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# Semi Prime Ideals in Meet Semilattices

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**Abstract.** The Concept of semi prime ideals was given by Y. Rav by generalizing the notion of 0-distributive lattices. An ideal $I$ of a lattice $L$ is called a *semi prime ideal* if for all $x, y, z \in L$, $x \wedge y \in I$ and $x \wedge z \in I$ imply $x \wedge (y \vee z) \in I$. In this paper, we extend the concept for meet semi lattices. Here we include several characterizations of these ideals in directed above meet semi lattices and provided a result related to prime separation theorem. We also include some results on minimal prime ideals.

***Keywords:*** Semi prime ideal, Maximal filter, Minimal prime ideal, Pseudo complemented meet semi lattice.

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## 1. Introduction

J.C.Varlet [6] first introduced the concept of 0-distributive lattices. Then many authors including [1,2,4] studied them for lattices and semi lattices. By [2], a meet semi lattice S with 0 is called 0-distributive if for all $a, b, c \in S$ with $a \wedge b = 0 = a \wedge c$ imply $a \wedge d = 0$ for some $d \geq b, c$. We also know that a 0-distributive meet semi lattice is directed above. A meet semi lattice S is called *directed above* if for all $a, b \in S$, there exists $c \in S$ such that $c \geq a, b$.

A non-empty subset I of a directed above meet semi lattice S is called a down set if for $x \in I$ and $y \leq x$ $(y \in S)$ imply $y \in I$. Down set $I$ is called an ideal if for $x, y \in I$, there exists $z \geq x, y$ such that $z \in I$.

A non-empty subset $F$ of $S$ is called an upset if $x \in F$ and $y \geq x$ $(y \in S)$ imply $y \in F$. An upset F of S is called a filter if for all $x, y \in F$, $x \wedge y \in F$. An ideal (down set) P is called a prime ideal (down set) if $a \wedge b \in P$ implies either $a \in P$ or $b \in P$. A filter $Q$ of S is called prime if $S - Q$ is a prime ideal.

A filter F of S is called a maximal filter if $F \neq S$ and it is not contained by any other proper filter of S. A prime down set P is called a minimal prime down set if it does not contain any other prime down set of S.

Y Rav [5] have generalized the concept of 0-distributive lattices and introduced the notion of semi prime ideals in lattices. An ideal $I$ of a lattice $L$ is called a *semi prime ideal* if for all $x, y, z \in L$, $x \wedge y \in I$ and $x \wedge z \in I$ imply $x \wedge (y \vee z) \in I$. Thus, for a lattice $L$ with 0, $L$ is called *0-distributive* if and only if $(0]$ is a semi prime ideal. In a distributive lattice $L$, every ideal is a semi prime ideal. Moreover, every prime ideal is semi prime. In a pentagonal lattice $\{0, a, b, c, 1; a < b\}$, $(0]$ is semi prime but not prime. Here (b] and (c] are prime, but (a] is not even semi prime. Again in $M_3 = \{0, a, b, c, 1; a \wedge b = b \wedge c = a \wedge c = 0; a \vee b = a \vee c = b \vee c = 1\}$, $(0]$, $(a]$, $(b], (c]$ are not semi prime. . In this paper, we extend the concept for directed above meet semi lattices and give several characterizations of these ideals.

In a directed above meet semi lattice *S*, an ideal *J* is called a semi prime ideal if for all $x, y, z \in S$, $x \wedge y \in J$, $x \wedge z \in J$ imply $x \wedge d \in J$ for some $d \geq y, z$. In a distributive semi lattice, every ideal is semi prime. Moreover, the semi lattice itself is obviously a semi prime ideal. Also, every prime ideal of *S* is semi prime.

**Theorem 1.** *A meet semilattice S with at least one proper semi prime ideal is directed above.*

**Proof**. Let $a, b \in S$ and P be a semi prime ideal of S. Then for any $p \in P$, $p \wedge a, p \wedge b \in P$. Since P is semi prime, so there exists $d \in S$ with $d \geq a, b$ such that $p \wedge d \in P$. Therefore, S is directed above. □

Following result is due to [2].

