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New Connectivity Topological Indices

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Abstract. New degree based graph indices called Kulli-Basava indices were introduced and studied their mathematical and chemical properties which have good response with mean isomer degeneracy. In this study, we introduce the sum connectivity Kulli-Basava index, product connectivity Kulli-Basava index, atom bond connectivity Kulli-Basava index and geometric-arithmetic Kulli-Basava index of a graph and compute exact formulas for some special graphs.

Keywords: Connectivity Kulli-Basava indices, wheel, gear, helm graphs

AMS Mathematics Subject Classification (2010): 05C05, 05C07, 05C12, 05C35

1. Introduction

Let G be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree of $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The degree of an edge $e = uv$ in G is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. The open neighborhood $N_G(v)$ of a vertex v is the set of all vertices adjacent to v. The edge neighborhood of a vertex v is the set of all edges incident to v and it is denoted by $N_e(v)$. Let $S_e(v)$ denote the sum of the degrees of all edges incident to a vertex v. We refer to [1] for undefined term and notation.

A topological index is a numerical parameter mathematically derived from the graph structure. Several topological indices have been considered in Theoretical Chemistry, see [2, 3].

The first and second Kulli-Basava indices were introduced in [4], defined as

$$KB_1(G) = \sum_{uv \in E(G)} \left[S_e(u) + S_e(v)\right],$$

$$KB_2(G) = \sum_{uv \in E(G)} S_e(u) S_e(v).$$

The first and second hyper Kulli-Basava indices were introduced by Kulli [5], defined as

$$HKB_1(G) = \sum_{uv \in E(G)} \left[S_e(u) + S_e(v)\right]^2, \qquad HKB_2(G) = \sum_{uv \in E(G)} \left[S_e(u) S_e(v)\right]^2.$$

We introduce the sum connectivity Kulli-Basava index, product connectivity Kulli-Basava index, atom bond connectivity Kulli-Basava index, geometric-arithmetic Kulli-Basava index and reciprocal Kulli-Basava index of a graph, defined as

$$SKB(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_e(u)+S_e(v)}},$$

$$PKB(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_e(u)S_e(v)}},$$

$$ABCKB(G)=\sum_{uv\in E(G)}\sqrt{\frac{S_e(u)+S_e(v)-2}{S_e(u)S_e(v)}},$$

$$GAKB(G)=\sum_{uv\in E(G)}\frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u)+S_e(v)},$$

$$RKB(G)=\sum_{uv\in E(G)}\sqrt{S_e(u)S_e(v)}.$$

Recently, some connectivity indices were studied [6,7,8,9,10]. In this paper, some connectivity Kulli-Basava indices for some graphs were computed.

2. Regular graphs

A graph G is r-regular if the degree of every vertex of G is r.

Theorem 1. Let G be an r-regular graph with n vertices and m edges. Then

(i) $SKB(G)=\dfrac{m}{2\sqrt{r(r-1)}}$. (ii) $PKB(G)=\dfrac{m}{2r(r-1)}$.

(iii) $ABCKB(G)=\dfrac{m\sqrt{4r(r-1)-2}}{2r(r-1)}$. (iv) $GAKB(G)=m$.

(v) $RKB(G)=2mr(r-1)$.

Proof: Let G be an r-regular graph with n vertices and m edges. Then $S_e(u)=2r(r-1)$ for any vertex $u\in V(G)$. Therefore

(i) $$SKB(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_e(u)+S_e(v)}}=\frac{m}{\sqrt{2r(r-1)+2r(r-1)}}=\frac{m}{2\sqrt{r(r-1)}}.$$

(ii) $$PKB(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_e(u)S_e(v)}}=\frac{m}{\sqrt{2r(r-1)2r(r-1)}}=\frac{m}{2r(r-1)}.$$

(iii) $$ABCKB(G)=\sum_{uv\in E(G)}\sqrt{\frac{S_e(u)+S_e(v)-2}{S_e(u)S_e(v)}}=\frac{m\sqrt{2r(r-1)+2r(r-1)-2}}{\sqrt{2r(r-1)2r(r-1)}}$$
$$=\frac{m\sqrt{4r(r-1)-2}}{2r(r-1)}.$$

