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A New Method for Solving Fuzzy Assignment Problems

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Abstract. A new method namely, parallel moving method is proposed to find an optimal solution to the fuzzy assignment problem considered in Lin and Wen [13]. We derive two theorems; one is related to an optimal solution to the fuzzy assignment problem and another is related to find an improved solution from the current solution to the fuzzy assignment problem. The parallel moving method provides an optimal solution to the fuzzy assignment problem in less number of iterations than the labeling algorithm [13]. A numerical example is given to demonstrate the procedure of the proposed method.

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1. Introduction

The assignment problem (AP) is a special type of a transportation problem and a linear zero-one programming problem [11]. It is a one of the well-studied optimization problems in Management Science and has been widely applied in both manufacturing and service systems. The main object of the AP is to find an assignment schedule in a jobs assignment problem where n jobs are allocated to n workers and each worker receives exactly just one job such that the total assignment cost is minimum. The classical AP can also be written as a 0–1 integer programming problem as follows:

$$\text{(P)} \qquad \text{Minimize } \; z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1; \quad i = 1,2,...,n \ ;$$

$$\sum_{i=1}^{n} x_{ij} = 1; \quad j = 1,2,...,n \ ;$$

$$x_{ij} = \{0,1\} \ ; \ i = 1,2,...,n \quad j = 1,2,...,n$$

where x_{ij} is the decision variable, that is , ith worker in jth job and c_{ij} is the cost of the ith worker who are doing the jth job.

The AP introduced by Votaw and Orden [21] can be solved using the linear programming technique, the transportation algorithm or the Hungarian method developed by Kuhn [12]. The Hungarian method is recognized to be the first practical method for solving the standard assignment problem. Balinski and Gomory [2] introduced a labeling algorithm for solving the transportation and assignment problems. Ford and Fulkerson [9] introduced the models and algorithms in Flows in Networks which are used widely today in the fields of transportation systems, manufacturing, inventory planning, image processing, and Internet traffic. Aggarwal et al. [1] developed two algorithms for solving bottleneck assignment problems.

Costs in many real life applications are not deterministic numbers. The fuzzy assignment problem (FAP) is more realistic than the AP because most real environments are uncertain. In recent years, many researchers have begun to investigate AP and its variants under fuzzy environments. For instance, Chen [8] solved a fuzzy assignment model that considers all individuals have same skills. Feng and Yang [10] studied a two objective fuzzy k-cardinality AP. Sakawa et al. [18] considered interactive fuzzy programming for two-level or multi level linear programming problems to obtain a satisfactory solution for decision making. Long-sheng Huang and Guang-hui Xu [14] proposed a solution procedure for the AP with restriction of qualification. By the max–min criterion suggested by Bellman and Zadeh[3], the fuzzy assignment problem can be treated as a mixed integer nonlinear programming problem. Lin and Wen [13] investigated a fuzzy assignment problem in which the cost depends on the quality of the job. Michéal ÓhÉigeartaigh [15] and Chanas et al. [4] solved transportation problems with fuzzy supply and demand values. An integer fuzzy transportation problem was solved in Tada and Ishii [20]. Chanas and Kuchta [5] proposed the concept of the optimal solution of the transportation problem with fuzzy coefficients expressed as L-R fuzzy numbers, and developed an algorithm for determining the solution. Additionally, Chanas and Kuchta [6] designed an algorithm for solving integer fuzzy transportation problem with fuzzy demand and supply values in the sense of maximizing the joint satisfaction of the fuzzy goal and the constraints. Pandian and Natarajan [16,17] have introduced two different methods for solving the fuzzy transportation problem. Fractional programming is a particular type of non-linear programming in which the objective function to be optimized is the ratio of two other objective function expressions. Shigeno et al. [19] proposed a solution procedure for the fractional assignment problem. Charnes and Cooper [7] introduced the variable transformation method for solving fractional programming model.

In this paper, we propose a new method namely, parallel moving method for finding an optimal solution to the FAP considered in Lin and Wen [13]. We construct two crisp APs from the given FAP. Using the optimal solutions of these two crisp APs, we develop the parallel moving method for solving the FAP. For ensuring the optimal solution to the FAP, we derive one theorem and for efficient section procedure of an improved solution from the current solution to the FAP, we derive another theorem. The proposed method provides an optimal solution to the FAP in less number of iterations than the labeling algorithm developed by Lin and Wen [13]. For computing the parameters in the parallel moving method, we follow the method of computation of the transportation algorithm. A numerical example is given to demonstrate the procedure of the proposed method. The parallel moving method enables the decision makers to evaluate the economical activities and make self satisfied managerial decisions when they are handling an AP having imprecise parameters.