**Lemma 2.** *A filter F of a meet semi lattice S is maximal if and only if S-M is a minimal prime down set.* □

**Lemma 3.** *Intersection of two prime (semi prime) ideals of a directed above meet semi lattice is a semi-prime ideal.*

**Proof:** Let $a, b, c \in S$ and $I = P_1 \cap P_2$. Let $a \wedge b \in I$ and $a \wedge c \in I$. Then $a \wedge b \in P_1, a \wedge c \in P_1$ and $a \wedge b \in P_2, a \wedge c \in P_2$. Since each $P_i$ is prime(semi prime) , so $a \wedge d_1 \in P_1$ and $a \wedge d_2 \in P_2$ for some $d_1, d_2 \geq b, c$. Choose $d = d_1 \wedge d_2 \geq b, c$. Then $a \wedge d \in P_1 \cap P_2$, and so $P_1 \cap P_2$ is semi prime. □

**Lemma 4.** *Every filter disjoint from an ideal $I$ is contained in a maximal filter disjoint from $I$.*

**Proof:** Let $F$ be a filter in $S$ with 0. Let Ŧ be the set of all filters containing F and disjoint from $I$. Then Ŧ is non-empty as F$\in$Ŧ. Let $C$ be a chain in Ŧ and let $M = \bigcup\left(X : X \in C\right)$.We claim that $M$ is a filter. Let $x \in M$ and $y \geq x$ .Then $x \in X$ for some $X \in C$. Hence $y \in X$ as $X$ is a filter. Therefore, $y \in M$. Let $x, y \in M$ .Then $x \in X$ and $y \in Y$ for some $X, Y \in C$.Since $C$ is a chain, either $X \subseteq Y$ or $Y \subseteq X$. Suppose $X \subseteq Y$. So $x, y \in Y$ .Then $x \wedge y \in Y$ and so $x \wedge y \in M$ . Moreover, $M \supseteq F$. So $M$ is a maximum element of $C$. Then by Zorn's Lemma, Ŧ has a maximal element, say $Q \supseteq F$ .□

**Lemma 5.** *Let $I$ be an ideal of a meet semilattice S. A filter M disjoint from $I$ is a maximal filter disjoint form $I$ if and only if for all $a \notin M$, there exists $b \in M$ such that $a \wedge b \in I$ .*

**Proof:** Let $M$ be a maximal filter such that it is disjoint from $I$ and $a \notin M$ .Let $a \wedge b \notin I$ for all $b \in M$. Consider $M_1 = \{y \in S\,;\, y \geq a \wedge b,\ b \in M\}$. Clearly $M_1$ is a filter. For any $b \in M$, $b \geq a \wedge b$ implies $b \in M_1$. So $M_1 \supseteq M$ .Also $M_1 \cap I = \phi$. For if not, let $x \in M_1 \cap I$. This implies $x \in I$ and $x \geq a \wedge b$ for some $b \in M$. Hence $a \wedge b \in I$, which is a contradiction. Hence $M_1 \cap I = \phi$. Now $M \subset M_1$ because $a \notin M$ but $a \in M_1$ . This contradicts the maximality of $M$. Hence there must exist $b \in M$ such that $a \wedge b \in I$.

