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Edge Regularity on m-Bipolar Fuzzy Graph

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Abstract. In this article, a new idea of m-bipolar fuzzy graph (m-BPFG) is initiated. Further, degree of an edge and total degree of an edge are defined and also determined necessary and sufficient condition under which edge regular m-BPFG and totally edge regular m-BPFG are equivalent.

Keywords: m-BPFG, Edge degree, Total edge degree.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Fuzzy sets are introduced for the parameters to solve problems related to vague and uncertain in real life situations were given by Zadeh [12] in 1965. The limitations of traditional model were overcome by the introduction of bipolar fuzzy set concept in 1994 by Zhang [13]. This was further improved by Chen et al. [4] to m-polar fuzzy set theory.

Free body diagrams using set of nodes connected by lines representing pairs are good problem solving tools in non-deterministic real life situations. Thus, Rosenfeld [10] first initiated the fuzzy graphs by taking fuzzy relations on fuzzy sets in 1975. Akram [1] introduced the notion of bipolar fuzzy graphs and studied some isomorphic properties on it. Pal and Rashmanlou [7] studied irregular interval-valued fuzzy graphs and several of their classifications. Rashmanlou et al. [11] studied categorical properties of bipolar fuzzy graphs. Radha and Kumarvel [9] initiated the notion of edge regular bipolar fuzzy graphs. Ghorai and Pal [5, 6] introduced generalized m-polar fuzzy graphs and studied some isomorphic properties and density of an m-polar fuzzy graph. Banasode and Umathar [2] introduced minimum total edge dominating energy of a graph. Bera and pal [3] introduced the concept of m-polar interval-valued fuzzy graph and studied the algebraic properties like density, regularity and irregularity etc. on m-PIVFG. Pal et al. [8] studied intersection graphs and provides tools for applying fuzzy mathematics and graph theory to real world problems.

This paper attempts to develop theory to analyze parameters combining concepts from m-polar fuzzy graphs and bipolar fuzzy graphs as a unique effort. The resultant graph is turned to m-BPFG and studied properties on it.

2. Preliminaries

In this section, basic terminologies of bipolar fuzzy graph (BPFG) and m-polar fuzzy graph (m-PFG) are studied.

For a given set V, define an equivalence relation $\leftrightarrow$ on $V \times V - \{(k, k) : k \in V\}$ as follows: $(k_1, l_1) \leftrightarrow (k_2, l_2) \Leftrightarrow$ either $(k_1, l_1) = (k_2, l_2)$ or $k_1 = l_2, l_1 = k_2$. The quotient set got in this way is denoted by $\overleftrightarrow{V^2}$.

Definition 2.1. A bipolar fuzzy graph of a graph $G^* = (V, E)$ is a pair $G = (V, S, T)$ where $S = [\psi_S^p, \psi_S^n]$ is a bipolar fuzzy set in V and $T = [\psi_T^p, \psi_T^n]$ is a bipolar fuzzy relation on $\overleftrightarrow{V^2}$ such that $\psi_T^p(s, t) \le \min\{\psi_S^p(s), \psi_S^p(t)\}$, $\psi_T^n(s, t) \ge \max\{\psi_S^n(s), \psi_S^n(t)\}$ for all $(s, t) \in \overleftrightarrow{V^2}$ and $\psi_T^p(s, t) = \psi_T^n(s, t) = 0$ for all $(s, t) \in \overleftrightarrow{V^2} - E$.

Definition 2.2. An m-polar fuzzy graph of a graph $G^* = (V, E)$ is a pair $G = (V, S, T)$ where $S : V \to [0, 1]^m$ is an m-polar fuzzy set in V and $T : \overleftrightarrow{V^2} \to [0, 1]^m$ is an m-polar fuzzy set in $\overleftrightarrow{V^2}$ such that $p_j \circ T(s, t) \le \min\{p_j \circ S(s), p_j \circ S(t)\}$ for all $(s, t) \in \overleftrightarrow{V^2}$, $j = 1, 2, \cdots, m$ and $T(s, t) = \langle 0, 0, \cdots, 0 \rangle$ for all $(s, t) \in \left(\overleftrightarrow{V^2} - E\right)$. Here, $p_j \circ S(s)$ and $p_j \circ T(s, t)$ represents the j^{th} component of the degree of membership value of the vertex 's' and the edge '(s, t)'.

