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On Total Vertex Irregularity Strength of Triangle Related Graphs

***Indra Rajasingh*[1], *Bharati Rajan*[2] *and V.Annamma*[3]**
[1] Department of Advanced Sciences, V.I.T. University, Chennai, India
[2] Department of Mathematics, Loyola College, Chennai, India
[3] Department of Mathematics, L.N.Government College, Ponneri, India
Email: annammasharo@yahoo.com

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Abstract. Let $G(V,E)$ be a simple graph. For a labeling $\partial : V \cup E \to \{1,2,3,...,k\}$, the weight of a vertex x is defined as $wt(x) = \partial(x) + \sum_{y \in N(x)} \partial(xy)$ where $N(x)$ is the set of neighbours of x. ∂ is called a vertex irregular total k -labeling if for every pair of distinct vertices x and y, $wt(x) \neq wt(y)$. The minimum k for which the graph G has a vertex irregular total k -labeling is called the *total vertex irregularity strength* of G and is denoted by $tvs(G)$. In this paper we obtain a bound for the total vertex irregularity strength of swing graph S_m^3, triangular graph $TR(k)$ and series-triangular graph $TS(k)$.

Keywords: total vertex irregularity strength (*tvs*), swing graph, triangular graph, series-triangular graph.

***AMS Mathematics Subject Classifications* (2010): 05C78**

1. Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs are becoming an increasingly useful family of mathematical models for a broad range of applications. They are also of interest in their own right due to their abstract mathematical properties arising from various structural considerations of the underlying graphs. An enormous body of literature has grown around the theme.The qualitative labeling of graphs have inspired research in diverse fields of human enquiry such as conflict resolution in social psychology, electrical circuit theory and energy crisis. Quantitative labeling of graphs have led to quite intricate fields of applications such as coding theory problems including the design of good radar location codes, missile guidance codes and convolution codes with optimal auto correlation properties. Labeled graphs have also been applied in determining ambiguities in X-ray crystallographic analysis to design communication network address system, network flows to determine optimal circuit layouts and radio astronomy. The theoretical applications of labeled graphs are numerous, not only within the theory of graphs but also in other areas of Mathematics such as combinatorial number theory, linear algebra and group theory.

Let $G(V,E)$ a simple graph. For a labeling $\partial : V \cup E \to \{1,2,3,...,k\}$ the weight of x is defined as $wt(x) = \partial(x) + \sum_{y \in N(x)} \partial(xy)$ where $N(x)$ is the set of neighbours of x. ∂ is called a vertex irregular total k-labeling if for every pair of distinct vertices x and y $wt(x) \neq wt(y)$. The minimum k for which the graph G has a vertex irregular total k-labeling is called the total vertex irregularity strength of G and is denoted by $tvs(G)$ [8]. Baca et al. [3] proved that $tvs(C_n) = \left\lceil \frac{n+2}{3} \right\rceil$, $n \geq 2$; $tvs(K_n) = 2$; $tvs(K_{1,n}) = \left\lceil \frac{n+1}{2} \right\rceil$; $tvs(C_n \times P_2) = \left\lceil \frac{2n+3}{4} \right\rceil$. Chunling et al. [4] showed that $tvs(K_p) = 2$ for $p \geq 2$ and for the generalized Petersen graph $P(n,k)$, $tvs(P(n,k)) = \left\lceil \frac{n}{2} \right\rceil + 1$, $k \leq \frac{n}{2}$. They also obtained the total vertex irregularity strength for ladder, Mobius ladder and Knodel graphs. For graphs with no isolated vertices, Pzrybylo [11] gave bounds for $tvs(G)$ in terms of the order, minimum and maximum degrees of G. For d-regular graphs $(d > 0)$, Przybylo [10] gave bounds for $tvs(G)$ in terms of d and the order of G. Ahmad et al. [1] determined the total vertex irregularity strength for five families of cubic plane graphs. They proved that for the circulant graph $C_n(1,2)$, $n \geq 5$, $tvs(C_n(1,2)) = \left\lceil \frac{n+4}{2} \right\rceil$. They also conjecture that for circulant graph $C_n(a_1, a_2, a_3 ... a_n)$ with degree atleast 5 and $n \geq 5$, $1 \leq a_i \leq \left\lfloor \frac{n}{2} \right\rfloor$, $tvs(C_n(a_1, a_2, a_3, ..., a_n)) = \left\lceil \frac{n+r}{1+r} \right\rceil$. In this paper we obtain the bound for the total vertex irregularity strength of swing graph S_m^3, triangular graph $TR(k)$ and series-triangular graph $TS(k)$.

