

Erratum in “On the Holographic Principle Description for the Viscous Incompressible Fluids” and “The Mathematical Description of Homogeneous Turbulence for Incompressible Fluids”

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Abstract. This note is to acknowledge errors made in previous article: “On the Holographic Principle Description for the Viscous Incompressible Fluids” due to the Laplacian of the Lagrangian fails to be zero as expected when a using G.I. Taylor's 2-D solution of the Navier-Stokes momentum equations. Unfortunately, the article “The Mathematical Description of Homogeneous Turbulence for Incompressible Fluids” was also affected due to the use of the Laplacian of the Lagrangian in order to obtain the power spectrum relationship of the pressure which obviously now is incorrect.

Keywords: Turbulence, Holographic Principle

AMS Mathematics Subject Classification (2010): 76F02

1. Introduction

The author has been informed that the Taylor's analytical 2D Navier Stokes solution does not satisfy the Laplace equation of the Lagrangian by Professor Filippo M. Denaro [3]. Professor Denaro has been very kind to me by sharing his knowledge with me, and I do appreciate Professor Denaro's effort on my behalf very much.

2. Erratum

The article "On the Holographic Principle Description for the Viscous Incompressible Fluids" [1] is incorrect. The main claim of the article, the Laplace equation of the Lagrangian is incorrect as shown in the following figures where kinematic viscosity and density are set to unity for simplicity [3,4].

Figure 1: G.I. Taylor's 2-dimensional solution

$$\begin{aligned}u &:= -\cos(x) \sin(y) e^{-2t} \\v &:= \sin(x) \cos(y) e^{-2t} \\p &:= -0.25 (\cos(2x) + \cos(2y)) e^{-4t}\end{aligned}$$

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Figure 2: Laplacian of the kinetic energy, K

$$\text{Laplacian}_K := e^{-4t} \cos(2x - 2y) + e^{-4t} \cos(2x + 2y)$$

Figure 3: Laplacian of the pressure, p

$$\text{Laplacian}_P := 1.00 \cos(2x) e^{-4t} + 1.00 \cos(2y) e^{-4t}$$

Therefore, Laplacian of the Lagrangian, $K-p$, fails to be zero as expected when using G.I. Taylor's 2-D exact solution of the Navier-Stokes's momentum equations provided by Professor Denaro [3]. Is interesting to observe that the Laplacian of the kinetic energy have the same functional form as the Laplacian of the pressure, except the arguments of the Laplacian of the kinetic energy seem to be rotated by 45 degrees with respect to the arguments of the pressure. This result could be just a coincidence, although it might be possible for someone with greater knowledge in PDEs and coordinate transformation group theory could find a causal relationship or effect. At the time when I wrote the article, I was not aware of Taylor's analytical 2D Navier Stokes solution at the time, nor I had the software to test with.

This will obviously impact my latest article, "The Mathematical Description of Homogeneous Turbulence for Incompressible Fluids" [2], also since it uses the Laplace equation of the Lagrangian as a reference, and all of the results of the spectrum of the pressure are therefore incorrect. Although, I still believe the three propositions of the article [2] are still correct and unaffected by the errors in [1].

3. Conclusion

The author takes full responsibility for the errors made in [1] and [2], and in no way hold APAM responsible for them. The first error in [1] is a very rookie mathematical or logical error which I obviously was aware of (and everyone else also for that matter), but like when one is scaling a cliff one takes risks to fulfill the desire to reach the summit. The author ignored and was careless by using the same equation (or derivative of the equation) multiple times which is not allowed in any mathematical or logical approach. That is, the author used Navier-Stokes's momentum equations 3 different times: 1) as NSE PDE, 2) as the divergence of NSE, which makes the Laplace equation, and 3) time integral of the Navier Stoke, i.e., Bernoulli equation. Additionally, like any new climber who takes risks while discovering a new path to the top of the mountain, the author thought by discovering and independent path based on well-known formulas involving the surface integral in Appendix A of [1] would compensate for this error. Obviously, the author deceived myself, just as a climber can deceive himself to climb to the top by taking unnecessary risks. The author thought this new path given by the surface integral conversion using well known kinematic formulas and divergence theorem would prove the hypothesis of the Laplace equation of the Lagrangian to be correct. Again, this is now clear to the author that this equation in isolation cannot be right, another elementary mistake. The gravest mistake was not testing this theory, since the author was unaware about Taylor's solution, and did not know how to test it. Again, the author took another more critical risk, because if the author had tested the Laplacian of the Lagrangian with the G.I. Taylor's 2-D Navier-Stokes's solution, the author would not have submitted the article for publication to APAM.

The author would humbly suggest to use article [1] as a learning tool for students in the classroom as a tutorial in research of fluid mechanics, to see if the students can catch

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the mistakes. The author still believe there must be a way to approach holographic principle mathematical description for fluids, but not with the path the author took, this new path is conjecture to be extremely challenging and provide for a deeper understanding of fluid mechanics, which the author will not pursue at this time, but the author would challenge the community of fluid mechanics to pursue a holographic mathematical description for the viscous incompressible fluids.

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