Annals of Pure and Applied Mathematics
Vol. 22, No. 2, 2020, 125-135
ISSN: 2279-087X (P), 2279-0888(Online)
Published on 3 December 2020
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v22n2a08800

Annals of Pure and Applied Mathematics

Derivation a New Numerical Methods to Calculate the Double Integration with Continuous Calls

***Rana Hassan Hilal*[1], *Nada A. M. Al-Karamy*[2] and *Sundus Najha Jabair*[3]**

Department of Mathematics, Faculty of Education for Girls,
University of Kufa, Najaf, Iraq
[2]email: nadaa.kharmi@uokufa.edu.iq
[3]email: sundusn.alamiry@uokufa.edu.iq
Corresponding author[*]. [1]email: ranah.salman@uokufa.edu.iq

Received 1 November 2020; accepted 3 December 2020

Abstract. The main objective of this research is to introduce a new methods for calculating numerically the double integers with continuous integrands. The correction limits (error formulas) have been derived. To improve the results, we used Romberg's acceleration based on the correction terms we found, AA, Aβ, βA, double integrations with continuous function gave high accuracy in results with relatively few partial periods.

Keywords: Numerical integration, Regardson Accelerating, Taylor series, Newton - Cotes formulas

AMS Mathematics Subject Classification (2010): 65D30, 65B99, 30K05

1. Introduction

Numerical analysis is characterized by the creation of a variety methods for finding approximate solutions to certain mathematical issues in an effective manner. The efficiency of these methods depends on both the accuracy and the ease with which they can be implemented. Modern numerical analysis is the numerical interface of the broad field of applied analysis. Since the dual integrals are important in finding the surface area and finding the middle centers and the intrinsic limitations of the flat surfaces and finding the size under the surface of the double integration, for example the volume resulting from the rotation of the curved heart as well as its importance in finding a curved surface area such as finding the surface area $x^2+y^2+z^2=4$, Located directly above the curved heart $\rho=(1-\cos\theta)$, or calculate the area of the ball segment $x^2+y^2+z^2=36$ Located inside the cylinder Frank Ayers [7].

This led many researchers to work in the field of bilateral integrals and researchers who highlighted the calculation of integrations with continuing calls in the formula $f(x,y)=f_1(x)f_2(y)$ Hans Jarre and Jacobsen [3] in 1973. In 2012, the researcher made Al- Karmi [2] to derive Numerical methods derived from Newton Cotes formulas to calculate the double l integrals and also in 2015 provided Khudair [5],the proposed method for numerical calculation of double integrals (SuSu) and characterized roads and high accuracy. In 2018 Hilal [4] derives a numerical method for calculating one and two integrals when the dimensions are not equal.

In this research, we present three new derivative rules to calculate approximate values of double integrations with continuous integrands and their correction limits. These methods result from the application of the two rules A and β below, which gave good results and high accuracy and our code AA, Aβ, Ba:

$$A=\int_{t_0}^{t_n} W(t)dt=\frac{\lambda}{12}\left(5\big(W(t_0)+W(t_n)\big)+14\sum_{i=1,3,\cdots}^{n-1} W(t_i)+10\sum_{i=2,4,\cdots}^{n-2} W(t_i)\right)+$$

$$\sum_{i=1}^{\infty}\alpha_i\lambda^{2i}\left(W^{(2i-1)}(t_n)-W^{(2i-1)}(t_0)\right)$$

$$\beta=\int_{t_0}^{t_n} W(t)dt=\frac{\lambda}{2}\left(\big(W(t_0)+W(t_n)\big)/2+2W(t_n-\tfrac{\lambda}{2})2\sum_{i=1}^{n-1}\left(W(t_i)+W(t_i-\tfrac{\lambda}{2})\right)\right)+$$

$$\sum_{i=1}^{\infty}\sigma_i\lambda^{2i}\left(W^{(2i-1)}(t_n)-W^{(2i-1)}(t_0)\right)$$

where $\lambda=\dfrac{t_n-t_0}{n}$, $\quad t_i=t_0+i\lambda$

2.1. Derivation of rules for the calculation of Double integrals and associated error formulas using the two rules

A, β: We now review three numerical rules to calculate binary integrals and associated error patterns when the integral is continuous within the integration area. These composite methods result from the application of a base and a base and then apply the method of Romberg 's acceleration Sifi [7] to the values resulting from the application of the composite rules

Assume that integration is defined as follows: $I=\int_{h_0}^{h_n}\int_{t_0}^{t_n} W(t,h)dtdh$.

