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A Note on *le*-Ternary Semigroups

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Abstract. Petraq Petro, Kostaq Hila, and Jani Dine given several properties in le- Γ -semigroups and especially in every Q-class satisfying the Green's condition. Rabah Kellil studied Green's Relations on Ternary Semigroups in 2013 and Parinyawat Choosuwa, Ronnason Chinram the notion of the quasi-ideals in ternary semigroups. In this paper we study that the characterizations of le–ternary semigroups and In particular we proved that the quasi-ideal element q of an le–ternary semigroup T has the intersection property if and only if $q = l(q) \wedge m(q) \wedge r(q)$.

Keywords: le-ternary semigroups, partially ordered (PO), ideal, quasi ideal

AMS Mathematics Subject Classification (2010): 18B40

1. Introduction

Likewise with plain semigroups, le-semigroups are being studied globally and locally. The first way aims to find out to what extent properties of certain subsets of an le-semigroup affect the structure of the le-semigroup as a whole, and the second way is somehow the converse of the first, that is, it aims to find out to what rate and under what conditions some properties of points (elements) are transmitted to subsets of the le-semigroup. In 1932, Lehmer introduced the concept of a ternary semigroup. He investigated certain ternary algebraic structures called triplexes. Santiago developed the theory of ternary semigroups and semiheaps. He studied regular and completely regular ternary semigroups. In [2] Dixit and Dewan studied the quani ideals and bi-ideals in ternary semigroups. Hila K.[4] introduced the notion of regular *le*-Γ-semigroup. In [6], D.H. Lehmer introduced the concept of a ternary semigroup. He investigated certain ternary algebraic structures called triplexes. In [15], Santiago developed the theory of ternary semigroups and semiheaps. He studied regular and completely regular ternary semigroups. Nagi Reddy,U. and Shobhalatha,G[7,8,] studied the Ideals in regular po Γ-ternary semigroups, they are also studied fuzzy weekly completely prime ideals, fuzzy Bi ideals in ternary semigroups in [9,10].

This paper contains some results on le – ternary semigroups. Many researchers conducted the researches on the generalizations of the notions of ideals in ternary semigroups with huge applications in computer, logics and many branches of pure and applied Mathematics.

2. Basic definitions and preliminaries

Definition 2.1. A non-empty set T is said to be ternary semigroup if there exists a ternary operation $\cdot : T\times T\times T\to T$ written as satisfies the following identity $ab(cde)=a(bcd)e=(abc)de$ for any $a,b,c,d,e\in T$.

Definition 2.2. A ternary semigroup T is said to be partially ordered (Po) ternary semigroup if there exists a partially ordered relation $\leq$ on T such that if for all $a,b\in T$, $a\leq b$ then $a\,cd\leq b\,cd$, $cad\leq cbd$ and $cda\leq cdb$, $c,d\in T$.

Definition 2.3. A poe ternary semigroup T is a po ternary semigroup with a greatest element "e". i.e., for all a$\in T$, $e\geq a$.

Definition 2.4. In a po ternary semigroup T, an element a is called a right(resp. lateral and left) ideal element if for all $b\in T, abc\leq a$,(resp. $bac\leq a$ and $bca\leq a$).

Definition 2.5. In a po ternary semigroup T, an element a is called a right(resp. lateral and left) ideal element if for all $aee\leq a$ (resp. $eae\leq a$ and $eae\leq a$) and *for* $A\subseteq T$,

We denote (A] $=\{t\in T/t\leq a \text{ for some } a\in A\}$.

Definition 2.6. An element a of a poe ternary semigroup T is called a quasi-ideal element, if $eea\wedge eae\wedge aee$ exists and $eea\wedge eae\wedge aee\leq a$

Definition 2.7. The zero of a poe ternary semigroup T is an element of T is denoted by 0 such that for every a, $b\in T$, e$\neq 0\leq a$ .and $0ab=a0b=ab0=0$.

Let T be a poe ternary semigroup with 0. A quasi-ideal element a of T is called minimal if $a\neq 0$ and there exist no quasi-ideal element t of T such that $0<t<a$.

Definition 2.8. An element a of a poe ternary semigroup T is called bi-ideal element of T, if $aeaea\leq a$.