3. Regularity on m-bipolar fuzzy graphs

All the vertices and edges of an m-polar fuzzy graph have m components and those components are fixed. But these components may be bipolar. Using this idea, m-BPFG has been introduced. Before defining m-bipolar fuzzy graph, we assume the following:

Definition 3.1. An m-bipolar fuzzy set (m-BPFS) S on V is defined by

$$S(s) = \left\{\left\langle \left[p_1 \circ \psi_S^p(s),\, p_1 \circ \psi_S^n(s)\right], \left[p_2 \circ \psi_S^p(s),\, p_2 \circ \psi_S^n(s)\right], \cdots, \left[p_m \circ \psi_S^p(s),\, p_m \circ \psi_S^n(s)\right]\right\rangle\right\}$$

for all $s \in V$ or shortly $S(s)=\left\{\left\langle\left[p_j \circ \psi_S^p(s),\ p_j \circ \psi_S^n(s)\right]_{j=1}^{m}\right\rangle \middle| s \in V\right\}$

where the functions $p_j \circ \psi_S^p : V \to [0, 1]$ and $p_j \circ \psi_S^n : V \to [-1, 0]$ denote the positive memberships and negative memberships of the element respectively.

Definition 3.2. Let S be an m-BPFS on a set V. An m-bipolar fuzzy relation on a set S is an m-BPFS T of $V \times V$,

$$T(s,t)=\left\{\left\langle\left[p_1 \circ \psi_T^p(s,t),\ p_1 \circ \psi_T^n(s,t)\right],\ \left[p_2 \circ \psi_T^p(s,t),\ p_2 \circ \psi_T^n(s,t)\right], \cdots,\right.\right.$$
$$\left.\left.\left[p_m \circ \psi_T^p(s,t),\ p_m \circ \psi_T^n(s,t)\right]\right\rangle\right\}$$

for all $s, t \in V$ or shortly $T(s,t)=\left\{\left\langle\left[p_j \circ \psi_T^p(s,t),\ p_j \circ \psi_T^p(s,t)\right]_{j=1}^{m}\right\rangle \middle| s, t \in V\right\}$ such that

$p_j \circ \psi_T^p(s, t) \le \min\left\{p_j \circ \psi_S^p(s), p_j \circ \psi_S^p(t)\right\}$, $\quad p_j \circ \psi_T^n(s, t) \ge \max\left\{p_j \circ \psi_S^n(s), p_j \circ \psi_S^n(t)\right\}$, for every $j = 1, 2, \cdots, m$ and $s, t \in V$.

Definition 3.3. An m-bipolar fuzzy graph (m-BPFG) of a graph $G^* = (V, E)$ is a pair $G=(V, S, T)$ where $S=\left\langle\left[p_j \circ \psi_S^p,\ p_j \circ \psi_S^n\right]_{j=1}^{m}\right\rangle$, $p_j \circ \psi_S^p : V \to [0, 1]$ and $p_j \circ \psi_S^n : V \to [-1, 0]$ is an m-BPFS on V; and $T=\left\langle\left[p_j \circ \psi_T^p,\ p_j \circ \psi_T^n\right]_{j=1}^{m}\right\rangle$, $p_j \circ \psi_T^p : \overleftrightarrow{V^2} \to [0, 1]$ and $p_j \circ \psi_T^n : \overleftrightarrow{V^2} \to [-1, 0]$ is an m-BPFS in $\overleftrightarrow{V^2}$ such that

$p_j \circ \psi_T^p(k, l) \le \min\left\{p_j \circ \psi_S^p(k),\ p_j \circ \psi_S^p(l)\right\}$, $p_j \circ \psi_T^n(k, l) \ge \max\left\{p_j \circ \psi_S^n(k),\ p_j \circ \psi_S^n(l)\right\}$

for all $(k, l) \in \overleftrightarrow{V^2}$, $j = 1, 2, \cdots, m$ and $p_j \circ \psi_T^p(k, l) = p_j \circ \psi_T^n(k, l) = 0$ for all $(k, l) \in \overleftrightarrow{V^2} - E$.