Continuous integration at each point of the integration area $[t_0,t_n]\times[h_0,h_n]$

1.1. A composite base (base A on both internal t and external dimensions h) when (n The number of partial periods to which the period is divided) is equal to m the number of partial periods to which the period is divided $[t_0,t_n]$) meaning that $\left(\overline{\lambda}=\dfrac{h_n-h_0}{n}\right)$

$\left(\lambda=\overline{\lambda}\right)$ we will mark the way with the symbol RAA whereas R refer to the method of Romberg 's acceleration either AA indicates the base of the vehicle base application A on all internal dimensions t and external h

In general it can be written integration of the following image:

$$I=\int_{h_0}^{h_n}\int_{t_0}^{t_n}W\left(t,h\right)dtdh=\mathrm{AA}\left(\lambda\right)+E\left(\lambda\right) \qquad ...(1)$$

The value AA of the integration is numerically represented by using the rule A on both dimensions ,and it is $E\left(\lambda\right)$ the series of correction limits that can be added to values $\mathrm{AA}\left(\lambda\right)$ Wan $\lambda=\frac{\left(t_n-t_0\right)}{n}=\frac{\left(h_n-h_0\right)}{n}$

for internal integration $\int_{t_0}^{t_n}W\left(t,h\right)dt$ it can be numerically calculated with a rule A On the dimension t and (dealing with h a fixed) using the mean value theorem in the calculus we get

$$\int_{h_0}^{h_n}\int_{t_0}^{t_n}W\left(t,h\right)dtdh=\int_{h_0}^{h_n}\mathrm{A}dh=\int_{h_0}^{h_n}\frac{\lambda}{12}\left[5\left(W(t_n,h)+W(t_0,h)\right)+14\sum_{i=1,3,...}^{n-1}W(t_i,h)+10\sum_{i=2,4,...}^{n-2}W(t_i,h)\right]dh$$
$$+\sum_{i=1}^{\infty}\alpha_i(t_n-t_0)\lambda^{2i}\frac{\partial^{2i}W(\mu_i,h)}{\partial t^{2i}} \qquad \cdots(2)$$

where $\mu_i\cdots\in(t_0,t_n)$ and $i=1,2,...,\ i=1,2,\cdots n-1 \quad t_i=a+i\lambda$ call formula (2) relative to h using the rule A and (dealing with t a constant) we get:

$$\mathrm{AA}=\int_{h_0}^{h_n}\int_{t_0}^{t_n}W\left(t,h\right)dtdh=\frac{\lambda^2}{144}\Bigg[25\left(W(t_n,h_n)+W(t_0,h_0)+W(t_n,h_0)+W(t_0,h_n)\right)$$
$$+70\sum_{i=1,3,...}^{n-1}\left(W(t_i,h_0)+W(t_0,h_i)+W(t_n,h_i)+W(t_i,h_n)\right)+$$
$$50\sum_{i=2,4,...}^{n-2}\left(W(t_i,h_0)+W(t_0,h_i)+W(t_n,h_i)+W(t_i,h_n)\right)$$
$$+196\sum_{i=1,3,...}^{n-1}\left(\sum_{j=1,3,...}^{n-1}W(t_i,h_j)\right)+100\sum_{i=2,4,...}^{n-2}\left(\sum_{j=2,4,...}^{n-2}W(t_i,h_j)\right)+$$
$$140\sum_{i=1,3,...}^{n-1}\left(\sum_{j=2,4,...}^{n-2}\left(W(t_i,h_j)+W(t_j,h_i)\right)\right)\Bigg]$$
$$+\int_{h_0}^{h_n}\left(\sum_{i=1}^{\infty}\alpha_i(t_n-t_0)\,\lambda^{2i}\frac{\partial^{2i}W(\mu_i,h)}{\partial t^{2i}}\right)dh+$$

