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On Multiplicative Inverse of Nirmala Indices

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Abstract. Recently, a novel degree based topological index was introduced, so called the Nirmala index. In this study, we introduce the multiplicative Nirmala index, the multiplicative first and second inverse Nirmala indices of a molecular graph. Furthermore we compute these Nirmala indices for certain nanotubes.

Keywords: multiplicative Nirmala index, multiplicative first and second inverse Nirmala indices, nanotube.

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*09

1. Introduction

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry, which has an important effect on the development of the Chemical Sciences. A topological index is a numerical parameter mathematically derived from the graph structure. Several such topological indices have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR study, see [1, 2].

Let $G=(V(G), E(G))$ be a finite, simple, connected graph. Let $d_G(u)$ be the degree of a vertex u in G. We refer [3] for undefined notations and terminologies.

In [4], Kulli introduced the Nirmala index of a graph G and it is defined as

$$N(G)=\sum_{uv\in E(G)}\sqrt{d_G(u)+d_G(v)}.$$

We now propose the multiplicative Nirmala index of a graph and it is defined as

$$NII(G)=\prod_{uv\in E(G)}\left[d_G(u)+d_G(v)\right]^{\frac{1}{2}}.$$

Recently, some Nirmala indices were studied, for example, in [5].

Inspired by work on Nirmala indices, we put forward the multiplicative first and second inverse Nirmala indices of a graph and they are defined as

$$IN_1II(G)=\prod_{uv\in E(G)}\left[\frac{1}{d_G(u)}+\frac{1}{d_G(v)}\right]^{\frac{1}{2}},$$

$$IN_2II(G) = \prod_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right]^{-\frac{1}{2}}.$$

Recently, some topological indices were studied in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. In this paper, we compute the multiplicative Nirmala index, multiplicative first and second inverse Nirmala indices for two families of nanotubes.

2. Results for $HC_5C_7[p, q]$ nanotubes

In this section, we focus on the family of nanotubes, denoted by $HC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. Let G be the graph of a nanotube $HC_5C_7[p,q]$.

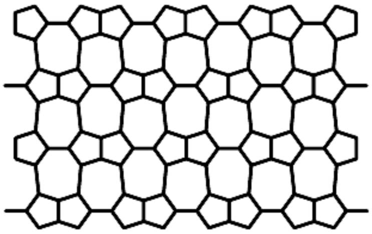

Figure 1: 2-D lattice of nanotube HC_5C_7 [8, 4]

The 2-D lattice of nanotube $HC_5C_7[p, q]$ is shown in Figure 1.By calculation, we obtain that G has $4pq$ vertices and $6pq – p$ edges. The graph G has two types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_1| = 4p$.

$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_2| = 6pq – 5p$.

Theorem 1. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$NII\left(HC_5C_7[p,q]\right) = 5^{2p} \times 6^{\frac{1}{2}(6pq-5p)}.$$

Proof: From definition and by cardinalities of the edge partition of $HC_5C_7[p, q]$, we deduce

$$NII\left(HC_5C_7[p,q]\right) = (2+3)^{\frac{1}{2}(4p)} \times (3+3)^{\frac{1}{2}(6pq-5p)}$$

$$= 5^{2p} \times 6^{\frac{1}{2}(6pq-5p)}.$$

Theorem 2. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

(i) $IN_1II(HC_5C_7[p,q]) = \left(\frac{5}{6}\right)^{2p} \times \left(\frac{2}{3}\right)^{\frac{1}{2}(6pq-5p)}$

(ii) $IN_2II(HC_5C_7[p,q]) = \left(\frac{5}{6}\right)^{2p} \times \left(\frac{3}{2}\right)^{\frac{1}{2}(6pq-5p)}$

Proof: From definitions and by cardinalities of the edge partition of $HC_5C_7[p, q]$, we deduce

(i) $IN_1(IIC_5C_7[p,q]) = \left(\frac{1}{2} + \frac{1}{3}\right)^{\frac{1}{2}4p} \times \left(\frac{1}{3} + \frac{1}{3}\right)^{\frac{1}{2}(6pq-5p)}$

$$= \left(\frac{5}{6}\right)^{2p} \times \left(\frac{2}{3}\right)^{\frac{1}{2}(6pq-5p)}$$