Example 3.1. An example of a 3-BPFG is as shown in Figure 1.

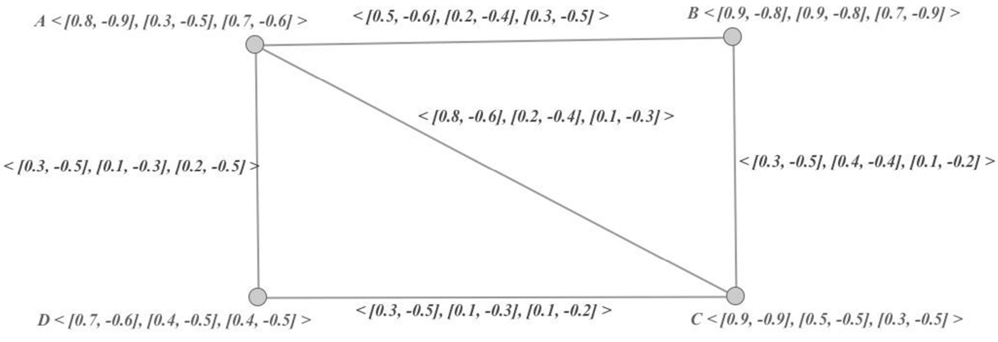

Figure 1: 3-Bipolar fuzzy graph G

Definition 3.4. The degree of a vertex r in an m-BPFG $G=(V,\ S,\ T)$ of $G^*=(V,\ E)$ is $d_G(r)=\left\langle \left[p_j\circ d_G^p(r),\ p_j\circ d_G^n(r)\right]_{j=1}^{m}\right\rangle$ where $p_j\circ d_G^p(r)=\sum_{\substack{r\neq s\\(r,s)\in E}} p_j\circ\psi_T^p(r,s)$ and $p_j\circ d_G^n(r)=\sum_{\substack{r\neq s\\(r,s)\in E}} p_j\circ\psi_T^n(r,\ s)$.

Definition 3.5. The degree of an edge $(r,\ s)\in E$ in an m-BPFG $G=(V,\ S,\ T)$ of $G^*=(V,\ E)$ is $d_G(r,\ s)=\left\langle \left[p_j\circ d_G^p(r,\ s),\ p_j\circ d_G^n(r,\ s)\right]_{j=1}^{m}\right\rangle$ where

$$p_j\circ d_G^p(r,\ s)=p_j\circ d_G^p(r)+p_j\circ d_G^p(s)-2p_j\circ\psi_T^p(r,\ s),$$
$$p_j\circ d_G^n(r,\ s)=p_j\circ d_G^n(r)+p_j\circ d_G^n(s)-2p_j\circ\psi_T^n(r,\ s).$$

Definition 3.6. The total degree of an edge $(r,\ s)\in E$ in an m-BPFG $G=(V,\ S,\ T)$ of $G^*=(V,\ E)$ is $td_G(r,\ s)=\left\langle \left[p_j\circ td_G^p(r,\ s),\ p_j\circ td_G^n(r,\ s)\right]_{j=1}^{m}\right\rangle$ where

$$p_j\circ td_G^p(r,\ s)=p_j\circ d_G^p(r)+p_j\circ d_G^p(s)-p_j\circ\psi_T^p(r,\ s),$$
$$p_j\circ td_G^n(r,\ s)=p_j\circ d_G^n(r)+p_j\circ d_G^n(s)-p_j\circ\psi_T^n(r,\ s).$$

Definition 3.7. If each vertex of an m-BPFG $G=(V,\ S,\ T)$ of $G^*=(V,\ E)$ is having the same degree $\left\langle \left[\delta_j^p,\delta_j^n\right]_{j=1}^{m}\right\rangle$, then G is called regular m-BPFG.

Definition 3.8. If each edge of an m-BPFG $G=(V,\ S,\ T)$ of $G^*=(V,\ E)$ is having the same degree $\left\langle \left[\delta_j^p,\delta_j^n\right]_{j=1}^{m}\right\rangle$, then G is called an edge regular m-BPFG.