(iv) $$GAKB(G)=\sum_{uv\in E(G)}\frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u)+S_e(v)}=\frac{m2\sqrt{2r(r-1)2r(r-1)}}{2r(r-1)+2r(r-1)}=m.$$

(v) $$RKB(G)=\sum_{uv\in E(G)}\sqrt{S_e(u)S_e(v)}=m\sqrt{2r(r-1)2r(r-1)}=2mr(r-1).$$

Corollary 1.1. If C_n is a cycle with n vertices, then

(i) $SKB(C_n)=\frac{n}{2\sqrt{2}}$. (ii) $PKB(C_n)=\frac{n}{4}$.

(iii) $ABCKB(C_n)=\frac{\sqrt{6}}{4}n.$ (iv) $GAKB(C_n)=n.$

(v) $RKB(C_n)=4n.$

Corollary 1.2. If K_n is a complete graph with n vertices, then

(i) $SKB(K_n)=\frac{n\sqrt{n-1}}{4\sqrt{n-2}}$. (ii) $SKB(K_n)=\frac{n}{4(n-2)}$.

(iii) $ABCKB(K_n)=\frac{n\sqrt{4(n-1)(n-2)-2}}{4(n-2)}$. (iv) $GAKB(K_n)=\frac{n(n-1)}{2}$.

(v) $RKB(K_n)=n(n-1)^2(n-2).$

3. Results for wheel graphs

A wheel W_n is the join of C_n and K_1. Clearly W_n has n+1 vertices and 2n edges. A wheel W_n is shown in Figure 1. The vertices C_n are called rim vertices and the vertex of K_1 is called apex.

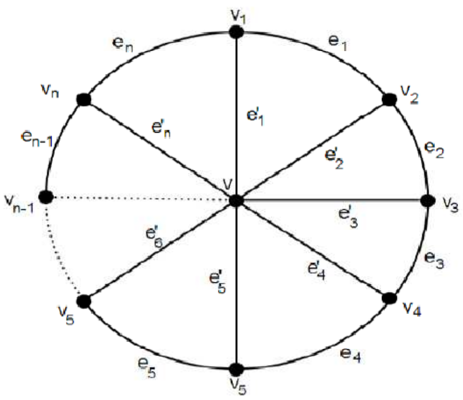

Figure 1: Wheel W_n

Lemma 2. Let W_n be a wheel with 2n edges, $n\geq 3$. Then

$E_1=\{uv\in E(W_n) | S_e(u) = n(n+1), S_e(v) = n+9\}$, $|E_1| = n$.

$E_2=\{uv\in E(W_n) | S_e(u) = S_e(v) = n+9\}$, $|E_2| = n$.

Theorem 3. Let W_n be a wheel with n+1 vertices and 2n edges, $n \geq 3$. Then

(i) $$SKB(W_n) = \frac{n}{\sqrt{n^2+2n+9}} + \frac{n}{\sqrt{2n+18}}.$$

(ii) $$PKB(W_n) = \frac{n}{\sqrt{n(n+1)(n+9)}} + \frac{n}{n+9}.$$

(iii) $$ABCKB(W_n) = n\sqrt{\frac{n^2+2n+7}{n(n+1)(n+9)}} + \frac{n\sqrt{2n+16}}{n+9}.$$

(iv) $$GAKB(W_n) = \frac{2n\sqrt{n(n+1)(n+9)}}{n^2+2n+9} + n.$$

(v) $$RKB(W_n) = n\sqrt{n^2+2n+9} + n(n+9).$$

Proof: Let W_n be a wheel with n+1 vertices and 2n edges. By using definitions and Lemma 2, we obtain