(ii) $IN_2II(HC_5C_7[p,q]) = \left(\frac{1}{2}+\frac{1}{3}\right)^{-\frac{1}{2}4p} \times \left(\frac{1}{3}+\frac{1}{3}\right)^{-\frac{1}{2}(6pq-5p)}$

$$= \left(\frac{5}{6}\right)^{2p} \times \left(\frac{3}{2}\right)^{\frac{1}{2}(6pq-5p)}$$

3. Results for $SC_5C_7[p,q]$ nanotubes

In this section, we focus on the family of nanotubes, denoted by $SC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternately. The 2-D lattice of nanotube $SC_5C_7[p,q]$ is presented in Figure 2.

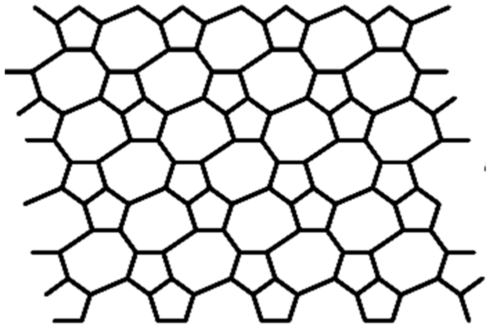

Figure 2: 2-*D* lattice of nanotube $SC_5C_7[p,q]$

Let G be the graph of $SC_5C_7[p,q]$. By calculation, we obtain that G has $4pq$ vertices and $6pq - p$ edges. Also by calculation, we get that G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = q$.

$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6q$.

$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 6pq - p - 7q$.

Theorem 3. Let G be the graph of a nanotube $SC_5C_7[p, q]$. Then

$$NII(SC_5C_7[p,q]) = 2^q \times 5^{3q} \times 6^{\frac{1}{2}(6pq-p-7q)}.$$

Proof: From definition and by cardinalities of the edge partition of $SC_5C_7[p, q]$, we deduce

$$NII(SC_5C_7[p,q]) = (2+2)^{\frac{1}{2}q} + (2+3)^{\frac{1}{2}6q} + (3+3)^{\frac{1}{2}(6pq-p-7q)}$$

$$= 2^q \times 5^{3q} \times 6^{\frac{1}{2}(6pq-p-7q)}.$$

Theorem 4. Let G be the graph of a nanotube $SC_5C_7[p, q]$. Then

(i) $IN_1II(SC_5C_7[p,q]) = \left(\frac{5}{6}\right)^{3p} \times \left(\frac{2}{3}\right)^{\frac{1}{2}(6pq-p-7q)}$

(ii) $IN_2II(SC_5C_7[p,q]) = \left(\frac{6}{5}\right)^{3p} \times \left(\frac{3}{2}\right)^{\frac{1}{2}(6pq-p-7q)}$

Proof: From definitions and by cardinalities of the edge partition of $SC_5C_7[p, q]$, we deduce

(i) $$IN_1II(SC_5C_7[p,q]) = \left(\frac{1}{2}+\frac{1}{2}\right)^{\frac{1}{2}q} \times \left(\frac{1}{2}+\frac{1}{3}\right)^{\frac{1}{2}6pq} \times \left(\frac{1}{3}+\frac{1}{3}\right)^{\frac{1}{2}(6pq-p-7q)}$$

$$= \left(\frac{5}{6}\right)^{3p} \times \left(\frac{2}{3}\right)^{\frac{1}{2}(6pq-p-7q)}$$

(ii) $$IN_2II(SC_5C_7[p,q]) = \left(\frac{1}{2}+\frac{1}{2}\right)^{-\frac{1}{2}q} \times \left(\frac{1}{2}+\frac{1}{3}\right)^{-\frac{1}{2}6q} \times \left(\frac{1}{3}+\frac{1}{3}\right)^{-\frac{1}{2}(6pq-p-7q)}$$

$$= \left(\frac{6}{5}\right)^{3p} \times \left(\frac{3}{2}\right)^{\frac{1}{2}(6pq-p-7q)}$$

4. Conclusion

In this study, we have introduced the multiplicative Nirmala index, multiplicative first and second inverse Nirmala indices of a molecular graph. Furthermore we have computed these multiplicative Nirmala indices for two families of nanotubes.

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