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Tripolar Fuzzy Bi ideal of a Near Ring

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Abstract. In this paper, we introduce the concept of tripolar fuzzy bi ideal of a near ring, which is a generalisation of tripolar fuzzy set, fuzzy bi ideal of a near ring. We would like to study few properties of it.

Keywords: Tripolar fuzzy set, bi-ideal, near ring

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1. Introduction

The notion of near ring was introduced by Dickson [3] in 1905. Fuzzy ideals in near rings were studied by Zaid [1]. The concept of bi ideal for semi groups was introduced by Good and Hughes [4]. The notion of bi ideals in associative rings were introduced by Lajos and Szasz [5]. Bi ideals in near rings was given by Chelvam and Ganesan [15]. Senapati, et al. [13] studied cubic subalgebras and cubic closed ideals of B-algebras. Rafi and Venkateshwarlu [11] gave a relation on almost distributive lattices. Concept of fuzzy set introduced by Zadeh [16]. Since then many extensions have been given like intuitionistic fuzzy sets, interval valued fuzzy sets, bipolar sets and so on. Intuitionistic fuzzy sets have been introduced by Attanosov [2]. Selvam and Nagalakshmi [14] discussed fuzzy PMS ideals in PMS algebras. Nagireddy et el. [8] gave a note on fuzzy bi ideals in ternary semigroups. Senapati, et al. [12] Initiated the notion of intuitionistic fuzzifications of ideals in BG-algebras. Lee [6] introduced the concept of bipolar valued fuzzy sets. Bipolar fuzzy set is an extension of fuzzy set whose membeshipp degree range is [-1,1].

The concept of tripolar fuzzy set has been introduced by Rao [7]. Tripolar fuzzy set is a generalisation of fuzzy set, intuitionistic fuzzy set and bipolar fuzzy set. He introduced the concept of tripolar fuzzy interior ideals of a gamma semi group. The tripolar concept is useful in studying the relevant, irrelevant and the implicit counter elements. Swamy, et al. [9] introduced the notion of tripolar fuzzy ideals of a near ring. In this paper we wish to introduce the concept of tripolar fuzzy bi ideal of a near ring which is a generalization of tripolar fuzzy set, fuzzy bi ideal of near ring. We also intend to study some of its properties. Throughout the paper R denotes a right near ring.

2. Preliminaries

Definition 2.1. [10] A Near ring (right) is a non-empty set *R* with two binary operations "+" and "." satisfying the following axioms:

i) $(R,+)$ is a group (not necessarily abelian)
ii) $(R,.)$ is a semi group
iii) $(y+z)x = yx + zx$ for all $x, y, z \in R$.

Definition 2.2. An ideal of a near ring *R* is a subset *I* of *R* such that

i) $(I,+)$ is a normal subgroup of $(R,+)$
ii) $IR \subseteq I$
iii) $x(y+i) - xy \in I$ for all $i \in I$ and $x, y \in R$.

Definition 2.3. [15] A subgroup *I* of $(R,+)$ satisfying $IRI \cap (IR) * I \subseteq I$ is called a bi ideal of *R*.

Definition 2.4. [16] A mapping $\mu : N \to [0,1]$ is called a fuzzy subset of *N*.

Definition 2.5. [1] A fuzzy subset μ of a near ring *R* is called a fuzzy sub near ring of *R* if

i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,
ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in R$.

Definition 2.6. A fuzzy sub near ring μ is a fuzzy ideal of a near ring *R* if

i) $\mu(y+x-y) \geq \mu(x)$,
ii) $\mu(xy) \geq \mu(x)$,
iii) $\mu(x(y+i) - xy) \geq \mu(i)$ for all $x, y, i \in R$.

Definition 2.7. A fuzzy subset μ of *R* is called a fuzzy bi ideal of *R*, if

i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,
ii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$.

Definition 2.8. Let *M* and *N* be two near rings. A mapping $f : M \to N$ is said to be a near ring homomorphism if

i) $f(x+y) = f(x) + f(y)$,
ii) $f(xy) = f(x) f(y)$ for all $x, y \in M$.

Definition 2.9. Let $\phi : M \to N$ be a homomorphism of near rings and *f* be a fuzzy subset of *M*. Then $\phi(f)(y) = \sup_{x \in \phi^{-1}(y)} f(x), if\, \phi^{-1}(y) \neq 0$

= 0, otherwise.