$$\lambda^2\left[a_i(h_n-h_0)\frac{\partial^2 W(t_0,\theta_{11})}{\partial h^2}+a_i(h_n-h_0)\frac{\partial^2 W(t_n,\theta_{21})}{\partial h^2}+\sum_{i=1}^{n-1}a_i(h_n-h_0)\frac{\partial^2 W(t_i,\theta_{2+i1})}{\partial h^2}\right]+$$
$$\lambda^4\left[b_i(h_n-h_0)\frac{\partial^4 W(t_0,\theta_{12})}{\partial h^2}+b_i(h_n-h_0)\frac{\partial^4 W(t_n,\theta_{22})}{\partial h^4}+\sum_{i=1}^{n-1}b_i(h_n-h_0)\frac{\partial^4 W(t_i,\theta_{2+i2})}{\partial h^4}\right]+\cdots \qquad \cdots(3)$$

where $j=1,2,\cdots n-1 \quad h_j=h_0+j\lambda \ i=1,2,\cdots n-1 \quad t_i=t_0+i\lambda$

and $\theta_{kl}\in(h_0,h_n)$ $l=1,2,3,\cdots$ and k=1, 2, 3, …, n+1.

and using the mean value theorem for integration we get:

$$\text{AA}=\int_{h_0}^{h_n}\int_{t_0}^{t_n}W(t,h)dtdh=\frac{\lambda^2}{144}\Bigg[25\big(W(t_n,h_n)+W(t_0,h_0)+W(t_n,h_0)+W(t_0,h_n)\big)$$
$$+70\sum_{i=1,3,\ldots}^{n-1}\big(W(t_i,h_0)+W(t_0,h_i)+W(t_n,h_i)+W(t_i,h_n)\big)+$$
$$50\sum_{i=2,4,\ldots}^{n-2}\big(W(t_i,h_0)+W(t_0,h_i)+W(t_n,h_i)+W(t_i,h_n)\big)+196\sum_{i=1,3,\ldots}^{n-1}\left(\sum_{j=1,3,\ldots}^{n-1}W(t_i,h_j)\right)+$$
$$100\sum_{i=2,4,\ldots}^{n-2}\left(\sum_{j=2,4,\ldots}^{n-2}W(t_i,h_j)\right)+140\sum_{i=1,3,\ldots}^{n-1}\sum_{j=2,4,\ldots}^{n-2}\big(W(t_i,h_j)+W(t_j,h_i)\big)\Bigg]$$
$$+(t_n-t_0)(h_n-h_0)\left(\sum_{i=1}^{\infty}\alpha_i(t_n-t_0)\lambda^{2i}\frac{\partial^{2i}W(\mu_i,\delta_i)}{\partial t^{2i}}\right)+$$
$$\lambda^2\left[\begin{array}{l}a_i(h_n-h_0)\dfrac{\partial^2 W(t_0,\theta_{11})}{\partial h^2}+a_i(h_n-h_0)\dfrac{\partial^2 W(t_n,\theta_{21})}{\partial h^2}+\\ \sum_{i=1}^{n-1}a_i(h_n-h_0)\dfrac{\partial^2 W(t_i,\theta_{2+i1})}{\partial h^2}\end{array}\right]$$
$$+\lambda^4\left[b_i(h_n-h_0)\frac{\partial^4 W(t_0,\theta_{12})}{\partial h^2}+b_i(h_n-h_0)\frac{\partial^4 W(t_n,\theta_{22})}{\partial h^4}+\sum_{i=1}^{n-1}b_i(h_n-h_0)\frac{\partial^4 W(t_i,\theta_{2+i2})}{\partial h^4}\right]+\cdots \qquad \cdots(4)$$

where $\delta_i\in(h_0,h_n)$ $i=1,2,3,\cdots$ this means that the value AA becomes:

$$\mathrm{AA}=\int_{h_0}^{h_n}\int_{t_0}^{t_n}W(t,h)\,dtdh=\frac{\lambda^2}{144}\Bigg[25\big(W(t_n,h_n)+W(t_0,h_0)+W(t_n,h_0)+W(t_0,h_n)\big)$$
$$+70\sum_{i=1,3,\ldots}^{n-1}\big(W(t_i,h_0)+W(t_0,h_i)+W(t_n,h_i)+W(t_i,h_n)\big)+$$
$$50\sum_{i=2,4,\ldots}^{n-2}\big(W(t_i,h_0)+W(t_0,h_i)+W(t_n,h_i)+W(t_i,h_n)\big)+196\sum_{i=1,3,\ldots}^{n-1}\left(\sum_{j=1,3,\ldots}^{n-1}W(t_i,h_j)\right)+$$
$$100\sum_{i=2,4,\ldots}^{n-2}\left(\sum_{j=2,4,\ldots}^{n-2}W(t_i,h_j)\right)+140\sum_{i=1,3,\ldots}^{n-1}\sum_{j=2,4,\ldots}^{n-2}\big(W(t_i,h_j)+W(t_j,h_i)\big)\Bigg]+\sum_{i=1}^{\infty}\omega_{\mathrm{AA}i}\lambda^{2i}\qquad\cdots(5)$$

where $\omega_{\mathrm{AA}i}$, $i=1,2,\ldots$ constants depend on the partial derivatives of the function W For both variables t,h do not depend on me λ .

2.1. A composite base (base A on the inner dimension t and base β on the external dimension h) when n (the number of partial periods to which the period is divided $[t_0,t_n]$) is equal to m (the number of partial periods divided by the period $[h_0,h_n]$) in the sense that $(\lambda=\overline{\lambda})$. Will Nrmzellatrivh $R\beta\mathrm{A}$ are indicated as R For the way to Romberg 's acceleration Sifi [7] , either $\beta\mathrm{A}$ Refer to the composite rule of the base application A On the inner dimension t Base β on the outer dimension h.

We adopt the same way of deriving the base $\beta\mathrm{A}$ First we calculate internal integration

$$\int_{t_0}^{t_n}W(t,h)\,dt$$

Numerically by base A on the dimension t and (dealing with h a constant)

So we apply the rule β (on the external dimension h) to every limit of the result we get:

$$\beta\mathrm{A}=\int_{h_0}^{h_n}\int_{t_0}^{t_n}W(t,h)\,dtdh=\frac{\lambda^2}{48}\Bigg[5\big(W(t_n,h_n)+W(t_0,h_0)+W(t_n,h_0)+W(t_0,h_n)\big)$$
$$+10\big(W(t_0,h_0+\tfrac{\lambda}{2})+W(t_n,h_0+\tfrac{\lambda}{2})\big)+10\sum_{i=1}^{n-1}\big(W(t_0,h_i)+W(t_0,h_i+\tfrac{\lambda}{2})+W(t_n,h_i)+W(t_n,h_i+\tfrac{\lambda}{2})\big)$$
$$+14\sum_{i=1,3,\dots}^{n-1}\left(W(t_i,h_0)+W(t_i,h_n)+2W(t_i,h_0+\tfrac{\lambda}{2})+2\sum_{j=1}^{n-1}\big(W(t_i,h_j+\tfrac{\lambda}{2})+W(t_i,h_j)\big)\right)$$
$$+10\sum_{i=2,4,\dots}^{n-2}\left(W(t_i,h_0)+W(t_i,h_n)+2W(t_i,h_0+\tfrac{\lambda}{2})+2\sum_{j=1}^{n-1}\big(W(t_i,h_j+\tfrac{\lambda}{2})+W(t_i,h_j)\big)\right)\Bigg]$$
$$+\sum_{i=1}^{\infty}\omega_{\beta\mathrm{A}i}\lambda^{2i} \qquad\cdots(6)$$

where $\omega_{\beta\mathrm{A}i}$, $i=1,2,\dots$ the constants depend on the partial derivatives of the function W for the variables t,h and does not depend on λ .

