Annals of Pure and Applied Mathematics
Vol. 22, No. 2, 2020, 143-150
ISSN: 2279-087X (P), 2279-0888(Online)
Published on 12 December 2020
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v22n2a10803

Annals of Pure and Applied Mathematics

On Semi Prime n-Ideals in Nearlattices

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Received 2 November 2020; accepted 11 December 2020

Abstract. For a central element n of a nearlattice S, we have discussed n-distribitive nearlattices and included several properties of semi prime n-ideals in nearlattices. In this paper, we have given a characterization of minimal prime n-ideals containing $\{a\}^{\perp^n K}$ for all $a \in S$. Finally, we have included a prime Separation Theorem with the help of annihilator n-ideal.

Keywords: Central element, Semi prime n-ideal, Minimal prime n-ideal, annihilator n-ideal, prime n-ideal, n-distributive nearlattice.

AMS Mathematics Subject Classification (2010): 16G30

1. Introduction

In generalizing the notion of pseudo complemented lattice, Varlet [11] introduced the notion of 0-distributive lattices. Then [7] have given several characterizations of these lattices. Also [9] have studied them in meet semi lattices. A lattice L with 0 is called 0-distributive if for all $a, b, c \in L$, $a \wedge b = 0 = a \wedge c$ imply $a \wedge (b \vee c) = 0$. Of course, every distributive lattice with 0 is 0-distributive. Rav [10] has given the concept of semi prime ideals in lattices by generalizing the notion of 0-distributive lattices. For a neutral element $n \in L$, Ali et.al.[5] and [6] have introduced the concept of n-distributive lattices and given the notion of semi prime n-ideals in lattices. In this paper, we generalize the concept of 0-distributive lattice and n-distributive lattice and give the notion of n-distributive nearlattice where n is a central element of this nearlattice. Here we give several characterizations of semi prime n-ideals of nearlattices.

A nearlattice S is a meet semilattice with the property that, any two elements possessing a common upper bound, have a supremum. Nearlattice S is distributive if for all $x, y, z \in S$, $x \wedge (y \vee z) = (x \wedge y) \vee (x \vee z)$ provided $y \vee z$ exists. For detailed literature on nearlattices, we refer the reader to consult [2,3,4] and [8]. An element n of a nearlattice S is called medial if $m(x, n, y) = (x \wedge y) \vee (x \wedge n) \vee (y \wedge n)$ exists in S for all $x, y \in S$. A nearlattice S is called a medial nearlattice if $m(x, y, z)$ exists for all $x, y, z \in S$.

An element s of a nearlattice S is called standard if for all $t, x, y \in S$,
$t \wedge [(x \wedge y) \vee (x \wedge s)] = (t \wedge x \wedge y) \vee (t \wedge x \wedge s)$. The element s is called neutral if
(i) s is standard and
(ii) for all $x, y, z \in S$, $s \wedge [(x \wedge y) \vee (x \wedge z)] = (s \wedge x \wedge y) \vee (s \wedge x \wedge z)$.

In a distributive nearlattice, every element is neutral and hence standard. An element n

in a nearlattice S is called sesquimedial if for all $x, y, z \in S$,

$$([(x \wedge n) \vee (y \wedge n)] \wedge [(y \wedge n) \vee (z \wedge n)]) \vee (x \wedge y) \vee (y \wedge z) \text{ exists in } S.$$

An element n of a nearlattice S is called a upper element if $x \vee n$ exists for all $x \in S$. Every upper element is of course a sesquimedial element. An element n is called a central element of S if it is neutral, upper and complemented in each interval containing it.

Let S be a nearlattice and $n \in S$. Any convex subnearlattice of S containing n is called an n-ideal of S. For two n-ideals I and J of a nearlattice S, [4] has given a description of $I \vee J$ while the set theoretic intersection is the infimum. Hence, the set of all n-ideals of a nearlattice S is a lattice which is denoted by $I_n(S)$. $\{n\}$ and S are the smallest and largest elements of $I_n(S)$.

An n-ideal generated by a finite number of elements $a_1, a_2, \cdots, a_m$ is called a finitely generated n-ideal and it is denoted by $< a_1, a_2, \cdots, a_m >_n$. The set of all finitely generated n-ideals is denoted by $F_n(S)$. Clearly, $< a_1, a_2, \cdots, a_m >_n = < a_1 >_n \vee < a_2 >_n \vee \cdots \vee < a_m >_n$. An n-ideal generated by a single element a is called a principal n-ideal denoted by $< a >_n$. The set of principal n-ideals is denoted by $P_n(S)$.

