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# Computation of Square Status and Inverse Sum Status Indices of Certain Graphs

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***Abstract.*** In this paper, we introduce the minus status index, square status index, inverse sum status index of a graph, Also we propose the minus status polynomial, square status polynomial of a graph. We determine exact formulas for wheel and friendship graphs.

***Keywords:*** minus status index, square status index, inverse sum status index, status polynomial, graph.

***AMS Mathematics Subject Classification (2010):*** 05*C*05, 05*C*07, 05*C*35

## 1. Introduction

Let $G$ be a simple, finite connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex $v$, denoted by $d_G(v)$, is the number of vertices adjacent to $v$ in a graph $G$. The distance $d(u, v)$ between any two vertices $u$ and $v$ in a graph $G$ is the length of shortest path containing $u$ and $v$. The status $\sigma(u)$ of a vertex $u$ in $G$ is the sum of distances of all other vertices from $u$ in a graph $G$. For undefined term and notation, we refer the book [1].

A graph index is numerical parameter mathematical derived from graph structure. In Chemical Graph Theory, graph indices [2] have found some applications in chemical documentation, isomer, discrimination, QSAR/QSPR research [3, 4].

The first and second status connectivity indices of a graph were introduced in [5] and they are defined as

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)], \qquad S_2(G) = \sum_{uv \in E(G)} \sigma(u)\sigma(v).$$

Recently, several status indices of a graph such as first and second hyper status [6], $F$-status index [7], $ABC$, augmented status indices [8], multiplicative first and second status indices [9], status Gourava indices [12], harmonic status index [13], geometric-arithmetic status index [14] were introduced and studied.

We introduce the minus status index, square status index of a graph as follows:

The minus status index of a graph $G$ is defined as

$$MS(G) = \sum_{uv \in E(G)} |\sigma(u) - \sigma(v)|.$$

The square status index of a graph $G$ is defined as

$$SS(G)=\sum_{uv\in E(G)}[\sigma(u)-\sigma(v)]^2.$$

We also propose the minus status polynomial and square status polynomial of a graph and they are defined as

$$MS(G,x)=\sum_{uv\in E(G)}x^{|\sigma(u)-\sigma(v)|}.$$

$$SS(G,x)=\sum_{uv\in E(G)}x^{[\sigma(u)-\sigma(v)]^2}.$$

The inverse sum indeg index of a graph $G$ was defined in [15] as

$$ISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)}.$$

Motivated by the definition of inverse sum indeg index, we introduce the inverse sum status index of a graph as follows:

$$ISS(G)=\sum_{uv\in E(G)}\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)}.$$

Recently, some new graph indices were studied in [16, 17, 18].

In this paper, we compute the minus status index, square status index and their polynomials of wheel graphs and friendship graphs. Also we determine the inverse sum status index of wheel and friendship graphs.

## 2. Results for wheel graphs

A wheel $W_n$ is the join of $C_n$ and $K_1$. Clearly $W_n$ has $n$+1 vertices and 2$n$ edges. A wheel graph $W_n$ is shown in Figure 1.

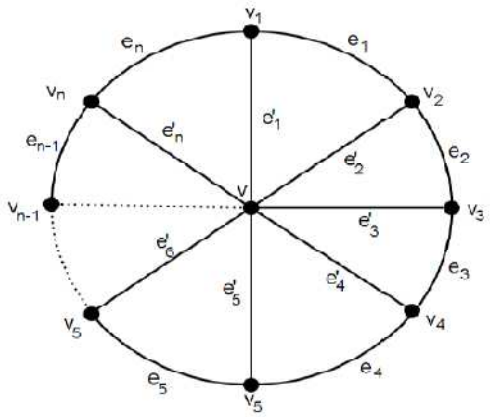


**Figure 1:** Wheel graph $W_n$

Let $G = W_n$. In $G$, there are two types of edges as follows:

$E_1=\{uv\in E(G) \mid d_G(u)= d_G(v)=3\}$, $|E_1| = n$.

$E_2=\{uv\in E(G) \mid d_G(u)=3, d_G(v)=2n\}$, $|E_2| = n$.

Hence there are two types of status edges as given in Table 1.

| $\sigma(u), \sigma(v)\backslash uv \in E(G)$ | $(4n-3, 4n-3)$ | $(n, 2n-3)$ |
|---|---|---|
| Number of edges | $n$ | $n$ |

**Table 1:** Status edge partition of $W_n$

**Theorem 1.** The minus status index of a wheel graph $W_n$ is

$$MS(W_n) = n(n-3).$$

**Proof:** From definition and by using Table 1, we derive

$$MS(W_n) = \sum_{uv \in E(G)} |\sigma(u) - \sigma(v)| = |(2n-3)-(2n-3)|n - |n-(2n-3)|n$$
$$= n(n-3).$$

**Theorem 2.** The square status index of a wheel graph $W_n$ is

$$SS(W_n) = n(n-3)^2.$$

**Proof:** From definition and by using Table 1, we derive

$$SS(W_n) = \sum_{uv \in E(G)} [\sigma(u) - \sigma(v)]^2$$
$$= [(2n-3)-(2n-3)]^2 n + [n-(2n-3)]^2 n$$
$$= n(n-3)^2.$$