2.2. In the same way, the third rule, which relies on the use of the rule β on the internal dimension t and the base A on the external dimension h when n (the number of partial periods to which the period is divided $[t_0,t_n]$), can be calculated equal to m (the number of partial periods to which the period is divided $[h_0,h_n]$). We will send the way to the symbol $R\mathrm{A}\beta$ where R it refers to how to speed up Romberg $\mathrm{A}\,\beta$ it refers to the composite rule of the base application On the inner dimension t and base A on the outer dimension h from this we get:

$$\mathrm{A}\beta=\int_{h_0}^{h_n}\int_{t_0}^{t_n}W(t,h)\,dtdh=\frac{\lambda^2}{48}\Bigg[5\big(W(t_n,h_n)+W(t_0,h_0)+W(t_n,h_0)+W(t_0,h_n)\big)$$
$$+10\big(W(t_0+\tfrac{\lambda}{2},h_n)+W(t_0+\tfrac{\lambda}{2},h_0)\big)$$
$$+14\sum_{i=1,3,\dots}^{n-1}\left(W(t_0,h_i)+W(t_n,h_i)+2W(t_0+\tfrac{\lambda}{2},h_i)+2\sum_{j=1}^{n-1}\big(W(t_j+\tfrac{\lambda}{2},h_i)+W(t_j,h_i)\big)\right)$$
$$+10\sum_{i=2,4,\dots}^{n-2}\left(W(t_0,h_i)+W(t_n,h_i)+2W(t_0+\tfrac{\lambda}{2},h_i)+2\sum_{j=1}^{n-1}\big(W(t_j+\tfrac{\lambda}{2},h_i)+W(t_j,h_i)\big)\right)$$
$$+10\sum_{i=1}^{n-1}\big(W(t_i,h_0)+W(t_i+\tfrac{\lambda}{2},h_0)+W(t_i,h_n)+W(t_i+\tfrac{\lambda}{2},h_n)\big)\Bigg]+\sum_{i=1}^{\infty}\omega_{\mathrm{A}\beta i}\lambda^{2i} \quad\cdots(7)$$

where $\omega_{\beta\mathrm{A}i}$, $i=1,2,\dots$ the constants depend on the partial derivatives of the function W for the variables t,h and does not depend on λ .

Examples and results:

We have applied the three rules ($\mathrm{AA}, \mathrm{A}\beta, \beta\mathrm{A}$) many of the specific integrations and we used to Romberg 's acceleration $\left(J = \dfrac{2^{2i}\alpha\left(\dfrac{\lambda}{2}\right) - \alpha(\lambda)}{2^{2i}-1} \right)$ to improve the results obtained by making use of the accompanying correction rules limits(Where $\alpha\left(\dfrac{\lambda}{2}\right), \alpha(\lambda)$ two successive values of one of the three rules $\mathrm{AA}, \mathrm{A}\beta, \beta\mathrm{A}$ when the number of second partial periods twice the number of first partial periods), and we got good results in terms of accuracy and the number of partial periods used.

We will mention four integrations similar to the application of the three rules.

3. Example

$1-\int_1^2\int_1^2 \ln(t^2+h^2)dtdh$ and its analytical valu**e**1.5036650239227 rounded to fourteen decimal places.

$2-\int_2^3\int_2^3 \dfrac{\cos(t+h)}{ht}dtdh$ and its analytical valu**e** 0.032856375881337 rounded to fifteen decimal places.

$3-\int_3^4\int_2^3 \dfrac{(t+h^2)}{(t+h)^4}dtdh$ and its analytical valu**e** 0.011635190281416 rounded to fifteen decimal places.

$4-\int_1^2\int_1^2 (t^h+h^t)\, dtdh$ not known analytical value.

3. Results

1- Integration $\int_1^2\int_1^2 \ln(t^2+h^2)\, dtdh$ shown in figure representation of integration (1) continuous integration within the region $[1,2]\times[1,2]$ We write down its results in tables (1) Note them when using rules $\mathrm{AA}, \mathrm{A}\beta, \beta\mathrm{A}$ The results were correct for nine decimal places when the number of partial periods was $n=64$ either when using Romberg 's acceleration with the three rules $R\mathrm{AA}, R\mathrm{A}\beta, R\beta\mathrm{A}$ Results became valid for fourteen decimal