Conversely, If $M$ is not maximal among the filters disjoint from $I$, then there exists a filter $N \supset M$ and disjoint from $I$. For any $a \in N - M$, there exists $b \in M$ such that $a \wedge b \in I$. Hence, $a, b \in N$ and this implies $a \wedge b \in I \cap N$, which is a contradiction . Hence $M$ must be a maximal filter disjoint from $I$ . □

Let $S$ be a meet semi-lattice with 0. For $A \subseteq S$, we define $A^{\perp} = \{\, x \in S \mid x \wedge a = 0 \quad \textit{for all} \quad a \in A \,\}$. $A^{\perp}$ is always a down set of $S$ but it is not necessarily an ideal. □

**Theorem 6.** *Let A be a non-empty subset of a meet semilattice directed above S and J be an ideal of S. Then*

$$A^{\perp_J} = \bigcap (P : P \textit{ is a minimal prime down set containing } J \textit{ but not containing } A)$$

**Proof:** Suppose

$X = \bigcap( P : A \not\subseteq P, P \textit{ is a minimal prime down set containing } J\,)$. Let $x \in A^{\perp_J}$. Then $x \wedge a \in J$ for all $a \in A$.Choose any $P$ of right hand expression. Since $A \not\subseteq P$, there exists $z \in A$ but $z \notin P$. Then $x \wedge z \in J \subseteq P$. So $x \in P$ , as $P$ is prime. Hence $x \in X$.

Conversely, Let $x \in X$. If $x \notin A^{\perp_J}$ , then $x \wedge b \notin J$ for some $b \in A$. Let $D = [x \wedge b)$.Hence $D$ is a filter disjoint from *J*. Then by Lemma 4, there is a maximal filter $M \supseteq D$ but disjoint form $J$ .Then by Lemma 2, *S-M is* minimal prime down set containing *J*. Now $x \notin S - M$ as $x \in D$ implies $x \in M$. Moreover, $A \not\subseteq S - M$ as $b \in A$, but $b \in M$ implies $b \notin S - M$, which is a contradiction to $x \in X$. Hence $x \in A^{\perp_J}$. □

**Theorem 7.** *Suppose S be a directed above meet semi lattice with 0 and J be an ideal of S. The following conditions are equivalent.*
*(i) J is semi prime.*
*(ii) For every* $a \in S$, $\{a\}^{\perp J} = \{x \in S : x \wedge a \in J\}$ *is a semi prime ideal containing J.*
*(iii)* $A^{\perp_J} = \{x \in S : x \wedge a \in J \ for\ all\ a \in A\}$ *is a semi prime ideal containing J, when A is finite.*
(iv) *Every maximal filter disjoint from J is prime.*

**Proof:** (i) $\Leftrightarrow$ (ii). Suppose (i) holds. $\{a\}^{\perp_J}$ is clearly a down set containing *J*. Now let $x, y \in \{a\}^{\perp_J}$ .Then $x \wedge a \in J$, $y \wedge a \in J$. Since $J$ is semi prime, so $a \wedge d \in J$ for some $d \geq x, y$. This implies $d \in \{a\}^{\perp J}$ and so $\{a\}^{\perp_J}$ is an ideal containing *J*. Now let $x \wedge y \in \{a\}^{\perp_J}$ and $x \wedge z \in \{a\}^{\perp_J}$ . Then $x \wedge y \wedge a \in J$ and $x \wedge z \wedge a \in J$. Thus, $(x \wedge a) \wedge y \in J$ and $(x \wedge a) \wedge z \in J$. Then $(x \wedge a) \wedge d \in J$ for some $d \geq y, z$, as *J* semi prime . This implies $x \wedge d \in \{a\}^{\perp_J}$ and so $\{a\}^{\perp_J}$ is semi prime.

(ii) $\Rightarrow$ (i). Suppose (ii) holds. Let $x \wedge y \in J$ and $x \wedge z \in J$. Then $y, z \in \{x\}^{\perp J}$. Since by (ii) $\{x\}^{\perp J}$ is an ideal, so there exists $d \geq y, z$ such that $d \in \{x\}^{\perp J}$. Thus $x \wedge d \in J$ and so J is semi prime.

( ii)$\Rightarrow$( iii ). This is trivial by Lemma1, as $A^{\perp_J} = \bigcap (\{a\}^{\perp_J} ; a \in A)$ .

