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C^3 Rational Quintic Spline

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Abstract. This paper discuss the construction of new C^3 rational quintic spline interpolation with quintic numerator and cubic denominator the idea has been extended to shape preserving interpolation for positive data using the constructed rational quintic spline interpolation. There are two parameter v_i and w_i play very crucial role to change shape of the resolution interpolating curves. However, as per spline interpolating with C^3 continuously is not able to preserve the positive data notably our scheme is easy to use and does not requires knots insertion and C^3 continuity unable to achieve by solving for the second derivations $i = 1, 2, \ldots, n-1$. An error analysis which the function to be interpolation is $f(t) \in C^4[t_0, t_n]$.

Key words: Rational, Interpolation, quintic, cubic, shape preserving, parameters.

AMS Mathematics Subject Classification (2010): 65D07

1. Introduction

Lower order Rational Spline have some weakness such that the interpolating curves may be give for unwanted behaviour of the original data due to wiggles along some interval. Thus uncracker behaviour may destroy the data, data are wastage. For some application any negative values is unaceptable. In my discovery any important information that may exist in the original data. Due to the fact that lower order spline is not able to produce completely the positive, monotonocity and convex interpolation curves on entire given interpolation. Fritch and Carlson [1] have discussed the monotonicity, positivity. Preserving by using cubic spline interpolation by modifying the first derivative values in which the shape valuation is found. Butt and Brodlie [19] have used cubic spline interpolation to preserve the positivity and convexity of the finite data by inserting extra knots in the interval in which the positively and/or convexity is not preserved by the cubic spline. There is no controlling to the final shape of the interpolating curves first user change data then correction will be final shape of the interpolation curve is possible the alteration of the first derivative parameters is require in the method. Sarfraz et al [6, 7] and Hussain and M. Hussain [9] studied positive preserving for curves and statistics by utilizing rational cubic spline with quadratic denominator. In works of by Hussain et al

[9] and Sarfraz et al [11] the rational cubic spline with quadratic denominator has been used for positivity, monotonicity and convexity preserving with C^2 continuity, Hussain [10] have only one free parameter meanwhile Sarfaraz et al [12] have no free parameter.

Abbass [13, 14, 15] have discussed the positivity by using new C^2 rational cubic spline with two free parameter. Delbourgo [16], Gregory [8], and Karim and Kong [19] have proposed new C^1 rational cubic spline (cubic/quadratic) with three parameters in which two parameter are free parameter. Hussain et al [2009] proposed a piecewise C^2 rational quintic function with two free parameters for the interpolation of curve data. In numerical result effect of free parameter is shown. C^2 rational quintic function change to the to the non-trivial prlynomial curve by taking very large and small values of free parameters. Hussain et al (2014) device a Oval quadratic trigonometric interpolant with two parameters for the shape. Preservation of curve data.

The proposed curve scheme is unique in its representation and it is equally applicable for the data with derivative or without derivatives. The proposed scheme is free from first derivative modification for shape preservation. The proposed scheme is must be produce positive interpolation in everywhere. Meanwhile in the scheme of Hussain [10] and Hussain et al leave positivity in some subintervals.

Methodology

In this paper a C^3 rational with two free parameter is introduced i.e. quantic upon cubic and its convergence is analysed further error bounds obtained.

2. C^3 rational quintic spline function

The piecewise rational functions are in use for interpolation scalar data (Hussain and Sarfarose 2008, Sarfaroz et al 2013).

Although there rational interpolation scheme are local. The order of continuity of these scheme are local, the order of continuity of there scheme at knots to C^1 Duan et al (2011) C^2 continuity were used C^3 rational interpolation function is introduced here.

Let $x_i < x_{i+1}, i = 0,1,......n-1$ the partition if interval $[0,1]$ with $x_i = a$ and x_n=*b. Take* f_i, d_i *and* D_i *are function values* D_i two functional values i.e.

$$s(x_i) = f_i,\ s(x_{i+1}) = f_{i+1}, s'(x_i) = d_i\ \ s'(x_{i+1}) = d_{i+1}.$$

The purpose C^3 rational quintic function RQF, $s(x)$ is defined over each subinterval as :

$$s(x) \cong s_i(x) = \frac{P_i(\theta)}{q_i(\theta)} \tag{1}$$

where

$$P(\theta) = A_0 p_1(\theta) + A_1 P_2(\theta) + A_3 p_3(\theta) + A_4 p_4(\theta) + A_5 p_5(\theta)$$

is a piecewise quintic polynomial.