We call $\phi(f)$ is the image of *f* under ϕ.

Definition 2.10. [6] A bipolar fuzzy set $\mu = (\mu^+, \mu^-) = \{(x, \mu^+(x), \mu^-(x)) : x \in X\}$, where $\mu^+ : X \to [0,1]$ and $\mu^- : X \to [-1,0]$ are membership functions. $\mu^+(x)$ is a degree of membership of the bipolar fuzzy set $\mu = \{(x, \mu^+(x), \mu^-(x)) : x \in X\}$ and $\mu^-(x)$ is a degree of non-membership of the bipolar fuzzy set $\mu = \{(x, \mu^+(x), \mu^-(x)) : x \in X\}$.

Definition 2.11. [7] A fuzzy subset *T* of a universe set *X* is said to be a tripolar fuzzy set, if
$T = \{(x, \alpha_T(x), \beta_T(x), \gamma_T(x)) / x \in X \; and \; 0 \leq \alpha_T(x) + \beta_T(x) \leq 1\}, where$
$\alpha_T : X \to [0,1], \beta_T : X \to [0,1], \gamma_T : X \to [-1,0]$. The membership degree $\alpha_T(x)$ characterises the extent that the element *x* satisfies to the property corresponding to tripolar fuzzy set *T*, $\beta_T(x)$ characterises the extent that the element *x* satisfies to the not property (irrelevant) corresponding to tripolar fuzzy set *T* and $\gamma_T(x)$ characterises the extent that the element x satisfies to the implicit counter property of tripolar fuzzy set *T*. $T = \{(x, \alpha_T(x), \beta_T(x), \gamma_T(x)) / x \in X \quad and \quad 0 \leq \alpha_T(x) + \beta_T(x) \leq 1\}$ is denoted by $T = (\alpha_T, \beta_T, \gamma_T)$. Thus, a tripolar fuzzy set *T* is a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set.

Definition 2.12. [9] A tripolar fuzzy set $T = (\alpha_T, \beta_T, \gamma_T)$ of a near ring *R* is called a tripolar fuzzy sub near ring of *R* if it satisfies the following conditions:

i) $\alpha_T(x - y) \geq \min\{\alpha_T(x), \alpha_T(y)\}$,
ii) $\alpha_T(xy) \geq \min\{\alpha_T(x), \alpha_T(y)\}$,
iii) $\beta_T(x - y) \leq \max\{\beta_T(x), \beta_T(y)\}$,
iv) $\beta_T(xy) \leq \max\{\beta_T(x), \beta_T(y)\}$,
v) $\gamma_T(x - y) \leq \max\{\gamma_T(x), \gamma_T(y)\}$,
vi) $\gamma_T(xy) \leq \max\{\gamma_T(x), \gamma_T(y)\}$.

Definition 2.13. [9] A tripolar fuzzy sub near ring $T = (\alpha_T, \beta_T, \gamma_T)$ of a near ring *R* is called a tripolar fuzzy ideal of *R* if *T* satisfies the following conditions:

i) $\alpha_T(xy) \geq \{\alpha_T(x)\}$,

ii) $\alpha_T(x(y+i)-xy) \geq \{\alpha_T(i)\}$,

iii) $\alpha_T(y+x-y) \geq \{\alpha_T(x)\}$,

iv) $\beta_T(y+x-y) \leq \{\beta_T(x)\}$,

v) $\beta_T(xy) \leq \{\beta_T(x)\}$,

vi) $\beta_T(x(y+i)-xy) \leq \{\beta_T(i)\}$,

vii) $\gamma_T(y+x-y) \leq \{\gamma_T(x)\}$,

viii) $\gamma_T(xy) \leq \{\gamma_T(x)\}$,

ix) $\gamma_T((x+i)y-xy) \leq \{\gamma_T(i)\}$ for all $x, y, i \in R$.

3. Tripolar fuzzy bi ideal

In this section we introduce the concept of tripolar fuzzy bi ideal and study some of their properties.