2. Fuzzy Assignment Problem

We consider the following FAP studied in Lin and Wen[13].

Consider a consultant company, which provides different services for customers for predetermined charges in each case. The quality of the job (case), i.e. the satisfaction of the customer or the job performance of the worker (consultant) doing that case, may assume to be positively correlated to the input time (cost) of the worker. The highest quality of the case is different because of the difference among individual workers. Furthermore, the manager of the company usually restricts the total labor hours or costs to a range as his fuzzy goal. Hence, we assume a minimum cost for a worker to perform a job, and greater cost spent may result in higher quality until it reaches an upper bound, after that an increase in cost does not increase the quality. In this case, the costs are no longer deterministic numbers and we will denote them as $\tilde{c}_{ij}$. Further, we define as α_{ij}, the least cost associated with the worker i performing the job j and as β_{ij}, the least cost associated with worker i performing the job j at the highest quality. Without loss of generality, we may assume that $\beta_{ij} > \alpha_{ij} > 0$. We further define the quality matrix $[q_{ij}]$, where q_{ij} represents the highest quality associated with the worker i performing the job j . In most real cases, we have $0 < q_{ij} < 1$. The membership function of $\tilde{c}_{ij}$ is given below:

$$\mu_{ij}(c_{ij}) = \begin{cases} q_{ij} & : if \;\; c_{ij} \geq \beta_{ij} \; ; x_{ij} = 1 \\ \dfrac{q_{ij}(c_{ij} - \alpha_{ij})}{(\beta_{ij} - \alpha_{ij})} & : if \;\; \alpha_{ij} \leq c_{ij} \leq \beta_{ij} \; ; x_{ij} = 1 \\ 0 & : \text{otherwise} \end{cases}$$

We use the notation $\langle \alpha_{ij}, \beta_{ij} \rangle$ to denote $\tilde{c}_{ij}$. The matrix $[\tilde{c}_{ij}]$ = $[\langle \alpha_{ij}, \beta_{ij} \rangle]$. In addition, all the α_{ij} 's form the matrix $[\alpha_{ij}]$ and β_{ij} 's form the matrix $[\beta_{ij}]$. From the definition of $\tilde{c}_{ij}$, any expense exceeding β_{ij} is useless since the quality (worker's job performance or customer's satisfaction) can no longer be increased at its upper limit q_{ij}.

Furthermore, let $\tilde{c}_T$ denote the total cost, which related to the job performance of the manager, and numbers *a* and *b* are defined as the lower and upper bounds of total cost, respectively. We define the membership function of $\tilde{c}_T$ as the linear monotonically decreasing function and use the notation $\langle a, b \rangle$ to denote fuzzy interval $\tilde{c}_T$. Numbers a and b are constants and are subjectively chosen by the manager. Using the idea from Werners [22], a number less than or equal to the minimum assignment of matrix $[\alpha_{ij}]$ as *a* and a number larger than or equal to the maximum assignment of matrix $[\beta_{ij}]$ as *b* are taken. The membership function of $\tilde{c}_T$ is given below:

$$\mu(c_T) = \mu\left(\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \right) = \begin{cases} 1 & : if \;\; c_T \leq a \\ \dfrac{b - c_T}{(\beta_{ij} - \alpha_{ij})} & : if \;\; a \leq c_T \leq b \\ 0 & : if \;\; c_T \geq b \end{cases}.$$

The main objective of the FAP is to find an assignment to the given FAP which minimize the total cost of workers and maximize the job performance of the manager.

The above FAP was modeled by Lin and Wen[13] as a 0-1 linear fractional programming problem as follows:

$$\text{(FP)} \quad \text{Maximize} \;\; f = \frac{b - \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_{ij}}{b - a + \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} x_{ij}}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1; \quad i = 1,2,...,n \tag{1}$$

$$\sum_{i=1}^{n} x_{ij} = 1; \quad j = 1,2,...,n \tag{2}$$

$$x_{ij} = \{0,1\}\ ; \ i = 1,2,...,n \quad j = 1,2,...,n \tag{3}$$

where $\gamma_{ij} = \dfrac{\beta_{ij} - \alpha_{ij}}{q_{ij}}$.

