of a space X is called:
1. D(c,*b) –set if $Int(A) = *bInt(A)$
2. D(c,**b) –set if $Int(A) = **bInt(A)$

Result 3.2.
1.Every regular open set is a t-set.
2.Every regular closed set is a t*-set.
3.Every locally closed set is a B-set.

Theorem 3.3. Let A be a subset of (X,τ).Then to prove the following:
1. A is a t-set iff it is semi closed.
2. A is a t*-set iff it is semi open.

Figure 2



Proof.

1. Let A be a t-set.
   Then $Int(A) = Int(Cl(A))$
   Therefore $Int(Cl(A)) \subseteq A$
   $\Rightarrow$ A is semi closed.
   Conversly, Assume that A is semi closed
   Then $Int(Cl(A)) \subseteq A$
   $Int(Int(Cl(A))) \subseteq Int(A)$
   $Int(Cl(A)) \subseteq Int(A)$ (1)
   Since $Int(A) \subseteq Int(Cl(A))$ (2)

From (1) and (2)
$Int(A) = Int(Cl(A))$
$\Rightarrow$ A is a t-set.

2 Let A be a t*-set.
Then $ClA) = Cl(Int(A))$
Therefore $A \subseteq Cl(Int(A))$
$\Rightarrow$ A is semi open.
Conversly, Assume that A is semi open
Then $A \subseteq Cl(Int(A))$
$Cl(A) \subseteq Cl(Cl(Int(A)))$
$Cl(A) \subseteq Cl(Int(A))$ (1)
Since $Cl(Int(A)) \subseteq Cl(A)$ (2)
From (1) and (2)
$Cl(A) = Cl(Int(A))$
$\Rightarrow$ A is a t*-set.

Theorem 3.4.

1. If A and B are t-sets, then $A \cap B$ is a t-set.
2. If A and B are t*-sets, then $A \cup B$ is a t*-set.

Proof.

1. Let A and B be t-set.
Then we have $Int(A) = Int(Cl(A))$; $Int(B) = Int(Cl(B))$
Since $Int(A \cap B) \subseteq Int(Cl(A \cap B))$
$\subseteq Int(Cl(A) \cap Cl(B))$
$= Int(Cl(A)) \cap Int(Cl(B))$
$= Int(A) \cap Int(B)$
$= Int(A \cap B)$
$Int(A \cap B) \subseteq Int(Cl(A \cap B)) \subseteq Int(A \cap B)$

$\Rightarrow Int(A \cap B) = Int(Cl(A \cap B))$

$\Rightarrow A \cap B$ is a t-set.

2. Let A and B be t*-set.
Then we have $Cl(A) = Cl(Int(A))$; $Cl(B) = Cl(Int(B))$
Since $Cl(A \cup B) = Cl(A) \cup Cl(B) = Cl(Int(A)) \cup Cl(Int(B))$
$= Cl(Int(A) \cup Int(B))$

$$\subset Cl(Int(A \cup B))$$

$$Cl(A \cup B) \subset Cl(Int(A \cup B)) \quad (1)$$

Since $Int(A \cup B) \subset A \cup B$

$$Cl(Int(A \cup B)) \subset Cl(A \cup B) \quad (2)$$

From (1) and (2)

$$Cl(A \cup B) = Cl(Int(A \cup B))$$

$\Rightarrow$ $A \cup B$ is a t*-set.

Theorem 3.5. A set A is a t-set iff its complement is a t*-set.
Proof.
Let A be a t-set.
Then $Int(A) = Int(Cl(A))$
$\Leftrightarrow$ $X - Int(A) = X - Int(Cl(A))$
$\Leftrightarrow$ $Cl(X - A) = Cl(Int(X - A))$
$\Leftrightarrow$ $Cl(A^c) = Cl(Int(A^c))$
$\Leftrightarrow$ A^c is a t*-set.

Theorem 3.6. For a subset A of a space (X, τ), the following are equivalent:

1. A is open.
2. A is pre open and a B-set.

Proof.
To prove: $(1) \Rightarrow (2)$
Let A be open
Then $A = Int(A)$
$\Rightarrow$ $Int(A) \subseteq Int(Cl(A))$
$\Rightarrow$ $A \subseteq Int(Cl(A))$
$\Rightarrow$ A is pre open
Let $U = A \in \tau$ and $V = X$ be a t-set containing A
$\Rightarrow$ $A = U \cap V$
$\Rightarrow$ A is a B-set.
Hence A is pre open and a B-set.
To prove: $(2) \Rightarrow (1)$
Let A be pre open and a B-set
Since A is a B-set
Therefore $A = U \cap V$ where U is open and V is a t-set.