(i)$\Rightarrow$(iv). Suppose *F* is a maximal filter disjoint from *J* . Suppose $f, g \in S - F$. Then $f, g \notin F$ By Lemma 5, there exist $a, b \in F$ such that $a \wedge f \in J$, $b \wedge g \in J$. Here *S-F* is a minimal prime down set containing J. Thus $a \wedge b \wedge f \in J$ and $a \wedge b \wedge g \in J$. Since j is semi prime, so there exists $e \geq f, g$ such that $e \wedge a \wedge b \in J \subseteq S - F$. But $a \wedge b \in F$ and so $e \in S - F$ as it is prime. Here *S-F* is a prime ideal. Hence *F* is a prime filter.

(iv)$\Rightarrow$ (i). Let (iv) holds. Suppose $a, b, c \in S$ with $a \wedge b \in J$, $a \wedge c \in J$. Suppose for all $d \geq b, c$ $a \wedge d \notin J$ Consider $F = \{y \in S : y \geq a \wedge d; d \geq b, c\}$ Then F is a filter disjoint from J. By Lemma 4, there is a maximal filter $M \supseteq F$ and disjoint from J. By (iv) M is prime. Thus S-M is a prime ideal containing J. Now $a \wedge b, a \wedge c \in S-M$. Since S-M is a prime ideal, so either $a \in S-M$ or $b,c \in S-M$. In any case $a \wedge d \in S-M$ for some $d \geq b,c$. This gives a contradiction as $a \wedge d \in M$ for all $d \geq b,c$. Therefore $a \wedge d \in J$ for some $d \geq b,c$. Hence $J$ is semi prime. □

**Corollary 8.** *In a meet semilattice S, every filter disjoint to a semi-prime ideal J is contained in a prime filter.*

**Proof:** This immediately follows from Lemma 4 and theorem 7.

**Theorem 9.** *If J is a semi-prime ideal of a directed above meet semi-lattice S and* $J \subset A = \bigcap\{J_\lambda : J_\lambda \text{ is an ideal containing } J\}$, *then* $A^{\perp_J} = \{x \in S : \{x\}^{\perp_J} \neq J\}$.

**Proof :** Let $x \in A^{\perp_J}$. Then $x \wedge a \in J$ for all $a \in A$. So $a \in \{x\}^{\perp_J}$ for all $a \in A$. Then $A \subseteq \{x\}^{\perp_J}$ and so $\{x\}^{\perp_J} \neq J$. Conversely, Let $x \in S$ such that $\{x\}^{\perp_J} \neq J$. Since $J$ is semi-prime, so $\{x\}^{\perp_J}$ is an ideal properly containing J. Therefore $A \subseteq \{x\}^{\perp_J}$, and so $A^{\perp_J} \supseteq \{x\}^{\perp_J \perp_J}$. This implies $x \in A^{\perp_J}$ which completes the proof. □

In [1], Balasubramani and Venkatanarasimhan, have provided a series of characterizations of 0-distributive lattices. Then [3] have generalized some of the results for semi prime ideals. Here we extend a part of those results for semi prime ideals.

**Theorem 10.** *Let S be a meet semi-lattice directed above and J be an ideal. Then the following conditions are equivalent.*

(i) *J is semi-prime.*

(ii) *Every maximal filter of S disjoint with J is prime.*

(iii) *Every minimal prime down set containing J is a minimal prime ideal containing J.*

(iv) *Every filter disjoint from J is disjoint from a minimal prime ideal containing J.*

**Proof:** (i)$\Leftrightarrow$(ii) follows from Theorem 7.

(ii)$\Rightarrow$ (iii). Let A be a minimal prime down set containing *J* .Then *S-A* is a maximal filter disjoint with *J* .Then by (ii), *S-A* is prime and so *A* is a minimal prime ideal.

(iii)$\Rightarrow$ (ii). Let F be a maximal filter disjoint with *J*. Then *S-F*is a minimal prime down set containing *J*. Then by (iii), *S-F* is a minimal prime ideal and so *F* is a prime filter.