We now, need the following result related to FAP which can be found in Lin and Wen [13].

Now, the linear programming model of the FAP, (LP) is given below:

(LP) Maximize $-\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij} y_{ij} + bt$

subject to

$$\sum_{j=1}^{n} y_{ij} = t, i = 1,2,...,n\ ;$$

$$\sum_{i=1}^{n} y_{ij} = t, j = 1,2,...,n\ ;$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} y_{ij} + (b-a)t = 1\ ;$$

$$t \geq 0,\ y_{ij} \geq 0,\ i = 1,2,...,n;\ j = 1,2,...,n;$$

where $t = \dfrac{1}{b - a + \sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} x_{ij}}$ and $y_{ij} = t x_{ij}$.

Now, the dual problem corresponding to (LP), (LD) is given below:

(LD) Minimize f

subject to

$u_i + v_j + \alpha_{ij} + f\gamma_{ij} \geq 0$, for all i and j

$-\sum_{i=1}^{n} u_i - \sum_{j=1}^{n} v_j + (b-a)f \geq b$

$u_1, u_2, ..., u_n$, $v_1, v_2, ..., v_n$ and f are real numbers.

Result 2.1. Let $X = \{x_{ij}, i, j = 1,2,...,n\}$ be a feasible solution to FAP and $W = \{u_i, v_j, f;\ \text{i, j} = 1,2,..., \text{n}\}$ be the complementary solution for the dual problem (LD) corresponding to $Y = \{y_{ij}, y_{ij} = tx_{ij}, i, j = 1,2,...,n\}$. Then,

(i) if $x_{ij} = 1$, that is, (i,j)th cell is allotted in FAP, then $u_i + v_j + \alpha_{ij} + f\gamma_{ij} = 0$ and

(ii) if $u_i + v_j + \alpha_{ij} + f\gamma_{ij} \geq 0$, for all non-allotted cells in the FAP, then $X = \{x_{ij}, i, j = 1,2,...,n\}$ is an optimal solution to FAP.

We now, construct two crisp APs (L) and (U) from the problem (FP) as follows:

$$\text{(L) Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_{ij}$$

subject to (1), (2) and (3) are satisfied

and

$$\text{(U) Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} x_{ij}$$

subject to (1), (2) and (3) are satisfied.

For easy computing and clear understanding , the parallel moving method will be applied directly on a table as that of classical transportation algorithm. Parameters corresponding to cell (i; j) are displayed in the Table 1..

α_{ij}		δ_{ij}
f		γ_{ij}

Table 1.

3. The Parallel Moving Method

Now, we derive the following theorem which connects the solutions of the problem (FP) and its deduction the problems (L) and (U) and is used in the parallel moving method.

Theorem 3.1. Let X_0 be optimal solution to the problem (L) and Y_0 be optimal solution to the problem (U). Then, X_0 and Y_0 are feasible solution to the fuzzy assignment problem and

$$\frac{P(X_0)}{Q(X_0)} \le \text{Max.}\, f(X) \le \frac{P(X_0)}{Q(Y_0)} \quad \text{and} \quad \frac{P(Y_0)}{Q(Y_0)} \le \text{Max.}\, f(X) \le \frac{P(X_0)}{Q(Y_0)}$$

where $P(X) = b - \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_{ij}$ and $Q(X) = b - a + \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} x_{ij}$ If X_0 and Y_0 are the same, then X_0 is an optimal solution to the fuzzy assignment problem and $\text{Max.}\, f(X) = \dfrac{P(X_0)}{Q(X_0)}$.

Proof: Now, since X_0 is optimal solution to the problem (L), then X_0 is optimal solution to the problem (N) where

(N) Maximize $P = b - \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_{ij}$

subject to (1), (2) and (3) are satisfied.

Now, since Y_0 is optimal solution to the problem (U), then Y_0 is optimal solution to the problem (D) where

(D) Minimize $Q = b - a + \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} x_{ij}$

subject to (1), (2) and (3) are satisfied.