2. Swing Graph

In this section we obtain a bound for the total vertex irregularity strength of swing graph S_m^3.

Definition 1. [9] Consider m copies of cycle C_3. If one vertex of each C_3 is joined to a common vertex say u, such a graph is called a swing graph S_m^3. Let $V=\{v_{ij},\ 1 \le i \le m, 1 \le j \le 3\} \cup \{u\}$ be the vertex set and $E=\{(v_{i1,}v_{i2}),(v_{i2,}v_{i3}),(v_{i3,}v_{i1}),(v_{i3,}u)/ 1 \le i \le 3\}$ the edge set of S_m^3. Then $|V| = 3m + 1$ and $|E| = 4m$. See Figure $1(a)$.

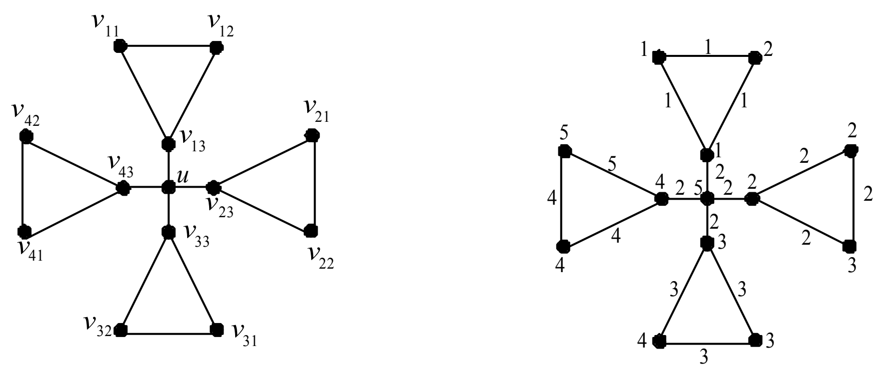

Figure 1: (a) S_3^4 (b) Labeled S_3^4

Theorem 1. [3] Let G be (p,q) graph with minimum degree δ and maximum degree Δ. Then $\left\lceil \frac{p+\delta}{\Delta+1} \right\rceil \le tvs(G) \le p+\Delta-2\delta+1.$

Theorem 2. [2] Let G be a graph of order n and minimum degree δ. Then $tvs(G) \le 3\left\lceil \frac{n}{\delta} \right\rceil + 1.$

Theorem 3. $\left\lceil \frac{n+2}{m+1} \right\rceil \le tvs(S_m^3) \le \frac{n-1}{3}+1, n=3m+1,\ m \ge 2.$

Proof. The lower bound follows from theorem 1. Now we give a procedure to label the vertices and edges of S_m^3 to get the upper bound.

Let $V(S_m^3)=\{v_{ij}, 1 \le i \le m, 1 \le j \le 3\} \cup \{u\}$ and $E(S_m^3)=\{e_{i1,}e_{i2,}e_{i3,}e_{i4}\}$ where $e_{i1}=(v_{i1,}v_{i2}),\ e_{i2}=(v_{i2,}v_{i3}),\ e_{i3}=(v_{i3,}v_{i1}),\ e_{i4}=(v_{i3,}u), 1 \le i \le m.$