(i) $$SKB(W_n) = \sum_{uv\in E(W_n)} \frac{1}{\sqrt{S_e(u)+S_e(v)}} = \frac{n}{\sqrt{n(n+1)+n+9}} + \frac{n}{\sqrt{n+9+n+9}}$$
$$= \frac{n}{\sqrt{n^2+2n+9}} + \frac{n}{\sqrt{2n+18}}.$$

(ii) $$PKB(W_n) = \sum_{uv\in E(W_n)} \frac{1}{\sqrt{S_e(u)S_e(v)}} = \frac{n}{\sqrt{n(n+1)(n+9)}} + \frac{n}{\sqrt{(n+9)(n+9)}}$$
$$= \frac{n}{\sqrt{n(n+1)(n+9)}} + \frac{n}{n+9}.$$

(iii) $$ABCKB(W_n) = \sum_{uv\in E(W_n)} \sqrt{\frac{S_e(u)+S_e(v)-2}{S_e(u)S_e(v)}}$$
$$= n\sqrt{\frac{n(n+1)+n+9-2}{n(n+1)(n+9)}} + n\sqrt{\frac{n+9+n+9-2}{(n+1)(n+9)}}$$
$$= n\sqrt{\frac{n^2+2n+7}{n(n+1)(n+9)}} + n\frac{\sqrt{2n+16}}{(n+9)}.$$

(iv) $$GAKB(W_n) = \sum_{uv\in E(W_n)} \frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u)+S_e(v)} = \frac{2n\sqrt{n(n+1)(n+9)}}{n(n+1)+(n+9)} + \frac{2n\sqrt{(n+9)(n+9)}}{(n+9)+(n+9)}$$
$$= \frac{2n\sqrt{n(n+1)(n+9)}}{n^2+2n+9} + n.$$

(v) $$RKB(W_n) = \sum_{uv\in E(W_n)} \sqrt{S_e(u)S_e(v)} = n\sqrt{n(n+1)(n+9)} + n\sqrt{(n+9)(n+9)}$$
$$= n\sqrt{n^2+2n+9} + n(n+9).$$

4. Results for gear graphs

A gear graph is obtained from W_n by adding a vertex between each pair of adjacent rim vertices and it is denoted by G_n. Clearly G_n has $2n$+1 vertices and $3n$ edges. A graph G_n is shown in Figure 2.

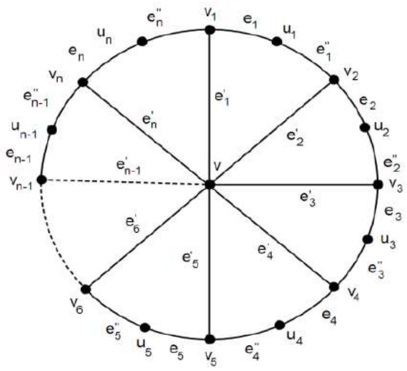

Figure 2: A gear graph G_n

Lemma 4. Let G_n be a gear graph with $2n$+1 vertices and $3n$ edges. Then G_n has two types of edges as follows:

$E_1=\{uv \in E(G_n) | S_e(u) = n(n+1), S_e(v) = n+7\}, \quad |E_1| = n.$

$E_2=\{uv \in E(G_n) | S_e(u) = n+7, S_e(v) = 6\}, \quad |E_2| = 2n.$

Theorem 5. Let G_n be a gear graph with $2n + 1$ vertices $3n$ edges. Then

(i) $$SKB(G_n) = \frac{n}{\sqrt{n^2+2n+7}} + \frac{2n}{\sqrt{n+13}}.$$

(ii) $$PKB(G_n) = \frac{n}{\sqrt{n(n+1)(n+9)}} + \frac{2n}{\sqrt{6(n+7)}}.$$

(iii) $$ABCKB(G_n) = n\left(\frac{n^2+2n+5}{n(n+1)(n+7)}\right)^{\frac{1}{2}} + 2n\left(\frac{n+11}{6(n+7)}\right)^{\frac{1}{2}}.$$

(iv) $$GAKB(G_n) = \frac{2n\sqrt{n(n+1)(n+7)}}{n^2+2n+7} + \frac{4n\sqrt{6(n+7)}}{n+13}.$$