Let S be a nearlattice and $n \in S$. For any $a \in S$,

$$< a >_n = \{y \in S : a \wedge n \le y = (y \wedge a) \vee (y \wedge n)\}$$

$$= \{y \in S:\ y = (y \wedge a) \vee (y \wedge n) \vee (a \wedge n)\}$$ whenever n is standard element in S.

If n is an upper element in a nearlattice S, then $< a >_n = [a \wedge n, a \vee n]$.

We know that when n is standard and medial, the set of all principal n-ideals $P_n(S)$ is a meet semilattice and $< a >_n \cap < b >_n = < m(a, n, b) >_n$ for all $a, b \in S$. Also, when n is neutral and sesquimedial, then $P_n(S)$ is a nearlattice. By [4] if S is medial nearlattice and n is a neutral element of S, then $P_n(S)$ is also a medial nearlattice.

For a distributive nearlattice S with an upper element n, $P_n(S)$ is a distributive nearlattice with the smallest element $\{n\}$.

A proper convex subnearlattice M of a nearlattice S is called a maximal convex subnearlattice if for any convex subnearlattice Q with $Q \supseteq M$ implies either $Q = M$ or $Q = S$. A proper convex subnearlattice M of a medial nearlattice S is called a prime convex subnearlattice if for any $t \in M$, $m(a, t, b) \in M$ implies either $a \in M$ or $b \in M$. For a medial element n, an n-ideal P of a nearlattice S is a prime n-ideal if $P \neq S$ and $m(x, n, y) \in P$ $(x, y \in S)$ implies either $x \in P$ or $y \in P$. Equivalently, P is prime if and only if $< a >_n \cap < b >_n \subseteq P$ implies either

$$< a >_n \subseteq P \text{ or } < b >_n \subseteq P.$$

Let n be a central element of a nearlattice S. For $a \in S$, we define $\{a\}^{\perp_n} = \{x \in S : m(x, n, a) = n\}$, known as an n-annihilator of $\{a\}$. Also for $A \subseteq S$, we define $A^{\perp_n} = \{x \in S : m(x, n, a) = n \text{ for all } a \in A\}$. $A^{\perp_n}$ is always a convex subnearlattice containing n. If S is a distributive nearlattice, then it is easy to check $\{a\}^{\perp_n}$ and $A^{\perp_n}$ are n-ideals. Moreover, $A^{\perp_n} = \cap_{a \in A} \{\{a\}^{\perp_n}\}$. If A is an n-ideal, then $A^{\perp_n}$ is called an annihilator n-ideal which is obviously the pseudocomplement of A in $I_n(S)$. Therefore, for a distributive nearlattice S with central element n, $I_n(S)$ is pseudocomplemented.

A nearlattice S with central element n, is called an n-distributive nearlattice if for all $a, b, c \in S$, $< a >_n \cap < b >_n = \{n\}$ and $< a >_n \cap < c >_n = \{n\}$ imply

$$< a >_n \cap [< b >_n \vee < c >_n] = \{n\}.$$

Equivalently, S is called n-distributive nearlattice if $a \wedge b \leq n \leq a \vee b$ and $a \wedge c \leq n \leq a \vee c$ imply $a \wedge (b \vee c) \leq n \leq a \vee (b \wedge c)$. In a directed above meet semilattice S,an ideal J is called a semi prime ideal if for all $x, y, z \in S$, $x \wedge y \in J$ and $x \wedge z \in J$ imply $x \wedge d \in J$ for some $d \geq y, z$. Let n be a central element of a nearlattice S. An n-ideal K of S is called a semi prime n-ideal if for all $a, b, c \in S$, $<a>_n \cap <b>_n \subseteq K$ and $<a>_n \cap <c>_n \subseteq K$ imply $<a>_n \cap (<b>_n \vee <c>_n) \subseteq K$. In a distributive nearlattice every n-ideal is semi prime. Moreover, every prime n-ideal is semi prime. A prime n-ideal P of a nearlattice S is a minimal prime n-ideal if there exists no prime n-ideal Q such that $Q \neq P$ and $Q \subseteq P$.