Definition 3.1. A tripolar fuzzy set $T = (\alpha_T, \beta_T, \gamma_T)$ of a near ring *R* is called a tripolar fuzzy bi ideal of *R*, if

i) $\alpha_T(x-y) \geq \min\{\alpha_T(x), \alpha_T(y)\}$

ii) $\alpha_T(xyz) \geq \min\{\alpha_T(x), \alpha_T(z)\}$,

iii) $\beta_T(x-y) \leq \max\{\beta_T(x), \beta_T(y)\}$,

iv) $\beta_T(xyz) \leq \max\{\beta_T(x), \beta_T(z)\}$,

v) $\gamma_T(x-y) \leq \max\{\gamma_T(x), \gamma_T(y)\}$,

vi) $\gamma_T(xyz) \leq \max\{\gamma_T(x), \gamma_T(z)\}$.

for every $x, y \in R$.

Example 3.1. Let $R = \{0, a, b, c\}$ be a near ring with the binary operations defined below:

+	0	*a*	*b*	*c*
0	0	*a*	*b*	*c*
a	*a*	0	*c*	*b*
b	*b*	*c*	0	*a*
c	*c*	*b*	*a*	0

•	0	*a*	*b*	*c*
0	0	0	0	0
a	0	0	*a*	*a*
b	0	0	*b*	*b*
c	0	0	*c*	*c*

Define a tripolar fuzzy set $T = (\alpha_T, \beta_T, \gamma_T)$ by

$T = \{(0, 0.8, 0.2, -0.4),\ (a, 0.6, 0.3, -0.2),\ (b, 0.6, 0.3, -0.2), (c, 0.6, 0.3, -0.2)\}$.

Then T is a tripolar fuzzy bi-ideal of the near ring *R*.

Theorem 3.1. Intersection of two tripolar fuzzy bi ideals of R is also its tripolar fuzzy bi ideal.

Proof: Let $f=(\alpha_f,\beta_f,\gamma_f), g=(\alpha_g,\beta_g,\gamma_g)$ be two tripolar fuzzy bi ideals of R and $x,y\in R$. Then

$$\alpha_{f\cap g}(x-y)=\min(\alpha_f(x-y),\alpha_g(x-y))\geq\min(\min(\alpha_f(x),\alpha_f(y)),\min(\alpha_g(x),\alpha_g(y)))$$
$$=\min(\min(\alpha_f(x),\alpha_g(x)),\min(\alpha_f(y),\alpha_g(y)))=\min(\alpha_{f\cap g}(x),\alpha_{f\cap g}(y)),$$
$$\alpha_{f\cap g}(xyz)=\min(\alpha_f(xyz),\alpha_g(xyz))\geq\min(\min(\alpha_f(x),\alpha_f(z)),\min(\alpha_g(x),\alpha_g(z)))$$
$$=\min(\min(\alpha_f(x),\alpha_g(x)),\min(\alpha_f(z),\alpha_g(z)))=\min(\alpha_{f\cap g}(x),\alpha_{f\cap g}(z)),$$
$$\beta_{f\cap g}(x-y)=\max(\beta_f(x-y),\beta_g(x-y))\geq\max(\max(\beta_f(x),\beta_f(y)),\max(\beta_g(x),\beta_g(y)))$$
$$=\max(\max(\beta_f(x),\beta_g(x)),\max(\beta_f(y),\beta_g(y)))=\max(\beta_{f\cap g}(x),\beta_{f\cap g}(y)),$$
$$\beta_{f\cap g}(xyz)=\max(\beta_f(xyz),\beta_g(xyz))\geq\max(\max(\beta_f(x),\beta_f(z)),\max(\beta_g(x),\beta_g(z)))$$
$$=\max(\max(\beta_f(x),\beta_g(x)),\max(\beta_f(z),\beta_g(z)))=\max(\beta_{f\cap g}(x),\beta_{f\cap g}(z)),$$
$$\gamma_{f\cap g}(x-y)=\max(\gamma_f(x-y),\gamma_g(x-y))\geq\max(\max(\gamma_f(x),\gamma_f(y)),\max(\gamma_g(x),\gamma_g(y)))$$
$$=\max(\max(\gamma_f(x),\gamma_g(x)),\max(\gamma_f(y),\gamma_g(y)))=\max(\gamma_{f\cap g}(x),\gamma_{f\cap g}(y)),$$
$$\gamma_{f\cap g}(xyz)=\max(\gamma_f(xyz),\gamma_g(xyz))\geq\max(\max(\gamma_f(x),\gamma_f(z)),\max(\gamma_g(x),\gamma_g(z)))$$
$$=\max(\max(\gamma_f(x),\gamma_g(x)),\max(\gamma_f(z),\gamma_g(z)))=\max(\gamma_{f\cap g}(x),\gamma_{f\cap g}(z)).$$

Hence proved.