Definition 3.9. If each edge of an m-BPFG $G=(V,\ S,\ T)$ of $G^*=(V,\ E)$ is having the same total degree $\left\langle \left[\delta_j^p,\delta_j^n\right]_{j=1}^{m}\right\rangle$, then G is called totally edge regular m-BPFG.

Theorem 3.1. Let $G=(V,\ S,\ T)$ be an m-BPFG on a cycle $G^*=(V,\ E)$. Then $\sum_{v_l\in V} d_G(v_l)=\sum_{(v_l,v_k)\in E,\ l\neq k} d_G(v_l,v_k)$.

Proof: Suppose that $G=(V,\ S,\ T)$ is an m-BPFG and G^* is a cycle $v_1v_2v_3\cdots v_tv_1$.

Then $\sum_{l=1}^{t} d_G(v_l,v_{l+1})=\left\langle\left[\sum_{l=1}^{t} p_j\circ d_G^p(v_l,v_{l+1}),\ \sum_{l=1}^{t} p_j\circ d_G^n(v_l,v_{l+1})\right]_{j=1}^{m}\right\rangle$

Now for $j=1,\ 2,\ \cdots,\ m$

$$\sum_{l=1}^{t} p_j\circ d_G^p(v_l,v_{l+1})=p_j\circ d_G^p(v_1,v_2)+p_j\circ d_G^p(v_2,v_3)+\cdots+p_j\circ d_G^p(v_t,v_1)$$

where $v_{t+1}=v_1$

$$=p_j\circ d_G^p(v_1)+p_j\circ d_G^p(v_2)-2p_j\circ\psi_T^p(v_1,v_2)+p_j\circ d_G^p(v_2)+p_j\circ d_G^p(v_3)$$
$$-2p_j\circ\psi_T^p(v_2,v_3)+\cdots+p_j\circ d_G^p(v_t)+p_j\circ d_G^p(v_1)-2p_j\circ\psi_T^p(v_t,v_1)$$

$$=2\sum_{v_l\in V} p_j\circ d_G^p(v_l)-2\sum_{l=1}^{t} p_j\circ\psi_T^p(v_l,v_{l+1})$$

$$=\sum_{v_l\in V} p_j\circ d_G^p(v_l)+\sum_{v_l\in V} p_j\circ d_G^p(v_l)-2\sum_{l=1}^{t} p_j\circ\psi_T^p(v_l,v_{l+1})$$

$$=\sum_{v_l\in V} p_j\circ d_G^p(v_l)+2\sum_{l=1}^{t} p_j\circ\psi_T^p(v_l,v_{l+1})-2\sum_{l=1}^{t} p_j\circ\psi_T^p(v_l,v_{l+1})=\sum_{v_l\in V} p_j\circ d_G^p(v_l).$$

Similarly, $\sum_{l=1}^{t} p_j\circ d_G^n(v_l,v_{l+1})=\sum_{v_l\in V} p_j\circ d_G^n(v_l)$. Hence, $\sum_{v_l\in V} d_G(v_l)=\sum_{(v_l,v_k)\in E,l\neq k} d_G(v_l,v_k)$.

Remark 3.1. Let $G=(V,\ S,\ T)$ be an m-BPFG on $G^*=(V,\ E)$. Then

$$\sum_{(v_l,v_k)\in E} d_G(v_l,v_k)=\left\langle\left[\sum_{(v_l,v_k)\in E} d_{G^*}(v_l,v_k)\,p_j\circ\psi_T^p(v_l,v_k),\ \sum_{(v_l,v_k)\in E} d_{G^*}(v_l,v_k)\,p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^{m}\right\rangle$$

where $d_{G^*}(v_l,v_k)=d_{G^*}(v_l)+d_{G^*}(v_k)-2$ for all $(v_l,\ v_k)\in E$.

Theorem 3.2. Let $G=(V,\ S,\ T)$ be an m-BPFG on a c-regular graph $G^*=(V,\ E)$. Then $\sum_{(v_l,v_k)\in E} d_G(v_l,v_k)=(c-1)\sum_{v_l\in V} d_G(v_l)$.