$\Rightarrow Int(V) = Int(Cl(V))$
since A is pre open
$\Rightarrow A \subseteq Int(Cl(A)) = Int(Cl(U \cap V)) = Int(Cl(U)) \cap Int(V)$
$\therefore U \cap V \subset Int(Cl(U)) \cap Int(V)$

$$\begin{aligned} \text{Consider } U \cap V &= (U \cap V) \cap U \\ &\subset [Int(Cl(U)) \cap Int(V)] \cap U \\ &= U \cap Int(V) \end{aligned}$$

$\Rightarrow U \cap V \subset U \cap Int(V)$
$\Rightarrow V \subset Int(V)$
$\Rightarrow V = Int(V)$
$\Rightarrow A = U \cap V = U \cap Int(V)$
$\Rightarrow A = Int(A)$
$\Rightarrow A$ is open.

Theorem 3.7. For a subset A of a space (X, τ), the following are equivalent:
1. A is regular open
2. A is pre open and a t-set.

Proof.
To prove: (1)$\Rightarrow$(2)
Let A be regular open.
Then $A = Int(Cl(A))$
$\Rightarrow A$ is pre open
Since $A = Int(Cl(A))$
$\Rightarrow Int(A) = Int(Cl(A))$
$\Rightarrow A$ is a t-set.
To prove: (2)$\Rightarrow$(1)
Let A be pre open and a t-set.
Then $A \subseteq Int(Cl(A))$ (1)
Since A is a t-set
Then $Int(Cl(A)) \subseteq A$ (2)
From (1) and (2)
A is regular open.

Theorem 3.8. For a subset A of a space (X, τ), the following are equivalent:
1. A is regular closed
2. A is pre closed and a t*-set.

Proof.
To prove: (1)$\Rightarrow$(2)
Let A be regular closed

Then $A = Cl(Int(A))$
$\Rightarrow A$ is pre closed
Since $A = Cl(Int(A))$
$\Rightarrow Cl(A) = Cl(Int(A))$
$\Rightarrow A$ is a t*-set.
To prove: (2)$\Rightarrow$(1)
Let A be pre closed and a t*-set.
Then $Cl(Int(A)) \subseteq A$ (1)
Since A is a t-set
Then $A \subseteq Cl(Int(A))$ (2)
From (1) and (2)
A is regular closed.

Theorem 3.9.
1. The intersection of a b-open set and a *b-open set is a b-open set.
2. The intersection of a b**-open set and a **b-open set is a b**-open set.

Theorem 3.10.
1. The intersection of a locally b-closed set and a locally *b-closed set is a locally b-closed set.
2. The intersection of a locally b**-closed set and a locally **b-closed set is a locally b**-closed set.

Theorem 3.11. For a subset A of an extremally disconnected space (X,τ),the following are equivalent:
1. A is open
2. A is *b-open and locally closed
3. A is b-open and locally closed

Proof.
Let A be a subset of an extremally disconnected space (X,τ).
Then $Cl(Int(A)) = Int(Cl(A))$
To prove: (1)$\Rightarrow$(2)
From theorem 2.4[13],
To prove: (2)$\Rightarrow$(3)
Let A be *b-open and locally closed
Since Every *b-open set is b-open.
$\Rightarrow A$ is b-open and locally closed
To prove: (3)$\Rightarrow$(1)
From theorem 2.1[2],

Theorem 3.12. For a subset A of a space (X,τ),the following are equivalent:

1. A is open
2. A is *b-open and D(c,*b)-set
3. A is b-open and D(c,b)-set

Proof.
To prove: (1) $\Rightarrow$ (2)
From theorem 2.7[13],
To prove: (2) $\Rightarrow$ (3)
Let A be *b-open and D(c,*b)-set
Then $A = {}^*bInt(A)$ and $Int(A) = {}^*bInt(A)$ (1)
Since Every *b-open set is b-open.
$\Rightarrow$ A is b-open.
$\Rightarrow$ $A = bInt(A)$ (2)
From (1) and (2)
$A = bInt(A) = {}^*bInt(A)$ (3)
From (1) and (3)
$Int(A) = bInt(A)$
$\Rightarrow$ A is a D(c,b)-set.
To prove: (3) $\Rightarrow$ (1)
Let A be b-open and D(c,b)-set
To prove: A is open
Since $A = bInt(A)$ and $Int(A) = bInt(A)$
$\Rightarrow$ A is open.

Theorem 3.13. For a subset A of an extremally disconnected space (X, τ), the following are equivalent:
1. A is open
2. A is **b-open and locally closed
3. A is b**-open and locally closed

Proof.
Let A be a subset of an extremally disconnected space (X, τ).
Then $Cl(Int(A)) = Int(Cl(A))$
To prove: (1) $\Rightarrow$ (2)
From theorem 2.9 [10],
To prove: (2) $\Rightarrow$ (3)
Let A be **b-open and locally closed
Since Every **b-open set is b**-open.
$\Rightarrow$ A is b**-open and locally closed
To prove: (3) $\Rightarrow$ (1)
From theorem 2.1[6],

Theorem 3.14. For a subset A of a space (X, τ), the following are equivalent:

1. A is open
2. A is **b-open and D(c,**b)-set
3. A is b**-open and D(c,b**)-set

Proof.