2. Main results

To obtain the main results of this paper we need to prove the following lemmas.

Lemma 1. Let S be a nearlattice with a central element n and let I be an n-ideal of S. Every convex subnearlattice disjoint from an n-ideal I is contained in a maximal convex subnearlattice disjoint from I.

Proof: Let F be a convex subnearlattice in S disjoint from I. Let $\mathcal{F}$ be the set of all convex sub nearlattices containing F and disjoint from I. Then $\mathcal{F}$ is non-empty as $F \in \mathcal{F}$. Let C be a chain in $\mathcal{F}$ and $M = \cup (X | X \in C)$. Let $x, y \in M$. Then $x \in X$ and $y \in Y$ for some $X, Y \in C$. Since C is a chain, so either $X \subseteq Y$ or $Y \subseteq X$. Suppose $X \subseteq Y$, so $x, y \in Y$. Then $x \wedge y, x \vee y \in Y$ and so $x \wedge y, x \vee y \in M$. Thus M is a subnearlattice of a nearlattice containing F. Also it is convex as each $X \in C$ is convex. Moreove $F \subseteq M$. Hence M is a maximal element of C. Therefore, by Zorn's Lemma, $\mathcal{F}$ has a maximal element, say Q with $F \subseteq Q$.

Lemma 2. For a central element n of a nearlattice, every maximal convex subnearlattice disjoint from an n-ideal I is either a maximal ideal or a maximal filter

Proof: Let F be a maximal convex subnearlattice disjoint from an n-ideal I. Since $F = (F] \cap [F)$, so either $(F] \cap I = \phi$ or $[F) \cap I = \phi$. If not, let $x \in (F] \cap I$ and $y \in [F) \cap I$. Then $x \in I$ and $x \leq f_1$ for some $f_1 \in F$ and $y \in I$ and $y \geq f_2$ for some $f_2 \in F$. Now $f_2 \leq x \vee f_2 \leq f_1 \vee f_2$ implies by convexity that $x \vee f_2 \in F$ Also $x \leq x \vee f_2 \leq x \vee y$ implies by convexity that $x \vee f_2 \in I$. It follows that $x \vee f_2 \in F \cap I$, which is a contradiction. Thus either $(F] \cap I = \phi$ or $[F) \cap I = \phi$. Since F is maximal so $F = (F]$ or $F = [F)$. That is, F must be either a maximal ideal or a maximal filter.

Lemma 3. Let S be a nearlattice with a central element n and let I be an n-ideal of S. A convex subnearlattice M disjoint from I is a maximal convex subnearlattice disjoint from I if and only if for all $a \notin M$, there exists $b \in M$ such that $m(a, n, b) \in I$.

Proof: Suppose M is a maximal convex subnearlattice and disjoint from I. Also let $a \notin M$. Suppose for all $b \in M$, $m(a, n, b) \notin I$. Set $M_1 = \{y \in S: y \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq y \vee n; b \in M\}$. Obviously, M_1 is a convex subnearlattice as n is central. Also $M_1 \cap I = \phi$. If not, let $x \in M_1 \cap I$. Then $x \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq x \vee n$ for some $b \in M$ and $x \in I$. Thus $x \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee (a \wedge n) \vee (b \wedge n) \leq (a \wedge b) \vee n \leq x \vee n$ implies $m(a, n, b) \in I$ which gives a contradiction to the assumption. For $b \in M$, $b \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq b \vee n$ implies $b \in M_1$ and so $M \subseteq M_1$. Also, $a \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq a \vee n$ implies $a \in M_1$ but $a \notin M$ so $M \subseteq M_1$.

Therefore, we have a contradiction to the maximality of M and so there exists some $b \in M$ such that $m(a,n,b) \in I$.

Conversely, if M is not maximal and disjoint from I then by Lemma 1, M properly contained in a maximal convex subnearlattice N and N disjoint with I. Then for any element $a \in N - M$ there exists an element $b \in M$ such that $m(a,n,b) \in I$. Now $a, b \in N$ implies $a \wedge b, a \vee b \in N$. Thus by Lemma 2, N is either an ideal or a filter. Hence $(a \wedge b) \vee n \in N$ or $(a \vee b) \wedge n \in N$ but not both. For otherwise, $n \in N$ would give a contradiction to $I \cap N = \phi$. Now any of the above causes will imply $m(a,n,b) \in N$ and so $m(a,n,b) \in I \cap N$ which is again a contradiction. Hence M must be a maximal convex subnearlattice disjoint from I.

