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Certain Notions of Intuitionistic Fuzzy Graphs

***Zahra Sadri Irani*[1] and *Hossein Rashmanlou*[2*]**

[1]Department of Chemistry, Faculty of Science, Falavarjan Branch
Islamic Azad University, Iran. Email: sadri@iaufala.ac.ir
[2]Adib Mazandaran Institute of Higher Education, Sari, Iran
[*]Corresponding author. Email: rashmanlou.1987@gmail.com

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Abstract. Fuzzy graph models enjoy the ubiquity of being in natural and human-made structures, namely dynamic process in physical, biological and social systems. As a result of inconsistent and indeterminate information inherent in real-life problems which are often uncertain, it is highly difficult for an expert to model those problems based on a fuzzy graph. Intuitionistic fuzzy graph (IFG) can deal with the uncertainty associated with the inconsistent and indeterminate information of any real-world problem, where fuzzy graphs may fail to reveal satisfactory results. Likewise, IFG has an important role in neural networks, computer network, and clustering. In the design of a network, it is important to analyze connections by the levels. In this paper, we describe d_m-regular, td_m-regular, m-highly irregular and m-highly totally irregular IFGs and prove the necessary and sufficient conditions which under this conditions the d_m-regular and td_m-regular IFGs are equivalent. Also, a comparative study between m-highly irregular IFG and m-highly totally irregular IFG are given.

Keywords: Intuitionistic fuzzy set, d_m-degree, intuitionistic fuzzy graph, d_m-regular intuitionistic fuzzy graph.

AMS Mathematics Subject Classification (2010): 05C99, 03E72

1. Introduction

The concept of a graph is one of the most powerful and widely employed tools for the representation, modelling, analyses, and solution of a multitude of real world problems. An immediate result of a rise of popularity of fuzzy sets theory has been the fuzzification of graph theory which has been initiated by Rosenfeld who has introduced the concept of a fuzzy graph. Basically, a fuzzy graph is a weighted graph in which the weights are from [0, 1] and are defined over a fuzzy set of vertices. In 1965, Zadeh [27] proposed fuzzy theory and introduced fuzzy set theory. The most important feature of a fuzzy set is that it consists of a class of objects that satisfy a certain (or several) property. Fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. Rosenfeld [18] in (1975), introduced fuzzy graph. It has been growing fast and has numerous applications in various field. Atanassove, [1, 2, 3] in 1986 proposed the concept of intuitionistic fuzzy set, by replacing the value of an element in a set with a subinterval of [0, 1]. The node degree in the graph is a quick way to get the relation number of nodes, so to analyze a graph it is important to look at the degree of nodes. Nagoor Gani and Latha [15, 16] in (2012), introduced irregular fuzzy graph, total degree and totally irregular fuzzy graph, and regular fuzzy graphs.. In a fuzzy set each element is associated with a point value selected from the unit interval [0,1], which is termed the grade of membership in the set. Instead of using point based membership as in fuzzy sets, interval based membership is used in a vague set. Rashmanlou et al. [6, 7, 17] defined new concepts of fuzzy graphs. Akram et al. [4, 5] introduced strong IFG and certain types of vague graphs. Ghorai and Pal [8, 9, 10, 11] introduced several concepts in m-polar fuzzy graphs. Mahapatra and Pal [12, 13] studied fuzzy colouring and applications of edge colouring on fuzzy graphs. Nayeem and Pal [14] described diameter constrained fuzzy minimum spanning tree problem. Recently, some research works have been done by the authors in continuation of previous works related to cubic graphs, vague graphs, bipolar fuzzy graphs, and intuitionistic fuzzy graphs which are mentioned in [19, 20, 21, 22, 23, 24, 25].

IFGs are the generalization of graph structures and extremely useful in the study of some structures, like graphs, colored graphs, signed graphs, and edge-labeled graphs. IFGs are more useful than fuzzy graphs (FGs) because they deal with the uncertainty and ambiguity of many real-word phenomena. Hence, in this paper, we represent $\mathrm{d_m}$-regular, $\mathrm{td_m}$-regular, m-highly irregular and m-highly totally irregular intuitionistic fuzzy graphs and some properties of them are discussed. Also, a comparative study between $\mathrm{d_m}$-regular (m-highly irregular) IFG and $\mathrm{td_m}$-regular (m-highly totally irregular) IFG are given.