1- Integration $\int_1^2\int_1^2 \ln(t^2+h^2)dtdh$ shown in figure representation of integration (1) continuous integration within the region $[1,2]\times[1,2]$ We write down its results in tables (1) Note them when using rules $\mathrm{AA}, \mathrm{A}\beta, \beta\mathrm{A}$ The results were correct for nine decimal places when the number of partial periods was $n=64$ either when using Romberg 's acceleration with the three rules $R\mathrm{AA}, R\mathrm{A}\beta, R\beta\mathrm{A}$ Results became valid for fourteen decimal

2- Integration $\int_2^3\int_2^3 \frac{\cos(t+h)}{ht}dtdh$ where continuous integration within the integration area $[2,3]\times[2,3]$ as shown graphically in figure (2) and write down of the results are shown in table (2). It shows us that the correct number of decimal places obtained when using the rules $\mathrm{AA}, \mathrm{A}\beta, \beta\mathrm{A}$ and take advantage of the accompanying correction limits have become valid results for fifteen decimal with al-Qaeda $R\mathrm{AA}$ The results were correct for sixteen decimal places with the two bases $R\mathrm{A}\beta, R\beta\mathrm{A}$

3- Integration $\int_3^4\int_2^3 \frac{(t+h^2)}{(t+h)^4}dtdh$ as shown in Fig. 3 and the results of which are presented in table (3). The results obtained in applying the three rules $\mathrm{AA}, \mathrm{A}\beta, \beta\mathrm{A}$ when applying the rules $\mathrm{AA}, \beta\mathrm{A}$ we got six decimal places are correct when the number of partial periods $n=64$, while at the base application $\mathrm{A}\beta$ the mattresses were correct for seven decimal places when the number of partial periods $n=256$ after using the correction boundary and the Romberg rule, the integrator value is valid for fifteen decimal places with the base $R\mathrm{AA}$ We obtained sixteen correct decimal places when applying this rule $R\beta\mathrm{A}$ when the number of partial periods $n=64$ when applying the rule $R\mathrm{A}\beta$, the result was correct for 10 decimal places only when the number of partial periods $n=256$.

4- Integration $\int_1^2\int_1^2 (t^h+h^t)dtdh$ is not known analytical value note from the results recorded in its table (4) that the value of integration is approaching a certain amount when applying the three rules $\mathrm{AA}, \mathrm{A}\beta, \beta\mathrm{A}$. After using the Romberg acceleration Sifi [7] and the correction boundary series, we find that the results are proven at a certain amount of (3.76133619282343) and that this amount is the end of the correction boundary series (where it is a convergent cascade series). Thus we conclude that the value of integration is the end value, (the number of partial periods) $n=64$, the graphic representation of the complement is shown in figure (4).

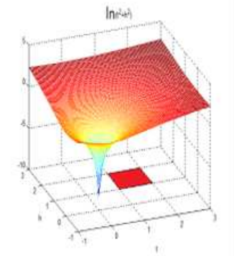


Figure (1) Graphic representation of the integral $\ln(t^2+h^2)$

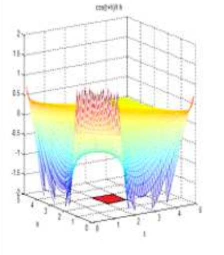
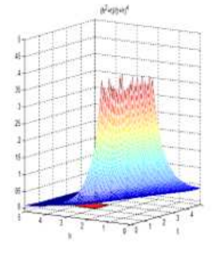


Figure (2) Graphic representation of the integral $\frac{\cos(t+h)}{ht}$



Figure(3) graphical representation of the integrator $\frac{(t+h^2)}{(t+h)^4}$

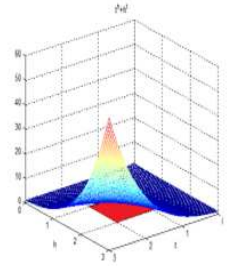


Figure (4) graphical representation of the integrator (t^h+h^t)

Table (1) to calculate the value of integration $\int_1^2\int_1^2 \ln(t^2+h^2)dtdh$

n	AA	Aβ	βA	$RA\beta$, RAA, $R\beta$A in λ^{10}
2	1.50374806919869	1.50370409469501	1.50370409469501	1.50366502392274
4	1.50367076863333	1.50366769364808	1.50366769364808	1.50366502392274
8	1.50366538570745	1.50366519190440	1.50366519190440	1.50366502392274
16	1.50366504654528	1.50366503442610	1.50366503442610	1.50366502392274
32	1.50366502533669	1.50366502457922	1.50366502457922	1.50366502392274
64	1.50366502401111	1.50366502396377	1.50366502396377	1.50366502392274