Theorem 4. For a central element n of a nearlattice S, K is a semi prime n-ideal of S if and only if (K] is a semi prime ideal and $[K)$ is a semi prime filter.

Proof: Let $x \vee y \in [K)$ and $x \vee z \in [K)$. Then $x \vee y \geq k_1$ and $x \vee z \geq k_2$ for some $k_1, k_2 \in K$. Thus $k_1 \wedge n \leq (x \vee y) \wedge n \leq n$ implies $(x \vee y) \wedge n \in K$ by convexity. So $m(x,n,y \wedge n) = (x \vee y) \wedge n \in K$ implies $< x >_n \cap < y \wedge n >_n \subseteq K$ Similarly, $< x >_n \cap < z \wedge n >_n \subseteq K$. Since K is semi prime, so $< x >_n \cap (< y \wedge n >_n \vee < z \wedge n >_n) = [x \wedge n, x \vee n] \cap [y \wedge z \wedge n, n] = [\big(x \vee (y \wedge z)\big) \wedge n, n] \subseteq K$ implies $\big(x \vee (y \wedge z)\big) \wedge n \in K$, and so $x \vee (y \wedge z) \in [K)$ Therefore $[K)$ is a semi prime filter. Similarly, we can prove that (K] is a semi prime ideal.

Conversely, let $< x >_n \cap < y >_n \subseteq K$ and $< x >_n \cap < z >_n \subseteq K$. That is $[(x \vee y) \wedge n, (x \wedge y) \vee n] \subseteq K$ and $[(x \vee z) \wedge n, (x \wedge z) \vee n] \subseteq K$. It follows that $(x \wedge y) \vee n \in K$ and $(x \wedge z) \vee n \in K$. Hence $(x \vee n) \wedge (y \vee n) \in K$ and $(x \vee n) \wedge (z \vee n) \in K$ as n is central. Then $x \wedge (y \vee n) \in (K]$ and $x \wedge (z \vee n) \in (K]$. So $x \wedge (y \vee z \vee n) \in (K]$ as $(K]$ is a semi prime ideal. This implies $(x \wedge (y \vee z)) \vee (x \wedge n) \in (K]$ and so $\big(x \wedge (y \vee z)\big) \vee (x \wedge n) \leq k_1$ for some $k_1 \in K$. Then $n \leq \big(x \wedge (y \vee z)\big) \vee n \leq k_1 \vee n$ implies $\big(x \wedge (y \vee z)\big) \vee n \in K$. Similarly, we can prove that $\big(x \vee (y \wedge z)\big) \wedge n \in K$ as $[K)$ is a semi prime filter. Therefore $< x >_n \cap (< y >_n \vee < z >_n) \subseteq K$ and so K is semi prime.

Theorem 5. For a medial element n, any prime ideal P containing n of a nearlattice S is a prime n-ideal.

Proof: Since every ideal P is a convex subnearlattice, so any ideal P containing n is an n-ideal. To show the primeness, let $m(a,n,b) \in P$. Then $a \wedge b \leq m(a,n,b)$ implies $a \wedge b \in P$. Since P is prime ideal so either $a \in P$ or $b \in P$. Hence P is a prime n-ideal.

Theorem 6. Let S be a nearlattice with a central element n . If the intersection of all prime (semi prime) n-ideals of S is equal to K, then K is a semi prime n-ideal.

Proof: Let $< a >_n \cap < b >_n \subseteq K$ and $< a >_n \cap < c >_n \subseteq K$. Let P be any prime n-ideal. If $a \in P$, then $< a >_n \subseteq P$ and so $< a >_n \cap [< b >_n \vee < c >_n] \subseteq P$. If $a \notin P$, then $< b >_n, < c >_n \subseteq P$ as P is prime n-ideal. Hence $< b >_n \vee < c >_n \subseteq P$. Therefore, $< a >_n \cap [< b >_n \vee < c >_n] \subseteq P$. That is, in either case, $< a >_n \cap [< b >_n \vee < c >_n] \subseteq P$ for all prime n-ideals P containing K. Therefore, $< a >_n \cap [< b >_n \vee < c >_n] \subseteq \cap P = K$. Thus K is semi prime.