Now, $$\frac{P(X)}{Q(X)} = \frac{b - \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ij} x_{ij}}{b - a + \sum_{i=1}^{m} \sum_{j=1}^{n} \gamma_{ij} x_{ij}} = f(X)$$

Let X be a feasible solution to the problem (FP).
Clearly, X is a feasible solution to the problems (N) and (D). Then, we have $P(X) \le P(X_0)$ and $Q(X) \ge Q(Y_0)$.

Since $Q(X) > 0$, we have $\dfrac{P(X)}{Q(X)} \le \dfrac{P(X_0)}{Q(Y_0)}$.

This implies that $\text{Max.} \left(\dfrac{P(X)}{Q(X)} \right) \le \dfrac{P(X_0)}{Q(Y_0)}$.

Therefore, $Max.f(X) \le \dfrac{P(X_0)}{Q(Y_0)}$. (4)

Now, since X_0 and Y_0 are feasible solutions to the problem (FP), we have

$f(X_0) \le Max.f(X)$ and $f(Y_0) \le Max.f(X)$.

(5)

Now, from (4) and (5), we have

$\dfrac{P(X_0)}{Q(X_0)} \le \text{Max. } f(X) \le \dfrac{P(X_0)}{Q(Y_0)}$ and $\dfrac{P(Y_0)}{Q(Y_0)} \le \text{Max. } f(X) \le \dfrac{P(X_0)}{Q(Y_0)}$.

(6)

Now, given that X_0 and Y_0 are the same.

From (6), we have

$$\frac{P(X_0)}{Q(X_0)} \le \text{Max. } f(X) \le \frac{P(X_0)}{Q(X_0)} \text{ and } \frac{P(X_0)}{Q(X_0)} \le \text{Max. } f(X) \le \frac{P(X_0)}{Q(X_0)}$$

This implies that, Max. $f(X) = \dfrac{P(X_0)}{Q(X_0)} = f(X_0)$.

Therefore, X_0 is an optimal solution to the problem (FP) and

Max. $f(X) = \dfrac{P(X_0)}{Q(X_0)}$.

Hence the theorem.

Now, we prove the following theorem corresponding to the selection of an improved solution from the current solution of the problem (FP) which is used in the proposed method.

Theorem 3.2. Let $X_0 = \{x_{ij}^{\circ}, i, j = 1,2,...,n\}$ be a solution to the problem (FP) in which (p,r) and (q,s) are two allotted cells, that is, $x_{pr}^{\circ} = x_{qs}^{\circ} = 1$ and (p,s) and (q,r) are non-allotted cells, that is, $x_{ps}^{\circ} = x_{qr}^{\circ} = 0$ such that $X_1 = \{x_{ij}^{1}, i, j = 1,2,...,n\}$ where $x_{ij}^{1} = x_{ij}^{\circ} = 1$, for all $(i, j) \ne (p,r)$ and (q,s) and $x_{ps}^{1} = x_{qr}^{1} = 1$, is a solution to the problem (FP). Then, $f(X_1) \ge f(X_0)$ provided $(\gamma_{pr} + \gamma_{qs}) - (\gamma_{ps} + \gamma_{qr}) \le 0$ and $(\alpha_{pr} + \alpha_{qs}) - (\alpha_{ps} + \alpha_{qr}) \ge 0$.

Proof: Now, we have, $f = f(X) = \dfrac{b - \sum\sum \alpha_{ij}x_{ij}}{(b-a) + \sum\sum \gamma_{ij}x_{ij}}$ where

$X = \{x_{ij}, i, i = 1,2,...,n\}$.

Now, $f_0 = f(X_0) = \dfrac{b - \sum\sum \alpha_{ij}x^0{}_{ij}}{(b-a) + \sum\sum \gamma_{ij}x^0{}_{ij}}$ and

$$f_1 = f(X_1) = \frac{b - \sum\sum \alpha_{ij}x^1{}_{ij}}{(b-a) + \sum\sum \gamma_{ij}x^1{}_{ij}}.$$

Now, $f_1 = \dfrac{b - \sum\sum \alpha_{ij}x^1{}_{ij}}{(b-a) + \sum\sum \gamma_{ij}x^1{}_{ij}}$

$$= \frac{b - \sum\sum \alpha_{ij}x^0{}_{ij} - \alpha_{ps} - \alpha_{qr} + \alpha_{pr} + \alpha_{qs}}{(b-a) + \sum\sum \gamma_{ij}x^0{}_{ij} + (\gamma_{ps} + \gamma_{qr} - \gamma_{pr} - \gamma_{qs})}$$

