Annals of Pure and Applied Mathematics
Vol. 21, No. 2, 2020, 113-118
ISSN: 2279-087X (P), 2279-0888(online)
Published on 2 May 2020
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v21n2a5646

Annals of
Pure and Applied Mathematics

The (*a*, *b*)-Status Index of Graphs

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Received 20 February 2020; accepted 21 April 2020

Abstract. The status of a vertex u is defined as the sum of the distances between u and all other vertices of a connected graph. In this paper, we introduce the (a, b)-status index of a graph. We also compute the (a, b)-status index of wheel and friendship graphs. Also we introduce F_1-status index, first and second status Gourava indices, symmetric division status index of a graph and compute exact formulas for wheel and friendship graphs.

Keywords: distance, (a, b)-status index, F_1-status index, symmetric division status index, graph.

AMS Mathematics Subject Classification (2010): 05C05, 05C12

1. Introduction

Let G be a finite, simple, connected graph. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The distance, denoted by $d(u, v)$, between any two vertices u and v is the length of shortest path connecting u and v. The status $\sigma(u)$ of a vertex u in G is the sum of distances of all other vertices from u in G. For undefined terms and notations, we refer [1].

A graph index is a numerical parameter mathematically derived from the graph structure. The graph indices have their applications in various disciplines of Science and Technology [2, 3]. Some of the graph indices can be found in [4, 5, 6, 7, 8, 9, 10].

The first and second status connectivity indices were introduced by Ramane at al. in [11], defined as

$$S_1(G)=\sum_{uv\in E(G)}[\sigma(u)+\sigma(v)], \qquad S_2(G)=\sum_{uv\in E(G)}\sigma(u)\sigma(v).$$

In [12], Kulli introduced the product connectivity status index, defined as

$$PS(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\sigma(u)\sigma(v)}}.$$

The reciprocal product connectivity status index and general second status index were introduced by Kulli in [12], and they are defined as

$$RPS(G)=\sum_{uv\in E(G)}\sqrt{\sigma(u)\sigma(v)}. \qquad S_2^a(G)=\sum_{uv\in E(G)}[\sigma(u)\sigma(v)]^a,$$

where a is a real number.

We introduce the F_1-status index of a graph and is defined as

$$F_1S(G)=\sum_{uv\in E(G)}\left[\sigma(u)^2+\sigma(v)^2\right].$$

We introduce the first and second status Gourava indices of a graph, defined as

$$SGO_1(G)=\sum_{uv\in E(G)}\left[\sigma(u)+\sigma(v)+\sigma(u)\sigma(v)\right],$$

$$SGO_2(G)=\sum_{uv\in E(G)}\sigma(u)\sigma(v)\left[\sigma(u)+\sigma(v)\right].$$

We define the symmetric division status index of a graph as

$$SDS(G)=\sum_{uv\in E(G)}\left[\frac{\sigma(u)}{\sigma(v)}+\frac{\sigma(v)}{\sigma(u)}\right].$$

Motivated by the work on status indices, we introduce the (*a, b*)-status index of a graph and it is defined as

$$S_{a,b}(G)=\sum_{uv\in E(G)}\left[\sigma(u)^a\,\sigma(v)^b+\sigma(u)^b\,\sigma(v)^a\right],$$

where *a* and *b* are real numbers.

Recently the first and second Gourava indices were studied in [13].Recently, some status indices were introduced and studied such as multiplicative vertex status index [14], multiplicative first and second status indices [15], multiplicative (*a, b*)-status index [16], *F*-status index [17], *ABC* status index [18], multiplicative *GA* status index [19], harmonic status index [20], status connectivity coindices [21].In this paper, the (*a, b*)-status index of wheel and friendship graphs are determined.

2. Observations

We observe the following relationships.

1. The first status index $S_1(G)=S_{1,0}(G)$.
2. The second status index $S_2(G)=\frac{1}{2}S_{1,1}(G)$.
3. The product connectivity status index $PS(G)=\frac{1}{2}S_{-\frac{1}{2},-\frac{1}{2}}(G)$.
4. The reciprocal product connectivity status index $RPS(G)=\frac{1}{2}S_{\frac{1}{2},\frac{1}{2}}(G)$.
5. The general second status index $S_2^a(G)=\frac{1}{2}S_{a,a}(G)$.
6. The F_1-status index $F_1S(G)=S_{2,0}(G)$.
7. The second status Gourava index $SGO_2(G)=S_{2,1}(G)$.
8. The symmetric division status index $SDS(G)=S_{1,-1}(G)$.

3. Results for wheel graphs

A wheel graph W_n is the join of C_n and K_1. Then W_n has n+1 vertices and 2n edges. A graph W_n is shown in Figure 1.

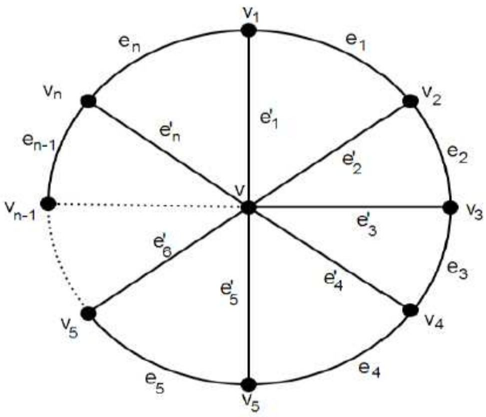

Figure 1: Wheel graph W_n

In W_n , there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\}$, $|E_2| = n$.

Therefore there are two types of status edges as given in Table 1.