Lemma 7. Let S be a nearlattice with a central element n. Then $p \in \{x\}^{\perp_n}$ if and only if $p \wedge x \leq n \leq p \vee x$.

Proof: $p \in \{x\}^{\perp_n}$ if and only if $m(p,n,x) = n$ if and only if $(p \wedge x) \vee (p \wedge n) \vee (x \wedge n) = (p \vee x) \wedge (p \vee n) \wedge (x \vee n) = n$, as n is central. This implies that $p \wedge x \leq n \leq p \vee x$.

Lemma 8. Let S be a nearlattice with a central element n. Then $p \in \{x\}^{\perp_n}$ if and only if $p \vee n \in \{x \vee n\}^{\perp_n}$ in $[n)$ and $p \wedge n \in \{x \wedge n\}^{\perp^d}$ in $(n]$.

Proof: Let $p \in \{x\}^{\perp_n}$. Then $p \wedge x \leq n \leq p \vee x$ and so $(p \vee n) \wedge (x \vee n) = (p \wedge x) \vee n = n$ and $(p \wedge n) \vee (x \wedge n) = (p \vee x) \wedge n = n$ as n is central element. Thus $p \vee n \in \{x \vee n\}^{\perp}$ in $[n)$ and $p \wedge n \in \{x \wedge n\}^{\perp^d}$ in $(n]$. Conversely, let $p \vee n \in \{x \vee n\}^{\perp}$ in $[n)$ and $p \wedge n \in \{x \wedge n\}^{\perp^d}$ in $(n]$. Then since n is central element, so $(p \vee n) \wedge (x \vee n) = n$ and $(p \wedge x) \vee n = n$. This implies $p \wedge x \leq n$. Also, $(p \wedge n) \vee (x \wedge n) = n$ implies $(p \vee x) \wedge n = n$ and so $n \leq p \vee x$. Hence $p \wedge x \leq n \leq p \vee x$. Therefore, by Lemma 7, $p \in \{x\}^{\perp_n}$.

Let S be a nearlattice with a central element n. For $A \subseteq S$, we define $A^{\perp_n} = \{x \in S: m(x,n,a) = n \text{ for all } a \in A\}$. $A^{\perp_n}$ is always a convex subnearlattice containing n.

Theorem 9. Let S be an n-distributive nearlattice. Then for $A \subseteq S$, $A^{\perp_n} = \{x \in S: m(x,n,a) = n \text{ for all } a \in A\}$. is a semi prime n-ideal.

Proof: By [1,Theorem 2.10] we already know that $A^{\perp_n}$ is an n-ideal. This is also equivalent to the condition $I_n(S)$ is pseudocomplemented. Let $< x >_n \cap < y >_n \subseteq A^{\perp_n}$ and $< x >_n \cap < z >_n \subseteq A^{\perp_n}$. As for any n-ideal $A \in I_n(S)$, $A^{\perp_n}$ is the pseudocomplement of A in $I_n(S)$. Then for all $a \in A$, this implies $< x >_n \cap < y >_n \cap < a >_n = \{n\} = < x >_n \cap < z >_n \cap < a >_n$ and $< y >_n \subseteq (< x >_n \cap < a >_n)^*$, $< z >_n \subseteq (< x >_n \cap < a >_n)^*$ and so $< y >_n \vee < z >_n \subseteq (< x >_n \cap < a >_n)^*$ and this implies $< x >_n \cap < a >_n \cap (< y >_n \vee < z >_n) = \{n\}$ for all $a \in S$. Hence $< x >_n \cap (< y >_n \vee < z >_n) \subseteq A^{\perp_n}$ and so $A^{\perp_n}$ is a semi prime n-ideal.

Let S be a nearlattice with a central element n. Let $A \subseteq S$ and K be an n-ideal of S. We define $A^{\perp^{n_K}} = \{x \in S: m(x,n,a) \in K \text{ for all } a \in A\}$. This is clearly a convex subset containing K. In presence of distributivity, this is an n-ideal. $A^{\perp^{n_K}}$ is called an n-annihilator of A relative to K. We denote $I_K(S)$, the set of all n-ideals containing K. Of course $I_K(S)$ is a bounded lattice with K and S as the smallest and the largest elements. If $A \in I_K(S)$, and $A^{\perp^{n_K}}$ is an n-ideal , then $A^{\perp^{n_K}}$ is called an annihilator n-ideal and it is the pseudocomplement of A in $I_K(S)$.