Definition 2.9. Let T be a ternary semi-lattice under $\vee$ with a greatest element end at the same time a po ternary semi group such that for all $a,b,c,d\in T$,

$$ab(c\vee d)=abc\vee abd.$$

and $$(a\vee b)\,cd=acd\vee bcd..$$

Then T is called a$\vee e$ - ternary semigroup.

Definition 2.10. A $\vee e$ - ternary semigroup T which is also a lattice is called an le – ternary semigroup.

Here T will stand for any le ternary semigroup and the usual order relation $\leq$ on T is denoted in following way $a\leq b\Leftrightarrow a\vee b=b$

Then we can show that for any $a, b \in T$, if $a \leq b$ then $a\,cd \leq b\,cd, cad \leq cbd$ and $cda \leq cdb$, $c, d \in T$.

Examples 2.1.

1) Let T be a partially ordered ternary semigroup . Let M be the set of all ideals of T . then $(M, \subseteq, \cap, \cup)$ is an le ternary semigroup.
2) Let $(X, \leq)$ *and* $(Y, \leq)$ be two finite chains . Let T be the set of all isotones mapping from X and Y . Let $f, g \in T$ we define fg to denote the usual mapping composition of f and g . Then T is a ternary semigroup and for any $f, g \in T$, the mapping $f \vee g$ and $f \wedge g$ are defined as $(f \vee g)(a) = \max\{f(a), g(a)\}$, $(f \wedge g)(a) = \min\{f(a), g(a)\}$, for each a $\in X$.
   The greatest element e is the mapping that sends every $a \in X$ to the greatest element of finite chains $(Y, \leq)$. Then T is an le-Γ-semi group.

Definition 2.11. Three mapping r, m and l are defined by for any $x \in T$ as follows:

$$r : T \to T, r(x) = xee \vee x.$$

$$m : T \to T, m(x) = exe \vee x.$$

and $l : T \to T, l(x) = ee\,x \vee x,$ for any $x \in T$

Definition 2.12. Let T be a le – ternary smigroup then we define the mapping b and q as follows:

$$b : T \to T, b(x) = x \vee xex.$$

$$q : T \to T, q(x) = x \vee (eex \wedge exe \wedge xee).$$ for all $x \in T$

Definition 2.13. Let T be le-ternary semigroup, the Green's relation are defined as follows:

$$\text{L} = \{(x, y) \in M^2 / ee\,x \vee x = ee\,y \vee y\}$$

Or

$$L = \{(x, y) \in M^2 / l(x) = l(y)\}$$

$$\Im = \{(x, y) \in M^2 / exe \vee x = eye \vee y\}$$

Or

$$\Im = \{(x, y) \in M^2 / m(x) = m(y)\}$$

$$\Re = \{(x, y) \in M^2 / xee \vee x = ye\,e \vee y\}$$

Or

$$\Re = \{(x, y) \in M^2 / r(x) = r(y)\}$$

H = L $\cap$ R.

Definition 2.14. An element x of an le - ternary semigroup T is called regular if $x \leq xexex$.

A le – ternary semigroup T is called regular, if every element of T is regular.

Definition 2.15. An element x of an le - ternary semigroup T is called intra-regular, if $x \leq ex^3e$.

An element x of a le- ternary semigroup T is called intra-regular, if every element of T is intra-regular.

Theorem 2.16. Let T be a le-ternary semigroup then $q(q(x)) = q(x)$ for all $x \in T$.

Proof: We know that, $q(x) = x \vee (eex \wedge exe \wedge xee)$, for all $x \in T$.

$$q(q(x)) = q(x \vee (eex \wedge exe \wedge xee))$$

$$= (x \vee (xee \wedge exe \wedge eex)) \vee ((ee(x \vee (xee \wedge exe \wedge eex))$$

$$\wedge e(x \vee (xee \wedge exe \wedge eex))e \wedge (x \vee (xee \wedge exe \wedge eex))ee)$$

$$q(q(x)) = (x \vee (xee \wedge exe \wedge eex)) \vee ((eex \vee ee(x \vee (xee \wedge exe \wedge eex))$$

$$\wedge (xee \vee e(xee \wedge exe \wedge eex)e) \wedge (xee \vee (xee \wedge exe \wedge eex)ee))$$

$$q(q(x)) = (x \vee (xee \wedge exe \wedge eex)) \vee (eex \wedge exe \wedge xee)$$

$$q(q(x)) = (x \vee (xee \wedge exe \wedge eex))$$

$q(q(x)) = q(x)$, for all $x \in T$.