2. Preliminaries

Definition 2.1. [1] *An intuitionistic fuzzy set A in an ordinary non-empty set X, is a pair (μ_A, ν_A) where $\mu_A: X \longrightarrow [0,1]$ and $\nu_A: X \longrightarrow [0,1]$ are membership and non-membership functions, respectively such that*

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \ for any \ x \in X.$$

Definition 2.2. *[1] Let X and Y be two ordinary non-empty sets. An intuitionistic fuzzy relation of X to Y is a intuitionistic subset of $X \times Y$, that is an expression R defined by;*

$$R = \{< x, y >, \mu_R < x, y >, \nu_R < x, y >\}$$

where $\mu_R: X \times Y \longrightarrow [0,1]$ and $\nu_R: X \times Y \longrightarrow [0,1]$, which satisfies the condition

$$0 \le \mu_R(x, y) + \nu_R(x, y) \le 1; \ for all \ (x, y) \in X \times Y.$$

Definition 2.3. *[5] Let $G^* = (V, E)$ be a graph. A pair $G = (A, B)$ is called an intuitionistic fuzzy graph (IFG) on G^* or an IFG where $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy set on V and $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy set on $E \subseteq V \times V$ such that for each $uv \in E$,*

$$\mu_B(uv) \le min(\mu_A(u), \mu_A(v)) \ , \ \nu_B(uv) \ge max(\nu_A(u), \nu_A(v)).$$

An intuitionistic fuzzy graph $G = (A, B)$ is called a complete-IFG if for every $u, v \in V$,

$$\mu_B(uv) = \mu_A(u) \wedge \mu_A(v) \quad , \quad \nu_B(uv) = \nu_A(u) \vee \nu_A(v)$$

A complete IFG with n nodes is denoted by K_n.

Definition 2.4. *[5] A path ρ in an IFG $G = (A, B)$ is a sequence of distinct nodes*

$v_0, v_1, \ldots, v_k$ *such that;*

$$(\mu_B(v_{i-1}v_i), \nu_B(v_{i-1}v_i)) > 0 \ , \ i = 1, \ldots, k.$$

Here k *is called the length of the path.*

Definition 2.5. *[5] Let* $G = (A, B)$ *be an IFG. If* u *and* v *are connected by means of a path of length* k *in* G *such as* $\rho: u = u_0, u_1, u_2, \ldots u_{k-1}, u_k = v$*, then* $\mu_B^k(uv)$ *and* $\nu_B^k(uv)$ *are defined as follows,*

$$\mu_B^k(uv) = sup\{\mu_B(uu_1) \wedge \mu_B(u_1u_2) \wedge \ldots \wedge \mu_B(u_{k-1}v)\},$$
$$\nu_B^k(uv) = inf\{\nu_B(uu_1) \vee \nu_B(u_1u_2) \vee \ldots \vee \nu_B(u_{k-1}v)\}.$$

The strength of connectedness between two nodes u *and* v *in IFG* G *is defined as follows,*

$$(\mu_B^\infty(uv) \ , \ \nu_B^\infty(uv)) = (sup\{\mu_B^k(uv) \mid k = 1,2,\ldots\}, inf\{\nu_B^k(uv) \mid k = 1,2,\ldots\}).$$

Definition 2.6 *[5] An IFG* $G = (A, B)$ *is called connected-IFG if for every nodes* $u, v \in V$*,* $\mu_B^\infty(uv) > 0$ *or* $\nu_B^\infty(uv) < 1$.

3. d_m-regular and td_m-regular intuitionistic fuzzy graphs

In this section, first we define d_m-degree and td_m-degree of nodes in an IFG. Then we introduce the notions of d_m-regular and td_m-regular IFGs and prove the necessary and sufficient conditions which under this conditions the d_m-regular and td_m-regular IFGs are equivalent.

Definition 3.1. *Let* $G = (A, B)$ *be an IFG. Then the* d_m*-degree of a node* u *in* G *is defined by,*

$$d_m(u) = (\textstyle\sum_{u \neq v \in V} \mu_B^m(uv), \sum_{u \neq v \in V} \nu_B^m(uv)),$$

where $u, u_1, u_2, \ldots u_{m-1}, v$ *is the shortest path connecting* u *and* v *of length* m.