Proof: From Remark 3.1., we have

$$\sum_{(v_l,v_k)\in E} d_G(v_l,v_k)=\left\langle\left[\sum_{(v_l,v_k)\in E} d_{G^*}(v_l,v_k)\,p_j\circ\psi_T^p(v_l,v_k),\ \sum_{(v_l,v_k)\in E} d_{G^*}(v_l,v_k)\,p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^{m}\right\rangle$$

$$=\left\langle\left[\sum_{(v_l,v_k)\in E}(d_{G^*}(v_l)+d_{G^*}(v_k)-2)p_j\circ\psi_T^p(v_l,v_k),\ \sum_{(v_l,v_k)\in E}(d_{G^*}(v_l)+d_{G^*}(v_k)-2)p_j\circ\psi_T^p(v_l,v_k)\right]_{j=1}^{m}\right\rangle.$$

Since G^* is a regular graph, we have the degree of every vertex in G^* is c. So

$$\sum_{(v_l,v_k)\in E} d_G(v_l,v_k)=(c+c-2)\left\langle\left[\sum_{(v_l,v_k)\in E} p_j\circ\psi_T^p(v_l,v_k),\ \sum_{(v_l,v_k)\in E} p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^m\right\rangle$$

$$\sum_{(v_l,v_k)\in E} d_G(v_l,v_k)=2(c-1)\left\langle\left[\sum_{(v_l,v_k)\in E} p_j\circ\psi_T^p(v_l,v_k),\ \sum_{(v_l,v_k)\in E} p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^m\right\rangle$$

$$\sum_{(v_l,v_k)\in E} d_G(v_l,v_k)=(c-1)\sum_{v_l\in V} d_G(v_l).$$

Theorem 3.3. Let $G=(V,\ S,\ T)$ be an m-BPFG on a crisp graph $G^*=(V,\ E)$. Then,

$$\sum_{(v_l,v_k)\in E} td_G(v_l,v_k)=\left\langle\left[\sum_{(v_l,v_k)\in E} d_{G^*}(v_l,v_k)\,p_j\circ\psi_T^p(v_l,v_k),\ \sum_{(v_l,v_k)\in E} d_{G^*}(v_l,v_k)\,p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^m\right\rangle+\sum_{(v_l,v_k)\in E}\left\langle\left[p_j\circ\psi_T^p(v_l,v_k),\ p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^m\right\rangle.$$

Proof: From the definition of total edge degree, we have

$$\sum_{(v_l,v_k)\in E} td_G(v_l,v_k)=\sum_{(v_l,v_k)\in E}\left(d_G(v_l,v_k)+\left\langle\left[p_j\circ\psi_T^p(v_l,v_k),\ p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^m\right\rangle\right)$$

$$=\sum_{(v_l,v_k)\in E} d_G(v_l,v_k)+\sum_{(v_l,v_k)\in E}\left\langle\left[p_j\circ\psi_T^p(v_l,v_k),\ p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^m\right\rangle$$

From Remark 3.1., we have

$$\sum_{(v_l,v_k)\in E} td_G(v_l,v_k)=\left\langle\left[\sum_{(v_l,v_k)\in E} d_{G^*}(v_l,v_k)\,p_j\circ\psi_T^p(v_l,v_k),\ \sum_{(v_l,v_k)\in E} d_{G^*}(v_l,v_k)\,p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^m\right\rangle+\sum_{(v_l,v_k)\in E}\left\langle\left[p_j\circ\psi_T^p(v_l,v_k),\ p_j\circ\psi_T^n(v_l,v_k)\right]_{j=1}^m\right\rangle.$$

Theorem 3.4. Let $G=(V,\ S,\ T)$ be an m-BPFG on a crisp graph $G^*=(V,\ E)$. Then $T=\left\langle\left[p_j\circ\psi_T^p,\ p_j\circ\psi_T^n\right]_{j=1}^m\right\rangle$ is a constant function if and only if the subsequent conditions are equivalent:

(i) G is an edge regular m-BPFG .

(ii) G is a totally edge regular m-BPFG .