Since $(\gamma_{pr} + \gamma_{qs}) - (\gamma_{ps} + \gamma_{qr}) \le 0$, we have

$$f_1 \ge \frac{b - \sum\sum \alpha_{ij}x^0{}_{ij} - \alpha_{ps} - \alpha_{qr} + \alpha_{pr} + \alpha_{qs}}{(b-a) + \sum\sum \gamma_{ij}x^0{}_{ij}}$$

$$= \frac{b - \sum \sum \alpha_{ij} x^0{}_{ij}}{(b-a) + \sum \sum \gamma_{ij} x^0{}_{ij}} + \frac{(\alpha_{pr} + \alpha_{qs}) - (\alpha_{ps} + \alpha_{qr})}{(b-a) + \sum \sum \gamma_{ij} x^0{}_{ij}} = f_0$$

$$+ \frac{(\alpha_{pr} + \alpha_{qs}) - (\alpha_{ps} + \alpha_{qr})}{(b-a) + \sum \sum \gamma_{ij} x^0{}_{ij}}$$

Since $(b-a) + \sum \sum \gamma_{ij} x^0{}_{ij} > 0$ and $(\alpha_{pr} + \alpha_{qs}) - (\alpha_{ps} + \alpha_{qr}) \geq 0$, we can conclude that

$f_1 - f_0 \geq 0$.

Hence the theorem.

Now, we extend the result of the Theorem 3.2. to a set of allotted cells to another set of non-allotted cells of the solution of the problem (FP). .

Theorem 3.3. Let $X_0 = \{x^\circ_{ij}, i, j = 1,2,...,n\}$ be a solution to the problem (FP) in which

E = (p_1, r_1), (p_2, r_2), …, (p_t, r_t); $x^\circ_{p_1 r_1} = \cdots = x^\circ_{p_t r_t} = 1$} and

D = $\{(q_1, s_1)$, (q_2, s_2),…, (q_t, s_t); $x^\circ_{q_1 s_1} = \cdots = x^\circ_{q_t s_t} = 0\}$

$(t \geq 2$ and $t \leq n)$,

are subsets of X_0 such that $X_1 = \{x^1_{ij}, i, j = 1,2,...,n\}$ where

$x^1_{ij} = x^\circ_{ij} = 1,$ for all $(i, j) \neq (p_a, r_a)$ and (q_a, s_a) ($a = 1,2,...,t$);

$x^1_{p_a s_a} = 1(a = 1,2,3,..,t)$; $x^1_{q_b r_{b+1}} = 1(b = 1,2,...,t-1)$ and $x^1_{q_t r_1} = 1$,

is a solution to the problem (FP). Then, $f(X_1) \geq f(X_0)$ provided

$$\sum_{a=1}^{t} (\gamma_{p_a r_a} + \gamma_{q_a s_a}) - \sum_{a=1}^{t-1} (\gamma_{p_a s_a} + \gamma_{q_a r_{a+1}}) - (\gamma_{p_t s_t} + \gamma_{q_t r_1}) \leq 0 \text{ and}$$

$$\sum_{a=1}^{t} (\alpha_{p_a r_a} + \alpha_{q_a s_a}) - \sum_{a=1}^{t-1} (\alpha_{p_a s_a} + \alpha_{q_a r_{a+1}}) - (\alpha_{p_t s_t} + \alpha_{q_t r_1}) \geq 0 .$$

Proof: It is similar to the proof of the Theorem 3.2.

We now, propose a new method namely, parallel moving method for solving the problem (FP).

The parallel moving method proceeds as follows.

Step 1: Construct the APs (L) and (U) from the given FAP. Then, solve them by the Hungarian method. Say X_0 is an optimal solution to the problem (L) and Y_0 is an optimal solution to the problem (U).

Step 2: If $X_0 = Y_0$, then X_0 is an optimal solution to the given FAP by the Theorem 3.1. and go to the Step 7. . If not, go to the Step 3..

Step 3: Fix U_0 where $U_0 = X_0$ or Y_0 as an initial solution to the given FAP. Then, compute the value of $f(U_0)$, f_0.