Example 3.2. *Consider the IFG* $G = (A, B)$ *as follows,*

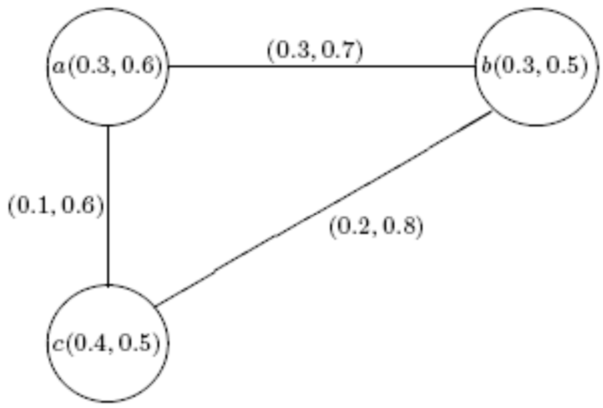

Figure 1: Intuitionistic fuzzy graph G.

Then the d_2-degree of nodes in G are as follow,

$$d_2(a) = (0.2 + 0.1, 0.8 + 0.8) = (0.3, 1.6),$$
$$d_2(b) = (0.1 + 0.1, 0.7 + 0.8) = (0.2, 1.5),$$
$$d_2(c) = (0.1 + 0.2, 0.7 + 0.8) = (0.3, 1.5).$$

Definition 3.3. *Let $G = (A, B)$ be an IFG. Then the td_m-degree or total d_m-degree of a node u in G is defined as follows,*

$$td_m(u) = (\sum_{u \neq v \in V} \mu_B^m(uv) + \mu_A(u) \quad , \sum_{u \neq v \in V} \nu_B^m(uv) + \nu_A(u)),$$

where $u, u_1, u_2, \ldots u_{m-1}, v$ is the shortest path connecting u and v of length m.

Example 3.4. *In the Figure 1, td_2-degree of nodes in G are as follows:*

$$td_2(a) = ((0.2 + 0.1) + 0.3, (0.8 + 0.8) + 0.6) = (0.6, 2.2),$$
$$td_2(b) = ((0.1 + 0.1) + 0.3, (0.7 + 0.8) + 0.5) = (0.5, 2.0),$$
$$td_2(c) = ((0.1 + 0.2) + 0.4, (0.7 + 0.8) + 0.5) = (0.7, 2.0).$$

Definition 3.5. *An IFG $G = (A, B)$ is said to be $(m, (d_1, d_2))$-regular IFG or d_m-regular if for all nodes v in $G, d_m(v) = (d_1, d_2)$.*

Example 3.6. *A $(2, (0.1, 0.3))$-regular IFG is given in the Figure 2.*

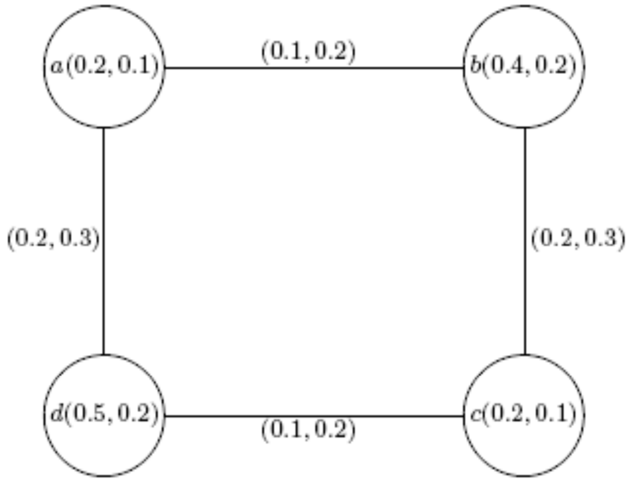

Figure 2: Intuitionistic fuzzy graph G.

Since,

$$d_2(a) = (0.1, 0.3) \quad , \quad d_2(b) = (0.1, 0.3)$$
$$d_2(c) = (0.1, 0.3) \quad , \quad d_2(d) = (0.1, 0.3).$$

Then G is a $(2, (0.1, 0.3))$-regular IFG.