Theorem 3.2. If a tripolar fuzzy set $T=(\alpha_T,\beta_T,\gamma_T)$ of near ring R is a tripolar fuzzy bi- ideal of R, then $(\alpha_T,\bar{\alpha}_T,\gamma_T)$, where $\bar{\alpha}_T=1-\alpha_T$ is a tripolar fuzzy bi ideal of R.

Proof: Let $x,y,z\in R$. Then $\bar{\alpha}_T(x-y)=1-\alpha_T(x-y)\leq 1-\min\{\alpha_T(x),\alpha_T(y)\}$ $=\max\{1-\alpha_T(x),1-\alpha_T(y)\}=\max\{\bar{\alpha}_T(x),\bar{\alpha}_T(y)\}$ and $\bar{\alpha}_T(xyz)=1-\alpha_T(xyz)$ $\leq 1-\min\{\alpha_T(x),\alpha_T(z)\}=\max\{1-\alpha_T(x),1-\alpha_T(z)\}=\max\{\bar{\alpha}_T(x),\bar{\alpha}_T(z)\}$.

Theorem 3.3. Let $f:M\to N$ be a near ring homomorphism. If $T=(\alpha_T,\beta_T,\gamma_T)$ is a tripolar fuzzy bi-ideal of N , then $f^{-1}(T)=(f^{-1}(\alpha_T),f^{-1}(\beta_T),f^{-1}(\gamma_T))$ is a tripolar fuzzy bi ideal of M.

Proof: Let $x,y\in M$.

$$\begin{aligned}f^{-1}(\alpha_T(x-y)) &= \alpha_T(f(x-y))\\ &=\alpha_T(f(x)-f(y))\\ &\geq\min\{\alpha_T(f(x)),\alpha_T(f(y))\}\\ &=\min\{f^{-1}(\alpha_T(x)),f^{-1}(\alpha_T(y))\},\end{aligned}$$

$$
\begin{aligned}
f^{-1}(\alpha_T(xyz)) &= \alpha_T(f(xyz)) \\
&= \alpha_T(f(x)f(y)f(z)) \\
&\geq \min\{\alpha_T(f(x)), \alpha_T(f(z))\} \\
&= \min\{f^{-1}(\alpha_T(x)), f^{-1}(\alpha_T(z))\}, \\
f^{-1}(\beta_T(x-y)) &= \beta_T(f(x-y)) \\
&= \beta_T(f(x)-f(y)) \\
&\leq \max\{\beta_T(f(x)), \beta_T(f(y))\} \\
&= \max\{f^{-1}(\beta_T(x)), f^{-1}(\beta_T(y))\}, \\
f^{-1}(\beta_T(xyz)) &= \beta_T(f(xyz)) \\
&= \beta_T(f(x)f(y)f(z)) \\
&\leq \max\{\beta_T(f(x)), \beta_T(f(z))\} \\
&= \max\{f^{-1}(\beta_T(x)), f^{-1}(\beta_T(z))\}, \\
f^{-1}(\gamma_T(x-y)) &= \gamma_T(f(x-y)) \\
&= \gamma_T(f(x)-f(y)) \\
&\leq \max\{\gamma_T(f(x)), \gamma_T(f(y))\} \\
&= \max\{f^{-1}(\gamma_T(x)), f^{-1}(\gamma_T(y))\}, \\
f^{-1}(\gamma_T(xyz)) &= \gamma_T(f(xyz)) \\
&= \gamma_T(f(x)f(y)f(z)) \\
&\leq \max\{\gamma_T(f(x)), \beta_T(f(z))\} \\
&= \max\{f^{-1}(\gamma_T(x)), f^{-1}(\gamma_T(z))\}.
\end{aligned}
$$

Thus $f^{-1}(T) = (f^{-1}(\alpha_T), f^{-1}(\beta_T), f^{-1}(\gamma_T))$ is a tripolar fuzzy bi ideal of M .

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