Theorem 10. Let S be a nearlattice with a central element n and K be an n-ideal of S. Then the following conditions are equivalent:

(i) K is semi prime

(ii) $\{a\}^{\perp^{n_K}} = \{x \in S: m(x,n,a) \in K\}$ is a semi prime n-ideal containing K.

(iii) $\{A\}^{\perp^{n_K}} = \{x \in S: m(x,n,a) \in K \; for \; all \; a \in A\}$ is a semi prime n-ideal containing K.

(iv) $I_K(S)$ is pseudocomplemented (v) $I_K(S)$ is 0 -distributive. (vi) Every maximal convex subnearlattice disjoint from K is prime.

Proof: (i)⇒(ii). $\{a\}^{\perp^n K}$ is clearly a convex subset containing K. Let $x, y \in \{a\}^{\perp^n K}$. Then $< x >_n \cap < a >_n \subseteq K$ and $< y >_n \cap < a >_n \subseteq K$. Since K is semi prime so $< a >_n \wedge (< x >_n \vee < y >_n) \in K$. Now $< x \wedge y >_n \cap < a >_n \subseteq K$ and $< x \wedge y >_n \subseteq < x >_n \vee < y >_n = [x \wedge y \wedge n, x \vee y \vee n]$. Also $< x \vee y >_n \subseteq < x >_n \vee < y >_n$. Thus $< x \wedge y >_n \cap < a >_n \subseteq K$ and $< x \vee y >_n \cap < a >_n \subseteq K$. Therefore $\wedge y, x \vee y \in \{a\}^{\perp^n K}$. This implies $\{a\}^{\perp^n K}$ is an n-ideal containing K. Again let $< x >_n \cap < y >_n \subseteq \{a\}^{\perp^n K}$ and $< x >_n \cap < z >_n \subseteq \{a\}^{\perp^n K}$. Then $< x >_n \cap < y >_n \cap < a >_n \subseteq K$ and $< x >_n \cap < z >_n \cap < a >_n \subseteq K$. Thus $(< x >_n \cap < a >_n) \cap < y >_n \subseteq K$ and $(< x >_n \cap < a >_n) \cap < z >_n \subseteq K$. Then $(< x >_n \cap < a >_n) \cap (< y >_n \vee < z >_n) \subseteq K$, as K is semi prime. This implies $< x >_n \cap (< y >_n \vee < z >_n) \subseteq \{a\}^{\perp^n K}$ and so $\{a\}^{\perp^n K}$ is semi prime.

(ii)⇒(iii).This is trivial by Theorem6, as $\{A\}^{\perp^n K} = \cap\, (\{a\}^{\perp^n K}; a \in A)$.

(iii)⇒(iv). Since for any $A \subseteq S$, $\{A\}^{\perp^n K}$ is an n-ideal, hence it is the pseudocomplement of A in $I_K(S)$ and so $I_K(S)$ is pseudocomplemented.

(iv)⇒(v). This is trivial as every pseudocomplemented nearlattice is 0-distributive.

(v)⇒(vi). Let $I_K(S)$ be 0-distributive. Suppose F is a maximal convex subnearlattice disjoint from K. Suppose $x, y \notin F$. Then by Lemma3, there exist $a \in F, b \in F$ such that $m(x,n,a) \in K$, $m(y,n,b) \in K$. Thus $< x >_n \cap < a >_n \subseteq K$, $< y >_n \cap < b >_n \subseteq K$ and so $< x >_n \cap < a >_n \cap < b >_n \subseteq K$, $< y >_n \cap < b >_n \cap < a >_n \subseteq K$. Hence $< x >_n \cap < m(a,n,b) >_n \subseteq K$ and $< y >_n \cap < m(a,n,b) >_n \subseteq K$. Since $I_K(S)$ is 0-distributive, so $< m(a,n,b) >_n \cap (< x >_n \vee < y >_n) \subseteq K$. By a routine calculation, $\left[\left(a \vee b \vee (x \wedge y)\right) \wedge n, \left(a \wedge b \wedge (x \vee y)\right) \vee n\right] \subseteq K$. This implies $\left(a \vee b \vee (x \wedge y)\right) \wedge n \in K$ and $\left(a \wedge b \wedge (x \vee y)\right) \vee n \in K$. Then by Lemma2, F is either an ideal or a filter. Suppose F is filter. If $x \vee y \in F$, then $\left(a \wedge b \wedge (x \vee y)\right) \vee n \subseteq F \cap K$ which is a contradiction. Hence $x \vee y \notin F$. Similarly by considering F as an ideal and if $x \wedge y \in F$, then $\left(a \vee b \vee (x \wedge y)\right) \wedge n \subseteq F \cap K$ which also gives a contradiction. Hence $x \wedge y \notin F$. When $x, y \notin F$ then $x \vee y \notin F$ and $x \wedge y \notin F$ so F must be prime.