Definition 3.7. *An IFG $G = (A, B)$ is said to be $(m, (k_1, k_2))$-totally regular intuitionistic fuzzy graph or td_m-regular if for all nodes v in G, $td_m(v) = (k_1, k_2)$.*

Example 3.8. *A $(2, (0.6, 2.0))$-totally regular IFG is given in the Figure 3.*

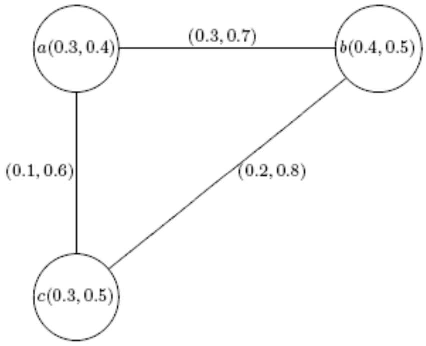

Figure 3: Intuitionistic fuzzy graph G.

Since,

$$td_2(a) = ((0.1+0.2)+0.3, (0.8+0.8)+0.4) = (0.6, 2.0),$$
$$td_2(b) = ((0.1+0.1)+0.4, (0.8+0.7)+0.5) = (0.6, 2.0),$$
$$td_2(c) = (0.2+0.1)+0.3, (0.8+0.7)+0.5) = (0.6, 2.0).$$

Then G is a $(2, (0.6, 2.0))$-totally regular IFG.

Theorem 3.9. *Let $G = (A, B)$ be an IFG. If t_A and f_A are constant functions, then G is a d_m-regular IFG if and only if G is a td_m-regular intuitionistic fuzzy graph (m is a positive integer).*

Proof: Suppose that for every node v in G, $(\mu_A(v), \nu_A(v)) = (c_1, c_2)$ and $d_m(v) = (d_1, d_2)$. Then

$$td_m(v) = d_m(v) + (\mu_A(v), \nu_A(v)) = (d_1 + c_1, d_2 + c_2),$$

Hence G is a td_m-regular IFG. If G is a td_m-regular IFG, then the proof is similar to the previous case.

Theorem 3.10. *Let $G = (A, B)$ be a $(m, (d_1, d_2))$-regular and a $(m, (k_1, k_2))$-totally regular IFG with n nodes. Then μ_A and ν_A are constant functions and*

$$O(G) = n(k_1 - d_1, k_2 - d_2).$$

Proof: If G is a $(m, (d_1, d_2))$-regular IFG and a $(m, (k_1, k_2))$-totally regular IFG then respectively for all $v \in V$ we get,

$$d_m(v) = (d_1, d_2) \rightarrow (\sum_{v \neq u \in V} \mu_B^m(uv), \sum_{v \neq u \in V} \nu_B^m(uv)) = (d_1, d_2), \quad (1)$$

and

$$td_m(v) = (k_1, k_2) \rightarrow (\sum_{v \neq u \in V} \mu_B^m(uv) + \mu_A(v), \sum_{v \neq u \in V} \nu_B^m(uv) + \nu_A(v)) = (k_1, k_2).$$

Therefore,

$$(\sum_{v \neq u \in V} \mu_B^m(uv), \sum_{v \neq u \in V} \nu_B^m(uv)) = (k_1 - \mu_A(v), k_2 - \nu_A(v)).$$

Hence by (1), $(d_1, d_2) = (k_1 - \mu_A(v), k_2 - \nu_A(v))$ and so $(\mu_A(v), \nu_A(v)) = (k_1 - d_1, k_2 - d_2)$. Then μ_A and ν_A are constant functions and since G has n nodes, then we get

$$O(G) = (\sum_{v \in V} \mu_A(v), \sum_{v \in V} \nu_A(v)) = n(k_1 - d_1, k_2 - d_2).$$

Definition 3.11. *Let $G=(A,B)$ be a connected IFG. Then*
(i) G is called an m-highly irregular IFG, if every node of G is adjacent to the other nodes with the distinct d_m-degree,
(ii) G is said to be an m-highly totally irregular IFG if every node of G is adjacent to the other nodes with distinct td_m-degree.

Example 3.12. *Let G be IFG in the Figure 1. Since for nodes a, b and c of G we get;*

$$d_2(a) \neq d_2(b) \quad , \quad d_2(b) \neq d_2(c) \quad , \quad d_2(c) \neq d_2(a),$$
$$td_2(a) \neq td_2(b) \ , \ td_2(b) \neq td_2(c) \ , \ td_2(c) \neq td_2(a).$$

Then G is an 2-highly irregular IFG and is an 2-highly totally irregular IFG.