$\sigma(u), \sigma(v) \setminus uv \in E(W_n)$	$(2n-3, 2n-3)$	$(n, 2n-3)$
Number of edges	n	n

Table 1: Status edge partition of W_n

Theorem 1. The (*a*, *b*)-status index of a wheel graph W_n is

$$S_{a,b}(W_n) = n\left[2(2n-3)^{a+b}\right] + n\left[n^a(2n-3)^b + n^b(2n-3)^a\right]. \qquad \text{(i)}$$

Proof : By using definition and Table 1, we deduce

$$S_{a,b}(W_n) = \sum_{uv \in E(W_n)} \left[\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a\right]$$

$$= n\left[(2n-3)^a(2n-3)^b + (2n-3)^b(2n-3)^a\right] + n\left[n^a(2n-3)^b + n^b(2n-3)^a\right]$$

$$= n\left[2(2n-3)^{a+b}\right] + n\left[n^a(2n-3)^b + n^b(2n-3)^a\right].$$

Corollary 1.1. From observations and by using equation (i), we establish the following results.

(1) $S_1(W_n) = 7n^2 - 9n.$

(2) $S_2(W_n) = 6n^3 - 15n^2 + 9n.$

(3) $PS(W_n) = \dfrac{n}{2n-3} + \dfrac{n}{\sqrt{n(2n-3)}}.$

(4) $RPS(W_n) = n(2n-3) + n\sqrt{n(2n-3)}.$

(5) $S_2^a(W_n) = (2n-3)^{2a} n + (2n^2 - 3n)^a n.$

(6) $F_1S(W_n) = 13n^3 - 36n^2 + 27n.$

(7) $SGO_2(W_n) = n(2n-3)(11n^2 - 27n + 18)$.

(8) $SDS(W_n) = \dfrac{9n^2 - 18n + 9}{2n-3}$.

Theorem 2. The first status Gourava index of a wheel graph W_n is

$$SGO_1(W_n) = 6n^3 - 8n^2.$$

Proof: By definition, we have

$$SGO_1(W_n) = \sum_{uv \in E(W_n)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]$$

Thus by using Table 1, we deduce

$$SGO_1(W_n) = n[(2n-3) + (2n-3) + (2n-3)(2n-3)] + n[n + (2n-3) + n(2n-3)] = 6n^3 - 8n^2.$$

5. Results for friendship graphs

A friendship graph F_n, $n \geq 2$, is a graph that can be constructed by joining n copies of C_3 with a common vertex. A graph F_4 is presented in Figure 2.

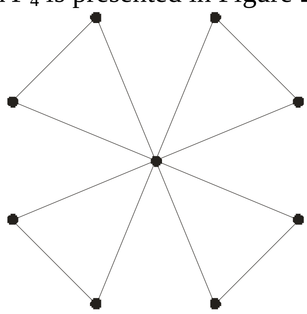

Figure 2: Friendship graph F_4

Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. By calculation, we obtain that there are two types of edges as follows:

$E_1 = \{uv \in E(F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}$, $|E_1| = n$.

$E_2 = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}$, $|E_2| = 2n$.

Therefore, in F_n, there are two types of status edges as given in Table 2.

$\sigma(u), \sigma(v) \setminus uv \in E(F_n)$	$(4n-2, 4n-2)$	$(2n, 4n-2)$
Number of edges	n	$2n$

Table 2: Status edge partition of F_n

Theorem 3. The (a, b)-status index of a friendship graph F_n is given by

$$S_{a,b}(F_n) = n\left[2(4n-2)^{a+b}\right] + 2n\left[(2n)^a(4n-2)^b + (2n)^b(4n-2)^a\right]. \quad \text{(ii)}$$

Proof: From definition and by using Table 2, we obtain

$$S_{a,b}(F_n) = \sum_{uv \in E(F_n)} \left[\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a \right]$$

$$= n\left[(4n-2)^a (4n-2)^b + (4n-2)^b (4n-2)^a\right] + 2n\left[(2n)^a (4n-2)^b + (2n)^b (4n-2)^a\right]$$

$$= n\left[2(4n-2)^{a+b}\right] + 2n\left[(2n)^a (4n-2)^b + (2n)^b (4n-2)^a\right].$$

Corollary 3.1. From observations and by using equation (ii), we derive the following results.

(1) $S_1(F_n) = 20n^2 - 8n.$

(2) $S_2(F_n) = 32n^3 - 24n^2 + 4n.$

(3) $PS(F_n) = \dfrac{n}{4n-2} + \dfrac{n}{\sqrt{n(2n-1)}}.$

(4) $RPS(F_n) = n(4n-2) + 4n\sqrt{n(2n-1)}.$

(5) $S_2^a(F_n) = (4n-2)^{2a}\, n + \left(8n^2 - 4n\right)^a\, 2n.$

(6) $F_1S(F_n) = 72n^3 - 64n^2 + 16n.$

(7) $SGO_2(F_n) = 2n(4n-2)\left(12n^2 - 2\right).$

(8) $SDS(F_n) = \dfrac{14n^2 - 10n + 2}{2n-1}.$

Theorem 4. The first status Gourava index of a friendship graph F_n is

$$SGO_1(F_n) = 32n^3 - 4n^2 - 4n.$$

Proof: By definition, we have

$$SGO_1(G) = \sum_{uv \in E(G)} \left[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)\right]$$

Thus by using Table 2, we obtain

$$SGO_1(F_n) = \left[(4n-2) + (4n-2) + (4n-2)(4n-2)\right] n$$
$$+ \left[2n + (4n-2) + 2n(4n-2)\right] 2n$$
$$= 32n^3 - 4n^2 - 4n.$$

4. Conclusion

In this study, the (*a*, *b*)-status index and some other status indices for particular values of *a* and *b* for wheel graphs and friendship graphs are computed.

Acknowledgement. The author is thankful to the referee for useful suggestions.

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