(i)$\Rightarrow$ (iv). Let *F* a filter of S disjoint from *J*. Then by Corollary 8,there is a prime(maximal) filter Q containing *F* and disjoint from *J.* Then $S-Q$ is a minimal prime ideal containing *J* and disjoint from *F*.

(iv)$\Rightarrow$ (ii). Let $Q$ be a maximal filter disjoint from *J.* Then by (iv), there exists a minimal prime ideal *P* containing *J* such that $Q \cap P = \varphi$. Then *S-P* is a (maximal) prime filter of *S* containing $Q$ and disjoint from *J*. By maximality of $Q$ , S-P must be equal to $Q$. Therefore $Q$ is prime. □

**Theorem 11.** *Let S be a directed above meet semilattice and J be ideal of S. Then each of the conditions of theorem, imply the conditions*
(i) *For each element* $a \notin J$, *there is a minimal prime ideal containing J but not containing a.*
(ii) *Each* $a \notin J$ *is contained in a prime filter disjoint from J.*
**Proof:** Let $a \notin J$. Then $[a) \cap J = \varphi$. So by (iv) of Theorem 10, $[a)$ is disjoint from a minimal prime ideal containing $J$ . Thus (i) holds. Since the complement of a prime ideal is a prime filter, so (i) implies (ii). □

**Remark:** By [3] we know that all the conditions of Theorem 10 and Theorem 11 are equivalent in case of lattices. But in meet semi lattices condition (ii)of Theorem 11 does not imply any of the equivalent conditions of Theorem 10. Observe that in the semi lattice of Figure 1, condition (ii) of Theorem 11 is satisfied , but (0] is not semi prime (i.e. S is not 0-distributive). Note that here each $(d_i]$ and $(a]$ are the only prime ideals. So $S$-$(d_i]$, S-$(a]$ are prime filters. That is, $[a)$ ,$[a_i)$ for each i and $[b \wedge c)$ are prime filters.

Since in case of lattices, the intersection of any number of prime ideals is a semi prime ideal, so we have the following result due to [3].

**Theorem 12.** *Let L be a lattice and J be an ideal of L. If* $J = \bigcap (P : P \text{ is a prime ideal containing } J)$, *then J is semi prime.* □

It should be mentioned that the above result is not true in case of directed above meet semi lattices. Observe that in Figure 1, $(0] = \bigcap (d_i] \cap (a]$ but S is not 0-distributive. In other words, $(0]$ is not semi prime.

Here each $(d_i]$ and $(a]$ are the only prime ideals. So $S$-$(d_i]$, S-$(a]$ are prime filter. i.e. $[a)$ and $[a_i)$ for each i and $[b \wedge c)$ are prime filters. □

Now we give another characterization of semi-prime ideals with the help of prime separation Theorem using annihilator ideals.

**Theorem 13.** *Let S be a meet semilattice with 0. J is a semi-prime if and only if for all filters F disjoint to* $\{x\}^{\perp_J}; x \in S$ *, there is a prime filter containing F disjoint to* $\{x\}^{\perp_J}$.

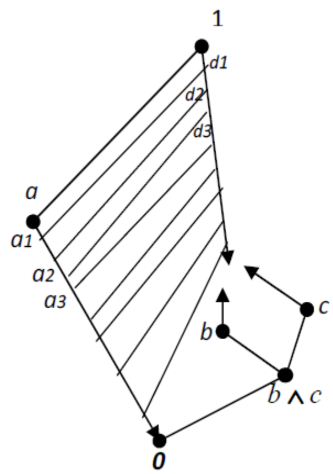


Figure 1

**Proof :** Suppose J is semi prime. Then by Theorem 7, $\{x\}^{\perp_J}$ is semi prime. Using Zorn's Lemma we can easily find a maximal filter $Q$ containing F and disjoint to $\{x\}^{\perp_J}$. We claim that $x \in Q$. If not, then $Q \vee [x) \supset Q$. By maximality of $Q$, $(Q \vee [x)) \cap \{x\}^{\perp_J} \neq \varphi$. If $t \in (Q \vee [x)) \cap \{x\}^{\perp_J}$, Then $t \geq q \wedge x$ for some $q \in Q$ and $t \wedge x \in J$. This implies $q \wedge x \in J$ and so $q \in \{x\}^{\perp_J}$ gives a contradiction. Hence $x \in Q$.