Proof: Let us suppose that T be a constant function.

Then $\left\langle\left[p_j\circ\psi_T^p(\alpha,\beta),\ p_j\circ\psi_T^n(\alpha,\beta)\right]_{j=1}^m\right\rangle=\left\langle\left[\gamma_j^p,\gamma_j^n\right]_{j=1}^m\right\rangle$ for all $(\alpha,\beta)\in E,$ where

$\gamma_j^p \in [0, 1]$, $\gamma_j^n \in [-1, 0]$. Let G be an edge regular m-BPFG. Then for all $(\nu_l, \nu_\gamma) \in E$, $d_G(\nu_l, \nu_\gamma) = \left\langle \left[\delta_j^p, \delta_j^n \right]_{j=1}^m \right\rangle$. Now we prove that G is a totally edge regular m-BPFG. Now

$$td_G(\nu_l, \nu_\gamma) = d_G(\nu_l, \nu_\gamma) + \left\langle \left[p_j \circ \psi_T^p(\nu_l, \nu_\gamma),\ p_j \circ \psi_T^n(\nu_l, \nu_\gamma) \right]_{j=1}^m \right\rangle$$

$= \left\langle \left[\delta_j^p, \delta_j^n \right]_{j=1}^m \right\rangle + \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle = \left\langle \left[\delta_j^p + \gamma_j^p, \delta_j^n + \gamma_j^n \right]_{j=1}^m \right\rangle$ for all $(\nu_l, \nu_\gamma) \in E$. Then G is a totally edge regular m-BPFG.

Now, let G be a $\left\langle \left[h_j^p, h_j^n \right]_{j=1}^m \right\rangle$ totally edge regular m-BPFG.

Then $td_G(\nu_l, \nu_\gamma) = \left\langle \left[h_j^p, h_j^n \right]_{j=1}^m \right\rangle$ for all $(\nu_l, \nu_\gamma) \in E$. So, we have

$$td_G(\nu_l, \nu_\gamma) = d_G(\nu_l, \nu_\gamma) + \left\langle \left[p_j \circ \psi_T^p(\nu_l, \nu_\gamma),\ p_j \circ \psi_T^n(\nu_l, \nu_\gamma) \right]_{j=1}^m \right\rangle = \left\langle \left[h_j^p, h_j^n \right]_{j=1}^m \right\rangle.$$

Hence, $d_G(\nu_l, \nu_\gamma) = \left\langle \left[h_j^p, h_j^n \right]_{j=1}^m \right\rangle - \left\langle \left[p_j \circ \psi_T^p(\nu_l, \nu_\gamma),\ p_j \circ \psi_T^n(\nu_l, \nu_\gamma) \right]_{j=1}^m \right\rangle$

$= \left\langle \left[h_j^p - \gamma_j^p, h_j^n - \gamma_j^n \right]_{j=1}^m \right\rangle$ for all $(\nu_l, \nu_\gamma) \in E$. Then G is an $\left\langle \left[h_j^p - \gamma_j^p, h_j^n - \gamma_j^n \right]_{j=1}^m \right\rangle$ -edge regular m-BPFG.

Conversely, we assume that conditions (i) and (ii) are equivalent. Now we have to show that the function T is constant. In a contrary way suppose that, the function T is not constant. Then $\left\langle \left[p_j \circ \psi_T^p(\nu_l, \nu_\gamma), p_j \circ \psi_T^n(\nu_l, \nu_\gamma) \right]_{j=1}^m \right\rangle \neq \left\langle \left[p_j \circ \psi_T^p(\nu_\delta, \nu_s), p_j \circ \psi_T^n(\nu_\delta, \nu_s) \right]_{j=1}^m \right\rangle$ for at least one pair of edges $(\nu_l, \nu_\gamma), (\nu_\delta, \nu_s) \in E$. Let G be a $\left\langle \left[\delta_j^p, \delta_j^n \right]_{j=1}^m \right\rangle$ -edge regular m-BPFG. Then $d_G(\nu_l, \nu_\gamma) = d_G(\nu_\delta, \nu_s) = \left\langle \left[\delta_j^p, \delta_j^n \right]_{j=1}^m \right\rangle$. Then for (ν_l, ν_γ), $(\nu_\delta, \nu_s) \in E$, we have $td_G(\nu_l, \nu_\gamma) = d_G(\nu_l, \nu_\gamma) + \left\langle \left[p_j \circ \psi_T^p(\nu_l, \nu_\gamma),\ p_j \circ \psi_T^n(\nu_l, \nu_\gamma) \right]_{j=1}^m \right\rangle$