Theorem 2.17. If an element a of T is a left ideal element, an element b of T is a lateral ideal element and an element c of T is a right ideal element, then $a \wedge b \wedge c$ is a quasi-ideal element.

Proof: Assume that a is a left ideal element, b is a lateral ideal element and c is a right ideal element of T. Then $eea \vee a = l(a) = a$, $ebe \vee b = m(b) = b$ and $eec \vee c = r(c) = c$, so , $eea \leq a$, $ebe \leq b$ and $cee \leq c$. Hence $ee(a \wedge b \wedge c)$ $\wedge e(a \wedge b \wedge c)e \wedge (a \wedge b \wedge c)ee \leq eea \wedge ebe \wedge cee \leq a \wedge b \wedge c$.

Therefore $a \wedge b \wedge c$ is a quasi-ideal element

Definition 2.18. A quasi ideal element of an le-ternary semigroup T has the intersection property if it is expressed as an intersection of a left-ideal element, a lateral-ideal element and a right ideal element.

Theorem 2.19. The quasi ideal element q of an le –ternary semigroup T has the intersection property if and only if $q = l(q) \wedge m(q) \wedge r(q)$.

Proof: Let q be a quasi ideal element of T. Then by Theorem 2.17,

$q = a \wedge b \wedge c$. where $a = l(a)$ $b = m(b)$ and $c = r(c)$

Then $l(q) = l(a \wedge b \wedge c) \leq l(a)$

$m(q) = m(a \wedge b \wedge c) \leq m(b)$ and $r(q) = r(a \wedge b \wedge c) \leq r(c)$

Consequently,

$q = l(a) \wedge m(b) \wedge r(c) \geq l(q) \wedge m(q) \wedge r(q).$

$q \geq l(q) \wedge m(q) \wedge r(q)$. (1)

On the other hand,

$q = q\ (qee \wedge qee \wedge e\, eq) \leq q \vee e\, eq = l(q).$

$q = q(qee \wedge eqe \wedge ee\, q) \leq q \vee eqe = m(q)$

and $q = q(qee \wedge eqe \wedge ee\, q) \leq q \vee qee = r(q)$

then $q \leq l(q) \wedge m(q) \wedge r(q)$. (2)

From equations (1) and (2) we get $q = l(q) \wedge m(q) \wedge r(q)$.

Conversely, assume that $q = l(q) \wedge m(q) \wedge r(q)$ for any left ideal l(q), lateral ideal elements m(q) and right ideal elements r(q) of T .

By the theorem 2.17 $q = l(q) \wedge m(q) \wedge r(q)$ quasi ideal element of T .

We observe here that if

$q = q(a) = a \vee (ae \wedge eae \wedge e\, a)$

Then $l(q) = l(a \vee (ae\, e \wedge ea\, e \wedge ee\, a))$

$= a \vee (ae\, e \wedge ea\, e \wedge ee\, a) \vee e\, e(a \vee (ae\, e \wedge ea\, e \wedge ee\, a)) = l(a)$

$m(q) = m(a \vee (ae\, e \wedge ea\, e \wedge ee\, a))$

$= a \vee (ae\, e \wedge ea\, e \wedge ee\, a) \vee e(a \vee (ae\, e \wedge ea\, e \wedge ee\, a))e = m(a)$

And $r(q) = r(a \vee (ae\, e \wedge ea\, e \wedge ee\, a))$

$= a \vee (ae\, e \wedge ea\, e \wedge ee\, a) \vee (a \vee (ae\, e \wedge ea\, e \wedge ee\, a))ee = r(a)$

$q = l(a) \wedge m(a) \wedge r(a)$.

3. Conclusions

We introduced the notion of left(lateral and Right) ideal, quasi ideal in a le - ternary semigroup and studied their properties and relations between them. In continuous of this paper we propose to ideals over le - ternary semigroups.

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