(v) $$RKB(G_n) = n\sqrt{n(n+1)(n+7)} + 2n\sqrt{6(n+7)}.$$

Proof: Let G_n be a gear graph with $2n$+1 vertices and $3n$ edges. By using definitions and Lemma 4, we obtain

(i) $$SKB(G_n) = \sum_{uv\in E(G_n)} \frac{1}{\sqrt{S_e(u)+S_e(v)}} = \frac{n}{\sqrt{n(n+1)+(n+7)}} + \frac{2n}{\sqrt{n+7+6}}$$

$$= \frac{n}{\sqrt{n^2+2n+7}} + \frac{2n}{\sqrt{n+13}}.$$

(ii) $$PKB(G_n) = \sum_{uv\in E(G_n)} \frac{1}{\sqrt{S_e(u)S_e(v)}} = \frac{n}{\sqrt{n(n+1)(n+7)}} + \frac{2n}{\sqrt{6(n+7)}}$$

(iii) $$ABCKB(G_n)=\sum_{uv\in E(G_n)}\sqrt{\frac{S_e(u)+S_e(v)-2}{S_e(u)S_e(v)}}$$

$$=n\left(\frac{n(n+1)+(n+7)-2}{n(n+1)(n+7)}\right)^{\frac{1}{2}}+2n\left(\frac{n+7+6-2}{(n+7)6}\right)^{\frac{1}{2}}$$

$$=n\left(\frac{n^2+2n+5}{n(n+1)(n+7)}\right)^{\frac{1}{2}}+2n\left(\frac{n+11}{6(n+7)}\right)^{\frac{1}{2}}.$$

(iv) $$GAKB(G_n)=\sum_{uv\in E(G_n)}\frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u)+S_e(v)}=\frac{n2\sqrt{n(n+1)(n+7)}}{n(n+1)+n+7}+\frac{2n2\sqrt{(n+7)6}}{n+7+6}$$

$$=\frac{2n\sqrt{n(n+1)(n+7)}}{n^2+2n+7}+\frac{4n\sqrt{6(n+7)}}{n+13}$$

(v) $$RKB(G_n)=\sum_{uv\in E(G_n)}\sqrt{S_e(u)S_e(v)}=n\sqrt{n(n+1)(n+7)}+2n\sqrt{6(n+7)}.$$

$$=n\sqrt{n^2+2n+9}+2n\sqrt{6(n+7)}.$$

5. Results for helm graphs

A helm graph H_n is a graph obtained from W_n by attaching an end edge to each rim vertex. Clearly H_n has $2n$+1 vertices and $3n$ edges. A graph H_n is depicted in Figure 3.

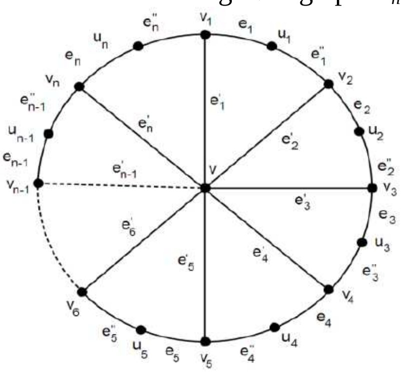

Figure 3: A graph H_n

Lemma 6. If H_n is a helm graph with $2n$+1 vertices and $3n$ edges, then H_n has three types of edges as

$E_1=\{uv\in E(H_n)|\ S_e(u) = n(n+2),\ S_e(v) = n + 17\}$ $\quad |E_1| = n$.

$E_2=\{uv\in E(H_n)|\ S_e(u) = S_e(v) = n + 17\}$, $\quad |E_2| = n$.

$E_3=\{uv\in E(H_n)|\ S_e(u) = n + 17,\ S_e(v) = 3\}$ $\quad |E_3| = n$.