(vi)⇒ (i). Let $a, b, c \in S$ with $< a >_n \cap < b >_n \subseteq K$ and $< a >_n \cap < c >_n \subseteq K$. Then $[(a \vee b) \wedge n, (a \wedge b) \vee n] \subseteq K$ and $[(a \vee c) \wedge n, (a \wedge c) \vee n] \subseteq K$. Hence $[(a \vee b) \wedge n, (a \wedge b) \vee n] \in K$ and $[(a \vee c) \wedge n, (a \wedge c) \vee n] \in K$. Now $< a >_n \cap (< b >_n \cap < c >_n) = [a \wedge n, a \vee n] \cap [b \wedge c \wedge n, b \vee c \vee n] = \left[\left(a \vee (b \wedge c)\right) \wedge n, \left(a \wedge (b \vee c)\right) \vee n\right]$. If $< a >_n \cap (< b >_n \cap < c >_n) \notin K$, then either $\left(a \vee (b \wedge c)\right) \wedge n \notin K$ or $\left(a \wedge (b \vee c)\right) \vee n \notin K$. Suppose $\left(a \wedge (b \vee c)\right) \vee n \notin K$. Let $F = [\left(a \wedge (b \vee c)\right) \vee n)$. Then $F \cap K = \phi$. If not, let $y \in F \cap K$, then $y \geq \left(a \wedge (b \vee c)\right) \vee n$ and so $y \in K$. Hence $n \leq \left(a \wedge (b \vee c)\right) \vee n \leq y$ this implies $\left(a \wedge (b \vee c)\right) \vee n \in K$ which is a contradiction. Then by Lemma1, there exists a maximal filter $M \supseteq [a \wedge (b \vee c))$ and disjoint from K. But a convex subnearlattice containing a filter is itself a filter. Thus by (vi), M is a prime filter and so $a \vee n \in M$, $b \vee c \vee n \in M$. Since M is a prime filter and $n \notin M$, so $a \in M$ and b or $c \in M$. Hence either $a \wedge b \in M$ or $a \wedge c \in M$. Thus $(a \wedge b) \vee n \in M \cap K$ or $(a \wedge c) \vee n \in M \cap K$, this is also a contradiction. Therefore $< a >_n \cap (< b >_n \cap < c >_n) \subseteq K$ and so K is a semi prime n-ideal.

Corollary 11. In a nearlattice S with a central element n, every convex subnearlattice disjoint to a semi prime n-ideal K is contained in a prime convex subnearlattice.
Proof: This immediately follows from Lemma 1 and Theorem 10.

Theorem 12. Let S be a nearlattice with a central element n. Let K be a semi prime n-ideal of S and $x \in S$. Then a prime ideal P containing $\{x\}^{\perp^n K}$ is a minimal prime n-ideal containing $\{x\}^{\perp^n K}$ if and only if for $p \in P$ there exists $q \in S - P$ such that $m(p,n,q) \in \{x\}^{\perp^n K}$.
Proof: Let P be a prime ideal containing $\{x\}^{\perp^n K}$ such that the given condition holds. Let J be a prime n-ideal containing $\{x\}^{\perp^n K}$ such that J $\subseteq P$ Let $p \in P$, then there is $q \in S - P$ such that $m(p,n,q) \in \{x\}^{\perp^n K}$. Thus $m(p,n,q) \in$ J. Since J is prime and $q \notin$ J so $p \in$ J. Hence $P \subseteq$ J and so J $= P$. Therefore P must be a minimal prime n-ideal containing $\{x\}^{\perp^n K}$.