Define a labeling $f : V \cup E \to \left\{1,2,3,\ldots,\frac{n-1}{3}+1\right\}$ in the following way:

$$f(v_{ij}) = \begin{cases} i, & 1 \le i \le m, \quad j = 1 \\ i+1, & 1 \le i \le m, \quad j = 2 \\ i, & 1 \le i \le m, \quad j = 3 \end{cases} \qquad \text{and } f(u) = \frac{n-1}{3}+1.$$

$$f(e_{ij}) = \begin{cases} i, & 1 \le i \le m, \quad j = 1 \\ i, & 1 \le i \le m-1, \quad j = 2 \\ i+1, & i = m, \quad j = 2 \\ i, & 1 \le i \le m, \quad j = 3 \\ 2, & 1 \le i \le m, \quad j = 4 \end{cases}$$

The weights of the vertices of S_m^3 are as follows:

$$wt(v_{ij}) = \begin{cases} 3i, & 1 \le i \le m, \quad j = 1 \\ 3i+1, & 1 \le i \le m-1, \quad j = 2 \\ 3i+2, & i = m, \quad j = 2 \\ 3i+2, & 1 \le i \le m-1, \quad j = 3 \\ 3i+3, & i = m, \quad j = 3 \end{cases}$$

and $wt(u) = 3m+1$.

The weights of the vertices v_{ij}, $1 \le i \le m$, $1 \le j \le 3$ and u constitute the set {3, 4, 5, . . ., 3m+3}. Thus the labeling $f : V \cup E \to \left\{1,2,3,\ldots,\frac{n-1}{3}+1\right\}$ provides the upper bound for the total vertex irregularity strength of S_m^3. Hence $tvs(S_m^3) \le \frac{n-1}{3}+1, m \ge 2$. See Figure 1($b$).

3. Triangular Graph

In this section we obtain a bound for the total vertex irregularity strength of triangular graph $TR(k)$.

Definition 2. *[7] For $k > 1$, a triangular graph $TR(k)$ is defined as follows: Let $C =$ $v_{11}v_{12}v_{13}v_{11}$ be a cycle on 3 vertices. Let v_{21}, v_{22}, v_{23} be three new vertices such that the vertex set $V = \{v_{11}, v_{12}, v_{13}, v_{21}, v_{22}, v_{23}\}$, the edge set $E = \{(v_{11}, v_{12})$, (v_{12}, v_{13}) (v_{13}, v_{11}), $\{(v_{21}, v_{22})$, (v_{22}, v_{23}), (v_{23}, v_{21}), (v_{11}, v_{22}), (v_{12}, v_{23}), (v_{13}, v_{21}), (v_{11}, v_{21}), (v_{12}, v_{22}), $(v_{13}, v_{23})\}$ and C form triangular graph $TR(2)$. Repeating the* *process k times we obtain a graph called triangular graph $TR(k)$.See Figure* $2(a)$.

Theorem 4. $\left\lceil \frac{n+4}{5} \right\rceil \le tvs(TR(k)) \le k+1, k>1.$

Proof. The lower bound follows from theorem 1. Now we give a procedure to label the vertices and edges of $TR(k)$ to get the upper bound.

Let $V(TR(k)) = \{v_{ij}, 1 \le i \le k, 1 \le j \le 3\}$ and $E(TR(k)) = \{e_{i1}, e_{i2,} e_{i3}, 1 \le i \le k\} \cup \{g_{i1,} g_{i2,} g_{i3}, 1 \le i \le k\}$ where $e_{i1} = (v_{i1}, v_{i2})$, $e_{i2} = (v_{i2}, v_{i3})$, $e_{i3} = (v_{i3}, v_{i1})$, $1 \le i \le k$ and $g_{i1} = (v_{i1}, v_{i+11})$, $g_{i2} = (v_{i2}, v_{i+12})$, $g_{i3} = (v_{i3}, v_{i+13})$, $1 \le i \le k-1$, $g_{k1} = (v_{11}, v_{k2})$, $g_{k2} = (v_{12}, v_{k3})$, $g_{k3} = (v_{13}, v_{k1})$.

Define a labeling $f : V \cup E \to \{1,2,3,...,k+1\}$ in the following way.