Theorem 7. If H_n is a helm graph with $2n$+1 vertices and $3n$ edges, then

(i) $$SKB(H_n)=\frac{n}{\sqrt{n^2+3n+17}}+\frac{n}{\sqrt{2n+34}}+\frac{n}{\sqrt{n+20}}.$$

(ii) $$PKB(H_n)=\frac{n}{\sqrt{n(n+2)(n+17)}}+\frac{n}{(n+17)}+\frac{n}{\sqrt{3(n+17)}}.$$

(iii) $$ABCKB(H_n)=n\left(\frac{n^2+3n+5}{n(n+2)(n+17)}\right)^{\frac{1}{2}}+n\frac{\sqrt{2n+32}}{n+17}+n\left(\frac{n+18}{3(n+17)}\right)^{\frac{1}{2}}.$$

(iv) $$GAKB(H_n)=\frac{2n\sqrt{n(n+1)(n+7)}}{n^2+3n+7}+n+\frac{2n\sqrt{3(n+17)}}{n+20}.$$

(v) $$RKB(H_n)=n\sqrt{n(n+2)(n+17)}+n(n+17)+n\sqrt{2(n+17)}.$$

Proof: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then by using definitions and Lemma 6, we deduce

(i) $$SKB(H_n)=\sum_{uv\in E(H_n)}\frac{1}{\sqrt{S_e(u)+S_e(v)}}$$
$$=\frac{n}{\sqrt{n(n+2)+n+17}}+\frac{n}{\sqrt{n+17+n+17}}+\frac{2n}{\sqrt{n+17+3}}$$
$$=\frac{n}{\sqrt{n^2+3n+17}}+\frac{n}{\sqrt{2n+34}}+\frac{n}{\sqrt{n+20}}.$$

(ii) $$PKB(H_n)=\sum_{uv\in E(H_n)}\frac{1}{\sqrt{S_e(u)S_e(v)}}$$
$$=\frac{n}{\sqrt{n(n+2)(n+17)}}+\frac{n}{\sqrt{(n+17)(n+17)}}+\frac{n}{\sqrt{(n+17)3}}$$
$$=\frac{n}{\sqrt{n(n+2)(n+17)}}+\frac{n}{(n+17)}+\frac{n}{\sqrt{3(n+17)}}.$$

(iii) $$ABCKB(H_n)=\sum_{uv\in E(H_n)}\sqrt{\frac{S_e(u)+S_e(v)-2}{S_e(u)S_e(v)}}$$
$$=|E_1|\sqrt{\frac{n(n+2)+n+17-2}{n(n+2)(n+17)}}+|E_2|\sqrt{\frac{n+17+n+17-2}{(n+17)(n+17)}}$$
$$+|E_3|\sqrt{\frac{n+17+n+3-2}{(n+17)3}}$$
$$=n\left(\frac{n^2+3n+15}{n(n+2)(n+17)}\right)^{\frac{1}{2}}+n\frac{\sqrt{2n+32}}{n+17}+n\left(\frac{n+18}{3(n+7)}\right)^{\frac{1}{2}}.$$

(iv) $$GAKB(H_n)=\sum_{uv\in E(H_n)}\frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u)+S_e(v)}$$
$$=|E_1|\frac{2\sqrt{n(n+2)(n+17)}}{n(n+2)+n+17}+|E_2|\frac{2\sqrt{(n+17)(n+17)}}{n+17+n+17}$$

$$+|E_3|\frac{2\sqrt{(n+17)3}}{n+17+3}$$

$$=\frac{2n\sqrt{n(n+2)(n+7)}}{n^2+3n+17}+n+\frac{2n\sqrt{3(n+17)}}{n+20}.$$

(v) $$RKB(H_n)=\sum_{uv\in E(H_n)}\sqrt{S_e(u)S_e(v)}$$

$$=|E_1|\sqrt{n(n+2)(n+17)}+|E_2|\sqrt{(n+17)(n+17)}+|E_3|\sqrt{(n+17)3}.$$

$$=n\sqrt{n(n+2)(n+17)}+n(n+17)+n\sqrt{3(n+17)}.$$

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