$= \left\langle \left[\delta_j^p, \delta_j^n \right]_{j=1}^m \right\rangle + \left\langle \left[p_j \circ \psi_T^p(\nu_l, \nu_\gamma),\ p_j \circ \psi_T^n(\nu_l, \nu_\gamma) \right]_{j=1}^m \right\rangle$ and

$$td_G(\nu_\delta, \nu_s) = d_G(\nu_\delta, \nu_s) + \left\langle \left[p_j \circ \psi_T^p(\nu_\delta, \nu_s),\ p_j \circ \psi_T^n(\nu_\delta, \nu_s) \right]_{j=1}^m \right\rangle$$

$$= \left\langle \left[\delta_j^p, \delta_j^n \right]_{j=1}^m \right\rangle + \left\langle \left[p_j \circ \psi_T^p(\nu_\delta, \nu_s),\ p_j \circ \psi_T^n(\nu_\delta, \nu_s) \right]_{j=1}^m \right\rangle.$$

Since

$$\left\langle \left[p_j \circ \psi_T^p(\nu_l, \nu_\gamma),\ p_j \circ \psi_T^n(\nu_l, \nu_\gamma) \right]_{j=1}^m \right\rangle \neq \left\langle \left[p_j \circ \psi_T^p(\nu_\delta, \nu_s),\ p_j \circ \psi_T^n(\nu_\delta, \nu_s) \right]_{j=1}^m \right\rangle,$$

we have $td_G(v_l, v_\gamma) \neq td_G(v_\delta, v_s)$. Hence G is not a totally edge regular m-BPFG. This is a contradiction to our assumption and so that the function T is constant.

Similarly, we can show that the function T is constant, when G is a totally edge regular m-BPFG.

Theorem 3.5. Let $G^* = (V,\ E)$ be a h-regular crisp graph and $G = (V,\ S,\ T)$ be an m-BPFG on G^*. Then, the function $T = \left\langle \left[p_j \circ \psi_T^p,\ p_j \circ \psi_T^n \right]_{j=1}^m \right\rangle$ is constant if and only if G is both regular m-BPFG and totally edge regular m-BPFG .

Proof: Let T be a constant function. Then $\left\langle \left[p_j \circ \psi_T^p(\alpha,\beta),\ p_j \circ \psi_T^n(\alpha,\beta) \right]_{j=1}^m \right\rangle = \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle$ for all $(\alpha, \beta) \in E$ where γ_j^p and γ_j^n are constants. From the definition of degree of a vertex, we get

$$d_G(v_l) = \left\langle \left[\sum_{\substack{v_l \neq v_\gamma \\ (v_l, v_\gamma) \in E}} p_j \circ \psi_T^p(v_l, v_\gamma),\ \sum_{\substack{v_l \neq v_\gamma \\ (v_l, v_\gamma) \in E}} p_j \circ \psi_T^n(v_l, v_\gamma) \right]_{j=1}^m \right\rangle = \left\langle \left[\sum_{\substack{v_l \neq v_\gamma \\ (v_l, v_\gamma) \in E}} \gamma_j^p,\ \sum_{\substack{v_l \neq v_\gamma \\ (v_l, v_\gamma) \in E}} \gamma_j^n \right]_{j=1}^m \right\rangle$$

$= \left\langle \left[h\gamma_j^p, h\gamma_j^n \right]_{j=1}^m \right\rangle$ for all $v_l \in V$. So $d_G(v_l) = \left\langle \left[h\gamma_j^p, h\gamma_j^n \right]_{j=1}^m \right\rangle$ for all $v_l \in V$.

Therefore, G is a regular m-BPFG.