Step 4: Compute the dual variables (MODI indices) u_{i} and v_{j} for all i and j using the relation $u_i + v_j + \alpha_{ij} + f_0\gamma_{ij} = 0$, for allotted cells by taking $u_{\mathrm{i}} = 0$, for all i.

Step 5: Construct MODI index table for the initial solution, U_0 to the FSP. Then, compute $\delta_{ij} = u_i + v_j + \alpha_{ij} + f_0\gamma_{ij}$ for all non-allotted cells. If $\delta_{ij} \geq 0$, for all non-allotted cells, go to the Step 7.. If not, go to the Step 6..

Step 6: Find a cell having the most negative value of δ_{ij}. Say (p,s). Then,

(a) if the assignments of p and s in the initial solution U_0 are (p,r) and (q,s) and if $(\gamma_{pr} + \gamma_{qs}) - (\gamma_{ps} + \gamma_{qr}) \leq 0$ and $(\alpha_{pr} + \alpha_{qs}) - (\alpha_{ps} + \alpha_{qr}) \geq 0$, assign (p,s) and (q,r) (see Table 2.) and obtain an improve solution $U_1 = (U_0 - \{(p,r),(q,s)\}) \cup \{(p,s),(q,r\}$ by the Theorem 3.2.. Then, go to the Step 2. for next iteration.

If the one of the conditions "$(\gamma_{pr} + \gamma_{qs}) - (\gamma_{ps} + \gamma_{qr}) \leq 0$ and $(\alpha_{pr} + \alpha_{qs}) - (\alpha_{ps} + \alpha_{qr}) \geq 0$" is not satisfied, go to Step 6.(b).

(b) Find a cell in the qth row , say (q,t) which satisfies the conditions $(\gamma_{pr} + \gamma_{qs} + \gamma_{zt}) - (\gamma_{ps} + \gamma_{qt} + \gamma_{zr}) \leq 0$ and $(\alpha_{pr} + \alpha_{qs} + \alpha_{zt}) - (\alpha_{ps} + \alpha_{qt} + \alpha_{zr}) \geq 0$ where (z,t) is the assignment of t in the initial solution U_0, assign (p,s), (q,t) and (z,r) (see the Table 3.) and obtain an improve solution $U_1 = (U_0 - \{(p,r),(q,s),(z,t)\}) \cup \{(p,s),(q,t),(z,r)\}$ by the Theorem 3.3. Then, go to the Step 2 for next iteration.

If the one of the conditions "$(\gamma_{pr}+\gamma_{qs}+\gamma_{zt})-(\gamma_{ps}+\gamma_{qt}+\gamma_{zr})\le 0$ and $(\alpha_{pr}+\alpha_{qs}+\alpha_{zt})-(\alpha_{ps}+\alpha_{qt}+\alpha_{zr})\ge 0$" is not satisfied, go to Step 6.(c).

(c) Continue for finding an assignment to the rth column as like as in the Step 6(b). After finite number of steps, an improve solution is obtained. Then, go to the Step 2. for the next iteration.

Step 7: The current solution is an optimal solution to the given FAP . We stop the computation process. The current value of f is the maximum value of f .

	R	…	*s*
P	X	…	#
⋮	⋮	⋮	⋮
Q	#	…	X

Table 2.

	r	…	*t*	…	*s*
P	X	…		…	#
⋮	⋮	⋮	⋮	⋮	⋮
Z	#		X		
⋮	⋮	⋮	⋮	⋮	⋮
Q		…	#	…	X

Table 3.

(Marks X denote current allotments and Marks # denote new allotments)

Now, the parallel moving method is illustrated with the help of the following numerical example.