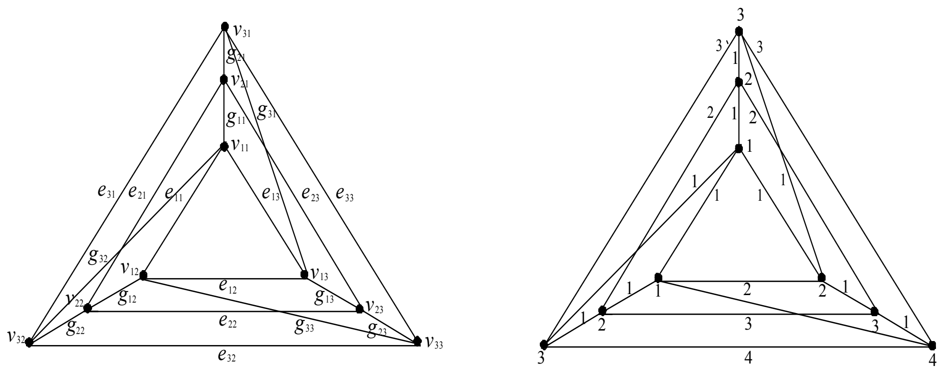

Figure 2: (a) *TR*(3); (b) Labeled *TR*(3)

$$f(v_{ij}) = \begin{cases} i, & 1 \le i \le k, \quad j=1 \\ i, & 1 \le i \le k, \quad j=2 \\ i, & 1 \le i \le k, \quad j=3 \end{cases}$$

$$f(e_{ij}) = \begin{cases} i, & 1 \le i \le k, \quad j=1 \\ i+1, & 1 \le i \le k, \quad j=2 \\ i, & 1 \le i \le k, \quad j=3 \end{cases}$$

$f(g_{ij}) = 1, 1 \le i \le k, 1 \le j \le 3.$

The weights of the vertices of $TR(k)$ are as follows:

$$wt_f(v_{ij}) = \begin{cases} 3i+2, & 1 \le i \le k, \quad j=1 \\ 3i+3, & 1 \le i \le k, \quad j=2 \\ 3i+4, & 1 \le i \le k, \quad j=3 \end{cases}$$

The weights of the vertices $v_{ij}, 1 \le i \le k, 1 \le j \le 3$ constitute the set {5, 6, .. . , 3k+1}. Thus the labeling $f : V \cup E \to \{1,2,3,...,k+1\}$ provides the upper bound for the total vertex irregularity strength of $TR(k)$. Hence $tvs(TR(k)) \le k+1$. See Figure $2(b)$.

4. Series Triangular Graph

In this section we obtain a bound for the total vertex irregularity strength of series-triangular graph $TS(k)$.

Definition 3. *[5] A graph is series-triangular if*
(i) *it is planar*
(ii) *every embedding of G is a triangulation and*
(iii) *there is a vertex v in V such that G/v is series-triangular.*

Definition 4. *[6] Let $C = v_1yzv_1$ be a triangle. Let v_2 be a new vertex in the exterior of C. Join v_2 with v_1 Fix the edge yz. Take a new vertex v_3 in the exterior of the triangle v_2yzv_2. Join v_3 with v_2, y and z. We continue this procedure k times to obtain a series-triangular graph $TS(k)$. In this construction yz of C is always an edge of all the triangles formed. The path $v_1, v_2, v_3, ..., v_n$ is called the spine of $TS(k)$. See Figure $3(a)$.*