Again,

$$td_G(v_l, v_\gamma) = \left\langle \left[\sum_{\substack{z \neq \gamma \\ (v_l, v_z) \in E}} p_j \circ \psi_T^p(v_l, v_z),\ \sum_{\substack{z \neq \gamma \\ (v_l, v_z) \in E}} p_j \circ \psi_T^n(v_l, v_z) \right]_{j=1}^m \right\rangle +$$

$$\left\langle \left[\sum_{\substack{z \neq l \\ (v_z, v_\gamma) \in E}} p_j \circ \psi_T^p(v_z, v_\gamma),\ \sum_{\substack{z \neq l \\ (v_z, v_\gamma) \in E}} p_j \circ \psi_T^n(v_z, v_\gamma) \right]_{j=1}^m \right\rangle + \left\langle \left[p_j \circ \psi_T^p(v_l, v_\gamma),\ p_j \circ \psi_T^n(v_l, v_\gamma) \right]_{j=1}^m \right\rangle$$

$$= \sum_{\substack{z \neq \gamma \\ (v_l, v_z) \in E}} \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle + \sum_{\substack{z \neq l \\ (v_z, v_\gamma) \in E}} \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle + \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle$$

$$= (h-1) \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle + (h-1) \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle + \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle$$

$= (2h-1) \left\langle \left[\gamma_j^p, \gamma_j^n \right]_{j=1}^m \right\rangle$ for all $(v_l, v_\gamma) \in E.$

Conversely, assume that G is both regular and totally edge regular m-BPFG. Now we have to prove that T is a constant function. Since G is regular, $d_G(\nu_l)=\left\langle\left[z_j^p,z_j^n\right]_{j=1}^m\right\rangle$ for all $\nu_l\in V$. Also G is totally edge regular. Hence, $td_G(\nu_l,\nu_\gamma)=\left\langle\left[h_j^p,h_j^n\right]_{j=1}^m\right\rangle$ for all $(\nu_l,\nu_\gamma)\in E$. From the definition of totally edge degree, we get $td_G(\nu_l,\nu_\gamma)=\left\langle\left[p_j\circ d_G^p(\nu_l),\, p_j\circ d_G^n(\nu_l)\right]_{j=1}^m\right\rangle+\left\langle\left[p_j\circ d_G^p(\nu_\gamma),\, p_j\circ d_G^n(\nu_\gamma)\right]_{j=1}^m\right\rangle$ $-\left\langle\left[p_j\circ\psi_T^p(\nu_l,\nu_\gamma),\, p_j\circ\psi_T^n(\nu_l,\nu_\gamma)\right]_{j=1}^m\right\rangle$ for all $(\nu_l,\nu_\gamma)\in E.$

$$\left\langle\left[h_j^p,h_j^n\right]_{j=1}^m\right\rangle=\left\langle\left[z_j^p,z_j^n\right]_{j=1}^m\right\rangle+\left\langle\left[z_j^p,z_j^n\right]_{j=1}^m\right\rangle-\left\langle\left[p_j\circ\psi_T^p(\nu_l,\nu_\gamma),\, p_j\circ\psi_T^n(\nu_l,\nu_\gamma)\right]_{j=1}^m\right\rangle,$$

$$\left\langle\left[p_j\circ\psi_T^p(\nu_l,\nu_\gamma),\, p_j\circ\psi_T^n(\nu_l,\nu_\gamma)\right]_{j=1}^m\right\rangle=2\left\langle\left[z_j^p,z_j^n\right]_{j=1}^m\right\rangle-\left\langle\left[h_j^p,h_j^n\right]_{j=1}^m\right\rangle=\left\langle\left[2z_j^p-h_j^p,2z_j^n-h_j^n\right]_{j=1}^m\right\rangle$$

for all $(\nu_l,\nu_\gamma)\in E.$ Hence T is a constant function.

4. Conclusions

In this article, edge degree and total edge degree of an m-BPFG are defined. Further, an equivalence condition for edge regular m-BPFG and totally edge regular m-BPFG is given. In future we intend to extend our work to density of m-BPFG and morphism between two m-BPFGs and study some of its properties.

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