Theorem 3.13. *Let $G=(A,B)$ be an IFG. If μ_A and ν_A are constant function and m is a positive integer. Then G is an m-highly totally irregular IFG if and only if G is an m-highly irregular IFG.*

Proof: Suppose that G is an m-highly totally irregular IFG. Then td_m-degree of every pair of adjacent nodes are distinct. Let u and v are a pair of adjacent nodes with distinct td_m-degree. We get,

$$td_m(u) = (\textstyle\sum_{u\neq w\in V} \mu_B^m(uw) + \mu_A(u), \sum_{u\neq w\in W} \nu_B^m(uw) + \nu_A(u))$$

and

$$td_m(v) = (\textstyle\sum_{v\neq w\in V} \mu_B^m(vw) + \mu_A(v), \sum_{v\neq w\in V} \nu_B^m(vw) + \nu_A(v)).$$

Since $td_m(u) \neq td_m(v)$, we have,

$$\textstyle\sum_{u\neq w\in V} \mu_B^m(uw) + \mu_A(u) \neq \sum_{v\neq w\in V} \mu_B^m(vw) + \mu_A(v)$$

or

$$\textstyle\sum_{u\neq w\in V} \nu_B^m(uw) + \nu_A(u) \neq \sum_{v\neq w\in V} \nu_B^m(vw) + \nu_A(v).$$

Since $\mu_A(u) = \mu_A(v)$ and $\nu_A(u) = \nu_A(v)$. Hence

$$\textstyle\sum_{u\neq w\in V} \mu_B^m(uw) \neq \sum_{v\neq w\in V} \mu_B^m(vw)$$

or

$$\textstyle\sum_{u\neq w\in V} \nu_B^m(uw) \neq \sum_{v\neq w\in V} \nu_B^m(vw)$$

and so

$$d_m(u) = (\textstyle\sum_{u\neq w\in V} \mu_B^m(uw), \sum_{u\neq w\in V} \nu_B^m(uw)) \neq (\sum_{v\neq w\in V} \mu_B^m(vw), \sum_{v\neq w\in V} \nu_B^m(vw)) = d_m(v).$$

Hence, any pair of adjacent nodes in G have distinct d_m-degree. Then G is an m-highly irregular IFG.

Conversely, let G is an m-highly irregular IFG. Then the d_m-degree of every pair of adjacent nodes such as u and v are distinct. This implies that $d_m(u) \neq d_m(v)$ and since $\mu_A(u) = \mu_A(v)$ and $\nu_A(u) = \nu_A(v)$. Hence

$$td_m(u) = d_m(u) + \mu_A(u) \neq d_m(v) + \mu_A(v) = td_m(v)$$

and so any two adjacent nodes in G have distinct td_m-degree. Therefore G is an m-highly totally irregular intuitionistic fuzzy graph.

Theorem 3.14. *Let $G=(A,B)$ be an IFG on cycle graph $G^*=(V,E)$ with $k\geq 3$ nodes and for all $i=1,\dots,k-1$ (where $v_{k+1}=v_1$),*

$$\mu_B(v_iv_{i+1}) < \mu_B(v_{i+1}v_{i+2}) \ \text{or} \ \nu_B(v_iv_{i+1}) > \nu_B(v_{i+1}v_{i+2}).$$

Then G is an $\{1,2,\dots,[k2]\}$-highly irregular IFG.

Proof: Suppose that $v_1v_2, v_2v_3, \ldots, v_kv_1$ is the arcs of G where,

$$\mu_B(v_1v_2) < \mu_B(v_2v_3) < \ldots < \mu_B(v_iv_{i+1}) < \mu_B(v_{i+1}v_{i+2}) < \ldots < \mu_B(v_kv_1)$$

and there exists $v_iv_{i+1} \in E$ such that for $1 \le m \le [k2]$, $d_m(v_i) = d_m(v_{i+1})$. Therefore we get

$$\sum_{v_i \neq v_j \in V} \mu_B^m(v_iv_j) = \mu_B(v_iv_{i+1}) + \mu_B(v_lv_{l+1})$$

and

$$\sum_{v_{i+1} \neq v_j \in V} \mu_B^m(v_{i+1}v_j) = \mu_B(v_iv_{i+1}) + \mu_B(v_{l+1}v_{l+2}).$$

Since $d_m(v_i) = d_m(v_{i+1})$ hence,

$$\mu_B(v_iv_{i+1}) + \mu_B(v_lv_{l+1}) = \mu_B(v_iv_{i+1}) + \mu_B(v_{l+1}v_{l+2})$$

and so,

$$\mu_B(v_lv_{l+1}) = \mu_B(v_{l+1}v_{l+2})$$

that this is a contradiction.