Example 3.1. Consider the following 3×3 fuzzy assignment problem whose fuzzy cost matrix, $[\widetilde{c}_{ij}]$ and quality matrix, $[q_{ij}]$ are given below:

$$[\widetilde{c}_{ij}]=\begin{bmatrix}[4,13] & [3,12] & [2,6]\\ [4,13] & [6,14] & [7,15]\\ [7,10] & [4,8] & [6,12]\end{bmatrix} \quad \text{and} \quad [q_{ij}]=\begin{bmatrix}0.9 & 0.6 & 0.8\\ 0.9 & 0.8 & 0.8\\ 0.6 & 0.8 & 0.6\end{bmatrix}.$$

Then, we obtain the following matrices:

$$[\alpha_{ij}]=\begin{bmatrix}4 & 3 & 2\\ 4 & 6 & 7\\ 7 & 4 & 6\end{bmatrix} \; ; \; [\beta_{ij}]=\begin{bmatrix}13 & 12 & 6\\ 13 & 14 & 15\\ 10 & 8 & 12\end{bmatrix} \quad \text{and} \quad [\gamma_{ij}]=\begin{bmatrix}10 & 15 & 5\\ 10 & 10 & 10\\ 5 & 5 & 10\end{bmatrix}$$

Now, by the Hungarian method, we obtain that the optimal solution to (L) is (1,3), (2,1) and (3,2) and the optimal solution to (U) are (1,3) (2,1) and (3,2) and (1,3) , (2,2) and (3,1).

Now, the common solution is (1,3), (2,1) and (3,2).

Now, the $(\alpha,\gamma)-$ table corresponding to FAP is given below:

4 10	3 15	2 5
4 10	6 10	7 10
7 5	4 5	6 10

<u>**Case (i)** Initial solution: (1,3), (2,1) and (3,2).</u>

Now, the value of f for the above allotment $= 0.6$. Hence, $f = 0.6$

Now, by the Theorem 1. and the assignment (1,3), (2,1) and (3,2) is the common solution to (L) and (U), the optimal allotment is (1,3), (2,1) and (3,2) and the optimal value of $f = 0.6$.

<u>**Case (ii)** Initial solution: (1,3), (2,2) and (3,1).</u>

Now, the value of f for the above allotment $= 0.5$. Hence, $f = 0.5$

Take $u_i = 0,$ for all i.

Now, $v_j = -\alpha_{ij} - f\gamma_{ij}$, for all allotted cells.

Now, $v_1 = -\alpha_{31} - f\gamma_{11} = -9.5$; $v_2 = -\alpha_{22} - f\gamma_{22} = -11$ and $v_3 = -\alpha_{13} - f\gamma_{13} = -4.5$.

To check the optimality- MODI index table:

	$v_1 = -9.5$	$v_2 = -11$	$v_3 = -4.5$
$u_1 = 0$	4 -0.5 0.5 10	3 -0.5 0.5 15	2 X 0.5 5
$u_2 = 0$	4 -0.5 0.5 10	6 X 0.5 10	7 7.5 0.5 10
$u_3 = 0$	7 X 0.5 5	4 -4.5 0.5 5	6 6.5 0.5 10

The cell corresponding to most negative of δ_{ij} for all non-allotted cells is (3,2).

Since we have the assignment (2,2) and (3,1) and $(\gamma_{22} + \gamma_{31}) - (\gamma_{32} + \gamma_{21}) = 0$; $(\alpha_{22} + \alpha_{31}) - (\alpha_{32} + \alpha_{21}) = 5 > 0$, we obtain new assignment to the problem is (1,3), (2,1) and (3,2).

<u>**First Iteration :**</u>

Now, the solution to the fuzzy assignment problem is (1,3), (2,1) and (3,2), and the value of f for the new allotment is 0.6. Hence, $f = 0.6$
Since the assignment is optimal assignment to (L) and (U), the assignment is optimal to FAP by the Theorem 1.. Therefore, the optimal value of f = 0.6.

Remark 3.1: In Lin and Wen [13], the Example 3.1. was solved by LW algorithm in 4 iterations , but by the parallel moving method, we solve it at most one iteration.

4. Conclusion

Assignment problem is one of the most important problem in decision-making. In many real life applications, costs of AP are not deterministic numbers. The FAP is more realistic than the AP because most real environments are uncertain. In recent years, many researchers have begun to investigate AP and its variants under fuzzy environments. In Lin and Wen [13], a fuzzy assignment problem in which the cost of each job, depending on the quality, is not a deterministic number, was studied and solved it by the labeling method [13]. In this paper, we propose a new method namely, parallel moving method for solving the fuzzy AP problem considered in Lin and Wen [13]. From the Example 3.1., we can observe that our proposed method performs satisfactorily and is better than labeling algorithm [13]. In near future, we extend our study to the sensitivity analysis in the fuzzy assignment problem considered in Lin and Wen [13].

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