Now let $z \notin Q$. Then $(Q \vee [z)) \cap \{x\}^{\perp_J} \neq \varphi$. Suppose $y \in (Q \vee [z)) \cap \{x\}^{\perp_J}$ then $y \geq q_1 \wedge z$ and $y \wedge x \in J$ for some $q_1 \in Q$. This implies $q_1 \wedge x \wedge z \in J$ and $q_1 \wedge z \in \{x\}^{\perp_J}$. Hence by Lemma 5, $Q$ is a maximal filter disjoint to $\{x\}^{\perp_J}$. Then by Theorem 7. $Q$ is prime.

Conversely, Let $x \wedge y \in J, x \wedge z \in J$. If $x \wedge d \notin J$ for all $d \geq y,z$, then $d \notin \{x\}^{\perp_J}$. Thus $[d) \cap \{x\}^{\perp_J} = \varphi$. So there exists a prime filter $Q$ containing $[d)$ and disjoint from $\{x\}^{\perp_J}$. As $y,z \in \{x\}^{\perp_J}$, so $y,z \notin Q$. Thus $d \notin Q$, for some $d \geq y,z$ as $Q$ is prime. This implies $[d) \not\subset Q$, a contradiction. Hence $x \wedge d \in J$, and so $J$ is semi prime. □

**Corollary 14.** *A directed above meet semi lattice S with 0 is 0-distributive if and only if every prime down set contains a minimal prime ideal.*

**Proof.** Let P be a prime down set of S. Then $P \neq S$. So there exists $x \in S$ such that $x \notin P$. If $t \in \{x\}^{\perp}$, then $t \wedge x = 0 \in P$. This implies $t \in P$, as P is prime.

Therefore $\{x\}^{\perp} \cap (S-P)=\varphi$, where S-P is a filter of S. Suppose S is 0-distributive (i,e. (0] is semi prime). Then by Theorem 13, there is a prime filter Q containing S-P and disjoint to $\{x\}^{\perp}$. It follows that S-Q is a minimal prime ideal contained in P. Proof of the converse is trivial from the proof of Theorem 13 by replacing J by (0]. □

**Theorem 15.** *Let J be a semi-prime ideal of a directed above meet semi-lattice S and $x \in S$. Then a prime ideal P containing $\{x\}^{\perp_J}$ is a minimal prime ideal containing $\{x\}^{\perp_J}$ if and only if for $p \in P$, there exists $q \in S-P$, such that $p \wedge q \in \{x\}^{\perp_J}$.*

**Proof :** Let P be a prime ideal containing $\{x\}^{\perp_J}$ such that the give condition holds. Let *K* be a prime ideal containing $\{x\}^{\perp_J}$ such that $K \subseteq P$. Let $p \in P$. Then there is $q \in S-P$ such that $p \wedge q \in \{x\}^{\perp_J}$. Hence $p \wedge q \in K$. Since $K$ is prime and $q \notin K$, so $p \in K$. Thus , $P \subseteq K$ and so $K = P$. Therefore, *P* must be a minimal prime ideal containing $\{x\}^{\perp_J}$.