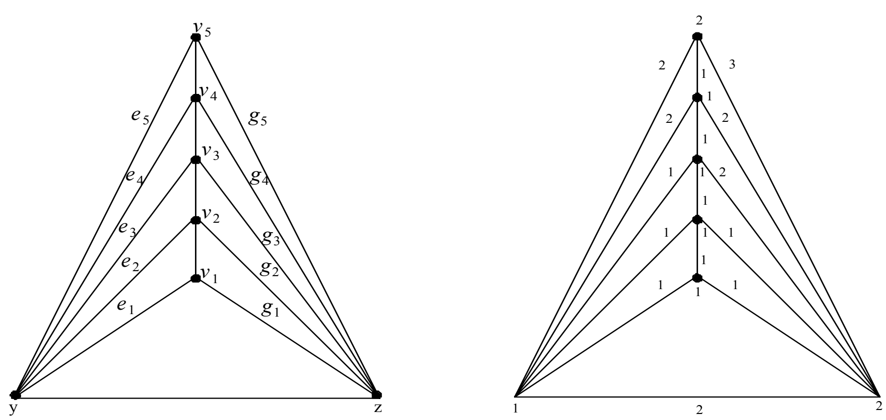

Figure 3: (a)$TS(5)$; (b) Labeled $TS(5)$

Theorem 5. *For* $, k > 3, \left\lceil \frac{n+3}{k} \right\rceil \le tvs(TS(k))$

$$\leq \begin{cases} \left\lceil \frac{k}{3} \right\rceil + 1 \text{when} k \equiv 0 (mod 3) \\ \left\lceil \frac{k}{3} \right\rceil \text{when} k \equiv 1 (mod 3) \\ \left\lfloor \frac{k}{3} \right\rfloor + 2 \text{when} k = 2 (mod 3) \end{cases}$$

Proof. The lower bound follows from theorem 1. We give a procedure to label the vertices and edges of $TS(k)$ to get the upper bound.

Let $V = \{v_i, 1 \leq i \leq k\} \cup \{y\} \cup \{z\}$ and $E = \{e_i, g_i, 1 \leq i \leq k\} \cup (y, z)$ where $e_i = (y, v_i)$ and $g_i = (z, v_i), 1 \leq i \leq k.$

Define a labeling $f : V \cup E \to \left\{1,2,3,..., \left\lfloor \frac{k}{3} \right\rfloor + 2\right\}$ in the following way:

$f(y) = 1$ and $f(z) = 2$

$$f(e_i) = \begin{cases} \left\lceil \frac{i}{3} \right\rceil, & 1 \leq i \leq k-1 \\ \left\lceil \frac{i}{3} \right\rceil + 1, & i = k, \quad k \equiv 0 (mod 3) \\ \left\lceil \frac{i}{3} \right\rceil, & i = k, \quad \text{otherwise} \end{cases}$$

$$f(g_i) = \begin{cases} \left\lfloor \frac{i}{3} \right\rfloor + 1, & 1 \leq i \leq k-1 \\ \left\lfloor \frac{i}{3} \right\rfloor + 1, & i = k, \quad k \equiv 0 (mod 3) \\ \left\lceil \frac{i}{3} \right\rceil, & i = k, \quad k \equiv 1 (mod 3) \\ \left\lfloor \frac{i}{3} \right\rfloor + 2, & i = k, \quad k \equiv 2 (mod 3) \end{cases}$$

and $f(y, z) = 2.$

The weights of the vertices of $TS(k)$ are:

$wt_f(v_i) = i + 3, i \leq i \leq k$

$wt_f(y) = 3 + \sum_{i=1}^{k} f(e_i)$

$$wt_f(z) = 4 + \sum_{i=1}^{k} f(g_i)$$

The weights of the vertices $v_i, 1 \le i \le k$ constitute the set {3, 4, 5, . . ., k+2}. Thus the labeling $f : V \cup E \to \left\{1,2,3,...,\left\lfloor \frac{k}{3} \right\rfloor + 2\right\}$ provides the upper bound for the vertex irregular total labeling of $TS(k)$. Hence

$$tvs(TS(k)) \le \begin{cases} \left\lceil \frac{k}{3} \right\rceil + 1 \text{when } k \equiv 0(mod 3) \\ \left\lceil \frac{k}{3} \right\rceil \text{ when } k \equiv 1(mod 3) \\ \left\lfloor \frac{k}{3} \right\rfloor + 2 \text{when } k = 2(mod 3) \end{cases}$$

See Figure $3(b)$.

5. Conclusion

In this paper, we have obtained a bound for the total vertex irregularity strength of swing graph S_m^3, triangular graph $TR(k)$ and the series triangular graph $TS(k)$. Total vertex irregular k-labeling for networks like hexagonal network, butterfly network and benes network is under investigation.

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