To prove: (1)$\Rightarrow$(2).
Let A be open
To prove: A is **b-open
From theorem 2.9[10] ,
To prove: A is a D(c,**b)-set
Since A is open and Every open set is **b-open.
Then $A = Int(A) = **bInt(A)$
$\Rightarrow$ A is a D(c,**b)-set.
To prove: (2)$\Rightarrow$(3)
Let A be **b-open and D(c,**b)-set
Then $A = **bInt(A)$ and $Int(A) = **bInt(A)$ (1)
Since Every **b-open set is b**-open.
$\Rightarrow$ A is b**-open.
$\Rightarrow$ $A = b**Int(A)$ (2)
From (1) and (2)
$A = b**Int(A) = **bInt(A)$ (3)
From (1) and (3)
$Int(A) = b**Int(A)$
$\Rightarrow$ A is a D(c,b**)-set.
To prove: (3)$\Rightarrow$(1)
Let A be b**-open and D(c,b**)-set
To prove: A is open
Since $A = b**Int(A)$ and $Int(A) = b**Int(A)$
$\Rightarrow$ A is open.

4. Applications of *b-continuous and **b-continuous functions

Definition 4.1.

A function $f : X \to Y$ is called:

1. **b-continuous if $f^{-1}(V)$ is **b-open in X for each open set V of Y.
2. t*-continuous if $f^{-1}(V)$ is t*-set in X for each open set V of Y.
3. B*- continuous if $f^{-1}(V)$ is B*-set in X for each open set V of Y.
4. locally **b-closed continuous if $f^{-1}(V)$ is locally **b-closed in X for each open set V of Y.
5. D(c,**b)-continuous if $f^{-1}(V)$ is D(c,**b)-set in X for each open set V of Y.

Example 4.2.
Let $X = \{a,b,c,d\}$

$\tau = \{X,\phi,\{c\},\{d\},\{c,d\},\{a,c,d\},\{\{b,c,d\}\}$

$\varsigma = \{X,\phi,\{a,b,d\},\{a,b,c\},\{a,b\},\{b\},\{a\}\ \}$

1.The collection of **b-open sets =

$\{X,\phi,\{c\},\{d\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$

Define a function $f : X \to X$ such that $f(a) = b, f(b) = a, f(c) = d, f(d) = c$

Then f is **b-continuous.

2.The collection of t*- sets =

$\{X,\phi,\{b\},\{c\},\{d\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$

Define a function $f : X \to X$ such tha $f(a) = b, f(b) = d, f(c) = a, f(d) = c$

Then f is t*-continuous.

3.The collection of B*-sets =

$\{X,\phi,\{b\},\{c\},\{d\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

Define a function $f : X \to X$ such tha $f(a) = b, f(b) = d, f(c) = a, f(d) = c$

Then f is B*-continuous.

4. The collection of locally **b-closed sets =

$\{X,\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},$
$\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

Define a function $f : X \to X$ such tha $f(a) = c, f(b) = a, f(c) = b, f(d) = d$

Then f is locally **b-closed continuous.

5. The collection of D(c,**b)-sets =

$\{X,\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},$

$\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

Define a function $f : X \to X$ such that $f(a) = c, f(b) = a, f(c) = f(d) = d$

Then f is D(c,**b)-continuous.

Theorem 4.3. Let $f : X \to Y$ be a function and let X be an extremally disconnected space. Then the following are equivalent:

1. f is continuous
2. f is *b-continuous and locally closed continuous
3. f is b-continuous and locally closed continuous

Proof.

It follows from theorem 3.11.

Theorem 4.4. Let $f : X \to Y$ be a function. Then the following are equivalent:

1. f is continuous
2. f is *b-continuous and D(c,*b)-continuous
3. f is b-continuous and D(c,b)-continuous

Proof.
It follows from theorem 3.12.

Theorem 4.5. Let $f : X \to Y$ be a function and let X be an extremally disconnected space. Then the following are equivalent:

1. f is continuous
2. f is **b-continuous and locally closed continuous
3. f is b**-continuous and locally closed continuous

Proof. It follows from theorem 3.13 .

Theorem 4.6. Let $f : X \to Y$ be a function. Then the following are equivalent:

1. f is continuous
2. f is **b-continuous and D(c,**b)-continuous
3. . f is b**-continuous and D(c,b**)-continuous

Proof. It follows from theorem 3.14.

Theorem 4.7. Let $f : X \to Y$ be a function. Then

1. f is continuous iff f is pre continuous and B-continuous
2. f is completely continuous iff f is pre continuous and t-continuous
3. . f is t*-continuous iff f is semi continuous

Proof.
1.It follows from theorem 3.6.
2.It follows from theorem 3.7.
3.It follows from theorem 3.3(2).

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