Conversely, let P be a minimal prime n-ideal containing $\{x\}^{\perp^n K}$. Let $p \in P$. Suppose $m(p,n,q) \notin \{x\}^{\perp^n K}$ for all $q \in S - P$. Then $[(p \vee q) \wedge n, (p \wedge q) \vee n] \notin \{x\}^{\perp^n K}$.Thus $(p \vee q) \wedge n \notin \{x\}^{\perp^n K}$ or $(p \wedge q) \vee n \notin \{x\}^{\perp^n K}$ Suppose $(p \vee q) \wedge n \notin \{x\}^{\perp^n K}$. Let $D = (S - P) \vee [p)$. We claim that $\{x\}^{\perp^n K} \cap D = \phi$. If not, let $y \in \{x\}^{\perp^n K} \cap D$. Then $p \wedge q \leq y \in \{x\}^{\perp^n K}$ for some $q \in S - P$. Hence $n \leq (p \wedge q) \vee n \leq y \vee n$ implies $(p \wedge q) \vee n \in \{x\}^{\perp^n K}$,which is a contradiction. Then by Theorem [10], there exists a maximal convex subnearlattice $Q \supseteq D$ and disjoint to $\{x\}^{\perp^n K}$. Now we prove that $x \in Q$. If $x \notin Q$ then $\big(Q \vee [x)\big) \cap \{x\}^{\perp^n K} \neq \phi$. Suppose $t \in \big(Q \vee [x)\big) \cap \{x\}^{\perp^n K}$. This implies $t \geq q_1 \wedge x$ and $m(t,n,x) \in K$ for some $q_1 \in Q$. Hence $q_1 \wedge x \leq t \wedge x$ and $(x \wedge t) \vee n \in K$. This implies $(q_1 \wedge x) \vee n \in K$. Thus $q_1 \vee n \in Q$ as Q is a filter. Again $m(q_1 \vee n, n, x) = (q_1 \wedge x) \vee n \in K$ implies $q_1 \vee n \in \{x\}^{\perp^n K}$, which is again a contradiction. Thus $x \in Q$. Let $M = S - Q$. Then M is a prime ideal, infact M is a prime n-ideal. Since $x \in Q$, so $x \notin M$. Let $r \in \{x\}^{\perp^n K}$. Then $m(r, n\, x) \in K \subseteq M$. This implies $r \in M$ as M is prime. Hence $\{x\}^{\perp^n K} \subseteq M$ and so $M \cap D = \phi$. This implies $M \cap (S - P) = \phi$ and so $M \subseteq P$. Also $M \neq P$, because $p \in D$ implies $p \notin M$ but $p \in P$. Thus M is a prime n-ideal containing $\{x\}^{\perp^n K}$ which is properly contained in P. This gives a contradiction to the minimal property of P. Hence the given condition holds.

We conclude this paper with the following Prime Separation Theorem for semi prime n-ideals in nearlattices

Theorem 13. Let S be a nearlattice with a central element n and K be an n-ideal of S. Then the following conditions are equivalent: (i) K is semi prime. (ii) For any proper convex subnearlattice F disjoint to K there is a prime convex subnearlattice P containing F such that $P \cap K = \phi$.
Proof: (i)$\Rightarrow$(ii). Since $F \cap K = \phi$, so by Lemma 1, there exists a maximal convex subnearlattice $P \supseteq F$ such that $P \cap K = \phi$. Hence by Theorem10, P is prime.

(ii)$\Rightarrow$(i). Let F be a maximal convex subnearlattice disjoint to K. Then by (ii), there exists a prime convex subnearlattice $P \supseteq F$ such that $P \cap K = \phi$. Since F is maximal, so $P = F$. Thus F is prime and so by Theorem 10, K must be semi prime.

3. Conclusion

In this paper, we extend the concept of semi prime n-ideals in nearlattices and include several interesting results on semi prime n-ideals in nearlattices. We also give a nice characterization of minimal prime n-ideals containing $\{a\}^{\perp^{n}K}$ for all $a \in S$.

Acknowledgements. I am grateful and thankful to the editor and referee for their valuable comments and suggestions for the improvement of this paper.

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