Table (2) to calculate the value of integration $\int_2^3\int_2^3 \frac{\cos(t+h)}{ht}dtdh$

n	AA	Aβ	βA	$RA\beta$, RAA, $R\beta$A in λ^{10}
2	0.029804834732241	0.0305635946206178	0.0305635946206178	0.032856375881337
4	0.032090133341743	0.0322814892054144	0.0322814892054144	0.032856375881337
8	0.032664652943843	0.0327125718921514	0.0327125718921514	0.032856375881337
16	0.032808435915057	0.0328204201867938	0.0328204201867938	0.032856375881337
32	0.032844390327941	0.0328473866715669	0.0328473866715669	0.032856375881337
64	0.032853379458114	0.0328541285611289	0.0328541285611289	0.032856375881337

n	AA	Aβ	βA	$RA\beta$, RAA RβA in λ^{10}	Table (2) to calculate the value of integration $\int_2^3\int_2^3 \frac{\cos(t+h)}{ht}dtdh$
2	0.029804834732241	0.0305635946206178	0.0305635946206178	0.032856375881337	
4	0.032090133341743	0.0322814892054144	0.0322814892054144	0.032856375881337	
8	0.032664652943843	0.0327125718921514	0.0327125718921514	0.032856375881337	
16	0.032808435915057	0.0328204201867938	0.0328204201867938	0.032856375881337	
32	0.032844390327941	0.0328473866715669	0.0328473866715669	0.032856375881337	
64	0.032853379458114	0.0328541285611289	0.0328541285611289	0.032856375881337	

n	AA	Aβ	βA	$RA\beta$, RAA , RβA in λ^{10}	Table (2) to calculate the value of integration $\int_2^3\int_2^3 \frac{\cos(t+h)}{ht}dtdh$
2	0.029804834732241	0.0305635946206178	0.0305635946206178	0.032856375881337	
4	0.032090133341743	0.0322814892054144	0.0322814892054144	0.032856375881337	
8	0.032664652943843	0.0327125718921514	0.0327125718921514	0.032856375881337	
16	0.032808435915057	0.0328204201867938	0.0328204201867938	0.032856375881337	
32	0.032844390327941	0.0328473866715669	0.0328473866715669	0.032856375881337	
64	0.032853379458114	0.0328541285611289	0.0328541285611289	0.032856375881337	

n	AA	Aβ	βA	$R\beta$A , $RA\beta$, RAA, in λ^{12}	Table (4) shows the value of Integration $\int_1^2\int_1^2 (t^h+h^t)dtdh$
2	3.78668364903250	3.78015876397483	3.78015876397483	3.76133619282343	
4	3.76753398350139	3.76597278526140	3.76597278526140	3.76133619282343	
8	3.76287691475192	3.76249099928381	3.76249099928381	3.76133619282343	
16	3.76172082700347	3.76162462250936	3.76162462250936	3.76133619282343	
32	3.76143231720724	3.76140828323926	3.76140828323926	3.76133619282343	
64	3.76136022178403	3.76135421436437	3.76135421436437	3.76133619282343	
128	3.76134219993012	3.76134069814223	3.76134069814223	3.76133619282343	
	Unknown analytical value				

4. Conclusion

We conclude from the results obtained in this research that the three rules of derivative AA, Aβ, βA. It produces good results in terms of accuracy, but using a relatively large number of partial periods. When making external adjustments using Rumbark's acceleration by taking advantage of the series of correction limits associated with each base derived, we obtained results closer to the analytical values of integrations and less than partial periods. The comparisons between the three rules show that: most of the rules gave good results in terms of accuracy and number of partial periods but the two bases RAA, RβA. The best of the rule Raβ is in some integrals as is evident from the third integration and is equal in the accuracy of the results in others.

Also, these three methods enable us to predict the values of integrations that can not be calculated analytically by proving the results at a certain amount is the value of integration no matter how much the correction limits used

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