Conversely, let *P* be a minimal prime ideal containing $\{x\}^{\perp_J}$. Let $p \in P$. Suppose for all $q \in S-P, p \wedge q \notin \{x\}^{\perp_J}$. Set $D=(S-P)\vee[p)$. We claim that $\{x\}^{\perp_J} \cap D = \varphi$. If not , let $y \in \{x\}^{\perp_J} \cap D$. Then $p \wedge q \le y \in \{x\}^{\perp J}$ for some $q \in S-P$, which is a contradiction to the assumption. Then by Theorem13, there exists a maximal (prime) filter $Q \supseteq D$ and disjoint to $\{x\}^{\perp_J}$. By the proof of Theorem 13 , $x \in Q$ Let $M = S-Q$. Then *M* is a prime ideal containing $\{x\}^{\perp J}$. Now $M \cap D = \varphi$ . This implies $M \cap (S-P)=\varphi$ and hence $M \subseteq P$. Also $M \neq P$, because $P \in D$ implies $p \notin M$ but $p \in P$. Hence *M* is a prime ideal containing $\{x\}^{\perp_J}$ which is properly contained in *P*. This gives a contradiction to the minimal property of *P*. Therefore, the given condition holds. □

An element $x^*$ in a directed above meet semi lattice S with 0 is called the pseudo complement of $x \in S$ if $x \wedge x^* = 0$ and for any $s \in S$, $x \wedge s = 0$ implies $s \le x^*$. S with 0 and 1 is called pseudo complemented if its every element has a pseudo complement.

An element $x$ in a pseudo complemented meet semi lattice is called a dense element if $x^* = 0$. The set of all dense elements is denoted by $D(S)$. It is easy to prove that $D(S)$ is a filter of *S*.

We conclude the paper by extending the result of [2].

**Theorem 16.** *Let S be a pseudo complemented meet semi lattice and let P be a prime ideal of S . Then the following conditions are equivalent:*

(i) *P is minimal* (ii) $x \in P$ *implies that* $x^* \notin P$.

(iii) $x \in P$ *implies that* $x^{**} \in P$. (iv) $P \cap D(S) = \varphi$.

**Proof:** (i)$\Rightarrow$(ii). Let *P* be a minimal prime ideal and let (ii) fail, that is $x^* \in P$ for some $x \in P$. Set $D = (S - P) \vee [x)$. We claim that $0 \notin D$ . For if $0 \in D$, then $0 = q \wedge x$ for some $q \in S - P$, which implies $q \le x^* \in P$ , which is a contradiction. Therefore, $0 \notin D$. Then by Lemma 4 and [2,Theorem 3.2], there is a prime filter $Q$ such that $D \subseteq Q$. Let $M = S - Q$. Then *M* is a prime ideal and $M \cap D = \phi$. Therefore, $M \cap (S - P) = \phi$ and hence $M \subseteq P$.Also $M \ne P$ , because $x \in D$ implies $x \in Q$ and hence $x \notin M$ but $x \in P$ . So *P* is not minimal, which is contradiction. Hence (ii) holds.

(ii)$\Rightarrow$(iii). Let $x \in P$. Since $x^* \wedge x^{**} = 0 \in P$, and by (ii) $x^* \notin P$, so $x^{**} \in P$ as P is prime.

(iii)$\Rightarrow$(iv).Let $x \in P \cap D(S)$, then $x \in P$ and $x^* = 0$. By (iii) $x^{**} \in P$ and so $1 = x^{**} \in P$ is a contradiction as *P* is prime.

(iv)$\Rightarrow$(i). Suppose (iv) holds. If *P* is not minimal, then there exists a prime ideal $Q$ such that $Q \subset P$. Let $x \in P - Q$. Since $Q$ is prime, so $x^* \wedge x^{**} = 0 \in Q$ implies $x^* \in Q \subset P$. Thus $x, x^* \in P$. Since *P* is an ideal. So there exists $d \in P$ such that $d \ge x, x^*$ implies $x^*, x^{**} \ge d^*$. Thus $d^* \le x^* \wedge x^{**} = 0$ implies $d \in D(S)$ which contradicts (iv). Therefore, *P* must be minimal. □

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