**Theorem 3.** The minus status polynomial of a wheel graph $W_n$ is

$$MS(W_n, x) = nx^0 + nx^{n-3}.$$

**Proof:** From definition and by using Table 1, we have

$$MS(W_n, x) = \sum_{uv \in E(W_n)} x^{|\sigma(u)-\sigma(v)|}$$
$$= nx^{|(2n-3)-(2n-3)|} + nx^{|n-(2n-3)|}$$
$$= nx^0 + nx^{n-3}.$$

**Theorem 4.** The square status polynomial of a wheel graph $W_n$ is

$$SS(W_n, x) = nx^0 + nx^{(n-3)^2}.$$

**Proof:** By using definition and Table 1, we obtain

$$SS(W_n, x) = \sum_{uv \in E(W_n)} x^{[\sigma(u)-\sigma(v)]^2}$$
$$= nx^{[(2n-3)-(2n-3)]^2} + nx^{[n-(2n-3)]^2}$$
$$= nx^0 + nx^{(n-3)^2}.$$

**Theorem 5.** The inverse sum status index of a wheel graph $W_n$ is

$$ISS(W_n) = \frac{n(2n-3)(5n-3)}{6(n-1)}.$$

**Proof:** From definition and by using Table 1, we deduce

$$ISS(W_n) = \sum_{uv \in E(W_n)} \frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)}$$

$$= \left[\frac{n(2n-3)(2n-3)}{2n-3+2n-3}\right] n + \left[\frac{n(2n-3)}{n+2n-3}\right] n$$

$$= \frac{n(2n-3)(5n-3)}{6(n-1)}.$$

### 3. Results for friendship graphs

A friendship graph $F_n$ is the graph obtained by taking $n \geq 2$ copies of $C_3$ with a vertex in common. A graph $F_4$ is presented in Figure 2.

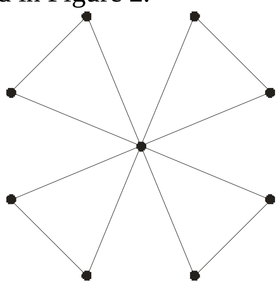

**Figure 2:** Friendship graph $F_4$

Let $G = F_n$. A graph $F_n$ has $2n+1$ vertices and $3n$ edges. In $G$, there are two types of edges as follows:

$E_1=\{uv \in E(G) \mid d_G(u)= d_G(v)=2\}$, $\quad |E_1| = n$.

$E_2=\{uv \in E(G) \mid d_G(u)=2n, d_G(v)=2\}$, $\quad |E_2| = 2n$.

Thus there are two types of status edges as given in Table 2.

| $\sigma(u), \sigma(v) \backslash uv \in E(G)$ | $(4n-2, 4n-2)$ | $(4n-2, 2n)$ |
|---|---|---|
| Number of edges | $n$ | $2n$ |

**Table 2:** Status edge partition of $F_n$

**Theorem 6.** The minus status index of a friendship graph $F_n$ is

$$MS(F_n) = 4n(n-1).$$

**Proof:** From definition and by using Table 2, we obtain

$$MS(F_n) = \sum_{uv \in E(F_n)} |\sigma(u) - \sigma(v)|$$

$$= |(4n-2)-(4n-2)|n + |(4n-2)-2n|2n$$

$$= 4n(n-1)$$

**Theorem** 7. The square status index of a friendship graph $F_n$ is

$$SS(F_n) = 8n(n-1)^2.$$

**Proof:** From definition and by using Table 2, we deduce

$$SS(F_n) = \sum_{uv \in E(F_n)} [\sigma(u) - \sigma(v)]^2$$
$$= [(4n-2)-(4n-2)]^2 n - [4n-2-2n]^2 2n$$
$$= 8n(n-1)^2.$$

**Theorem 8.** The minus status polynomial of $F_n$ is

$$MS(F_n, x) = nx^0 + 2nx^{2(n-1)}.$$

**Proof:** By using definition and Table 2, we have

$$MS(F_n, x) = \sum_{uv \in E(F_n)} x^{|\sigma(u)-\sigma(v)|}$$
$$= nx^{|(4n-2)-(4n-2)|} + 2nx^{|4n-2-2n|}$$
$$= nx^0 + 2nx^{2(n-1)}.$$

**Theorem 9.** The square status polynomial of $F_n$ is

$$SS(F_n, x) = nx^0 + 2nx^{4(n-1)^2}.$$

**Proof:** From definition and by using Table 2, we derive

$$SS(F_n, x) = \sum_{uv \in E(G)} x^{[\sigma(u)-\sigma(v)]^2}$$
$$= nx^{[(4n-2)-(4n-2)]^2} + 2nx^{[4n-2-2n]^2}$$
$$= nx^0 + 2nx^{4(n-1)^2}.$$

**Theorem** 10. The inverse sum status index of $F_n$ is

$$ISS(F_n) = \frac{n(2n-1)(7n-1)}{3n-1}.$$

**Proof:** By using definition and Table 2, we obtain

$$ISS(F_n) = \sum_{uv \in E(F_n)} \frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)}$$
$$= \left[\frac{(4n-2)(4n-2)}{4n-2+4n-2}\right] n + \left[\frac{(4n-2)2n}{4n-2+2n}\right] 2n$$
$$= \frac{n(2n-1)(7n-1)}{3n-1}.$$

## 4. Conclusion

In this study, the minus status index, square status index and inverse sum status index for wheel and friendship graphs are computed. Also minus status polynomial and square status polynomial for wheel and friendship graphs are determined.

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