Now, if for every $v_1v_2, v_2v_3, \ldots, v_kv_1 \in E$,

$$\nu_B(v_1v_2) > \nu_B(v_2v_3) > \ldots > \nu_B(v_iv_{i+1}) > \nu_B(v_{i+1}v_{i+2}) > \ldots > \nu_B(v_kv_1)$$

then similar to the proof of previous case, the contradiction is obtained. Therefore G is an $\{1,2,\ldots,[k2]\}$-highly irregular intuitionistic fuzzy graph.

Remark 3.15. *Let $G = (A, B)$ be an IFG on cycle graph $G^* = (V, E)$ with $k \ge 3$ nodes and for all $i = 1, \ldots, k-1$ (where $v_{k+1} = v_1$),*

$$\mu_B(v_iv_{i+1}) < \mu_B(v_{i+1}v_{i+2}) \text{ or } \nu_B(v_iv_{i+1}) > \nu_B(v_{i+1}v_{i+2}).$$

Then G is not an $\{1,2,\ldots,[k2]\}$-highly totally irregular IFG, in general.

Example 3.16. *Consider the IFG $G = (A, B)$ on graph $G^* = (V, E)$ in the Figure 4,*

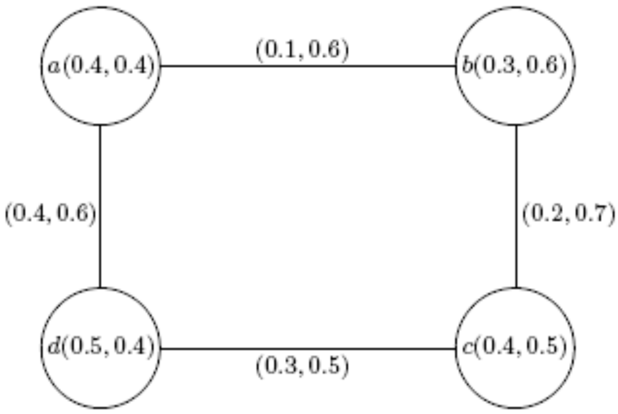

Figure 4: Intuitionistic fuzzy graph G.

We see that

$$\mu_B(ab) < \mu_B(bc) < \mu_B(cd) < \mu_B(da)$$

and we get

$$td_2(a) = (0.3 + 0.4, 0.6 + 0.4) = (0.7, 1.0),$$
$$td_2(b) = (0.2 + 0.3, 0.6 + 0.6) = (0.5, 1.2),$$
$$td_2(c) = (0.3 + 0.4, 0.6 + 0.5) = (0.7, 1.1),$$
$$td_2(d) = (0.2 + 0.5, 0.6 + 0.4) = (0.7, 1.0).$$

Since for two nodes a and d that are adjacent, $td_2(a) = td_2(d)$. Then G is not an 2-highly totally irregular IFG.

Theorem 3.17. *Let $G = (A, B)$ be an IFG on path graph $G^* = (V, E)$ with $k \geq 3$ nodes and for all $i = 1, \dots, k-1$ (where $v_{k+1} = v_1$),*

$$\mu_B(v_i v_{i+1}) < \mu_B(v_{i+1} v_{i+2}) \ or \ \nu_B(v_i v_{i+1}) > \nu_B(v_{i+1} v_{i+2}).$$

Then G is an $\{1,2, \dots, [k2]\}$-highly irregular IFG.

Proof: Suppose that $v_1v_2, v_2v_3, \dots, v_{k-1}v_k$ is the arcs of the path G such that,

$$\mu_B(v_1v_2) < \mu_B(v_2v_3) < \dots < \mu_B(v_iv_{i+1}) < \mu_B(v_{i+1}v_{i+2}) < \dots < \mu_B(v_{k-1}v_k)$$

Then for every $1 \leq m < [k2]$ we get,

$$\sum_{v_i \neq v_j \in V} \mu_B^m(v_i v_j) =$$

$$\begin{cases} \mu_B(v_i v_{i+1}) & if \quad i = 1,2, \dots, m \\ \mu_B(v_i v_{i+1}) + \mu_B(v_{i-m} v_{i-m+1}) & if \quad i = m+1, \dots, k-m \\ \mu_B(v_{i-m} v_{i-m+1}) & if \quad i = k-m+1, \dots, k \end{cases}$$

Hence for every two adjacent nodes as v_l and v_{l+1} in G, we get $d_m(v_l) \neq d_m(v_{l+1})$. Therefore G is an $\{1,2, \dots, [k2]-1\}$-highly irregular IFG. Now, suppose that $m = [k2]$. If k is even, then we get

$$\sum_{v_i \neq v_j \in V} \mu_B^m(v_i v_j) = \begin{cases} \mu_B(v_i v_{i+1}) & if \quad i = 1,2, \dots, m \\ \mu_B(v_{i-m} v_{i-m+1}) & if \quad i = m+1, \dots, k-m \end{cases}$$

and if k is odd, then we get

$$\sum_{v_i \neq v_j \in V} \mu_B^m(v_i v_j) = \begin{cases} \mu_B(v_i v_{i+1}) & if \quad i = 1,2, \dots, m \\ \mu_B(v_1 v_2) + t_B(v_{i-m} v_{i-m+1}) & if \quad i = m+1 \\ \mu_B(v_{i-m} v_{i-m+1}) & if \quad i = m+2, \dots, k \end{cases}$$

where in both cases for every two adjacent nodes v_l and v_{l+1} in G, we get $d_m(v_l) \neq d_m(v_{l+1})$ and so G is an [k2]-highly irregular IFG. Similarly, for the non-membership function of each arcs, it is established.

Definition 3.18. *A star-IFG $G = (A, B)$ is a complete bipartite IFG such that, one partition of V contains only one node.*

Theorem 3.19. *Let $G = (A, B)$ be a star IFG with $k \geq 4$ nodes, such that the node v_1 is incident to the other nodes. If for all $i = 2, \dots, k$,*

$$\mu_B(v_i v_1) < \mu_B(v_{i+1} v_1) \ or \ \nu_B(v_i v_1) > \nu_B(v_{i+1} v_1).$$

Then G is an $\{1,2\}$-highly irregular IFG.

Proof: Suppose that $v_2v_1, v_3v_1, \dots\dots, v_{k-1}v_1, v_kv_1$ are the arcs in G such that

$$\mu_B(v_2 v_1) < \mu_B(v_3 v_1) < \dots < \mu_B(v_{i+1} v_1) < \mu_B(v_{i+2} v_1) < \dots < \mu_B(v_k v_1).$$

Then,

$$\sum_{v_1 \neq v_j \in V} \mu_B^1(v_1 v_j) = \sum_{j=2}^{k} \mu_B(v_1 v_j) \ and \ \sum_{v_i \neq v_j \in V} \mu_B^1(v_i v_j) = \mu_B(v_1 v_i); \ i = 2, \dots, k.$$

i

$$d_1(v_1) \neq d_1(v_2) \neq d_1(v_3) \neq \dots \neq d_1(v_k).$$

Therefore G is an 1-highly irregular IFG. Also we see that

$$d_2(v_1) = (0,1) \ and \ \sum_{v_i \neq v_j \in V} \mu_B^2(v_i v_j) = (k-i)\mu_B(v_1 v_i) + \sum_{j=2}^{i-1} \mu_B(v_1 v_j).$$

Hence for every $i = 2, \ldots, k$, $d_2(v_i) \neq d_2(v_1)$ and so G is an 2-highly irregular IFG. Similarly, if the non-membership function of each arc is distinct from other arcs, then it is established.

4. Conclusion

An intuitionistic fuzzy graph has numerous applications in the modelling of real life systems where the level of information inherited in the system varies with respect to time and have the different level of precision. The IFG is the extension of FG having both membership and non-membership degrees. Intuitionistic fuzzy models are much better than FGs in precision, elasticity, and compatibility for the system. The IFG defines all types of complexity as an FG. In this paper, we introduce d_m-regular, td_m-regular, m-highly irregular and m-highly totally irregular intuitionistic fuzzy graphs and some properties of them are discussed. Likewise, a comparative study between d_m-regular (m-highly irregular) intuitionistic fuzzy graph and td_m-regular (m-highly totally irregular) intuitionistic fuzzy graph are given.

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