where

$$P_1(\theta) = (1-\theta)^3 \left(1 + 3\theta + 6\theta^2\right)$$

$$P_2(\theta) = \theta^3 \left(10 - 15\theta + 6\theta^2\right)$$

$$P_3(\theta) = \left(\theta - 6\theta^3 + 8\theta^4 - 3\theta^5\right) = \theta(1-\theta)\left(1 - 5\theta^2 + 3\theta^3\right)$$

$$P_4(\theta) = \left(-4\theta^3 + 7\theta^4 - 3\theta^5\right) = -\theta^3 (1-\theta)^3 (-4 + 3\theta)$$

$$P_5(\theta)=\frac{(\theta^2-3\theta^3+3\theta^4-\theta 5)}{2}=\theta^2(1-\theta)^3$$

$$P_6(\theta)=\frac{\theta^3(1-\theta)^2}{2}$$

Let $q_i(\theta)=\left[(1-\theta)^3+v_i\theta(1-\theta)^2+w_i\theta^2(1-\theta)+\theta^3\right]$ (2)

where v_i and w_i are free parameters is cubic piecewise polynomial and $\theta=\frac{x-x_i}{h_i}\in[0,1]$.

Consider rational quintic spline satisfy following interpolating conditions :-

$$s(x_i)=f(x_i), S(x_{i+1})=f(x_{i+1}). S'(x_i)=f'(x_i).$$

$$s''(x_i)=f''(x_i)\, Let\, s'(x_i)=d_i$$

$$s''(x_i)=D_i,\ s''(x_{1+i})=D_{i+1} \tag{3}$$

It follows from (2) that the rational quintic function (1) is a C^3 continuations function at knots. If $v_i=3, w_i=3$ the rational quintic function become quintic Hermite Interpolatant (Farin 2005). Rest of paper v_i and w_i, are assumed to be positive read free parameters.

Note:

(1) If $v_i=1$ and $w_i=1$ the rational quintic function (1) becomes cubic Hermitte interpolant (Farin 2005).

(2) Given 2D data (*f, d_i, D_i*), i=0, 1, 2,…,n defined over the interval $[x_0,x_n]$ if we take v_i=*3*, w_i=*1* in each subinterval and apply third order derivative continuity at knots.

By using interpolating condition we have

$$s(x_i)=h=A_0 \quad s(x_{1+i})=f_{i+1}=A_1 \quad d_ih_i+(-3+v_i)f_i=A_2$$

$$d_{i+1}h_i+(-w_1+3)f_{i+1}=A_3$$

$$D_ih_i^2+2(-3+v_i)d_ih_i+(6-4v_i+2w_i)f_i=A_4$$

$$A_5=D_{i+1}h_i^2+2(-w_1+3)d_{i+1}+(6+2u_i-4w_i)f_{i+1}$$

So $s(x)=[f_ip_1(t)+f_{i+1}P_2(t)+\{d_ih_it(-3+v_1)f_i\}]$

$$\beta(t)+\{d_{i=1}h_i+(-v_1+3)f_{i+1}\}p_4(t)+\{h_i^2D_i+2(d_ih_i(-3+1))+(6-4v_i+2w_i)f\}$$

$$P_5(t)+\{h_i^2D_{i+1}+2(d_{i+1}h_i(-w_{ii}+3))+(6+2v_i-4w_i)f_{i+1}\}P_6(t)/q_i(t)$$

$$f_i[P_1(t)+(-3+v_i)P_3(t)+(6-4v_i+2w_i)P_5(t)+f_{i+1}]$$

$$[P_2(t)+(3-w_i)P_4(t)+(2v_i-4w_i+6)P_6(t)]+d_ih_iP_3(t)+d_{i+1}h_iP_4(t)$$

$$+\{h_i^2D_1+2(d_ih_i(-3+v_i))P_5(t)+\{D_{i+1}h_i^2+2d_ih_{i+1}(-w_i+3)\}P_6(t)/q_i(t) \tag{4}$$

When simple calculation given $v_i\to 0, w_i\to 0$

$$s_i(x)=((1-t)f_i+tf_{i+1})+t(1-t))\,[\{\Delta f_i(2t-1)\}]+d_ih_i(1-t)^2-d_{i+1}h_it^2+h_i^2t\frac{(1-t)}{2}$$
$$\frac{\{D_i(1-t)+t\,D_{i+1}\}]}{(1-t)^3+t^3} \quad (5)$$

and where let $v_i\to\infty, w_i\to\infty$ is

$$s(x)=[t\,h_{1+i}+(1-t)f_i] \quad (6)$$

is a linear equation which is straight line i.e. increase v_i and w_i, $s_i(x)$ reduce to straight line.

Table 1: Generates from constant function *f*(*x*) = 2, constant data is interpolated by the C^3-continuous rational quintic spline (1) for different values of few parameter v_i and w_i.

x	0	1	2	3
t	2	2	2	2

Design

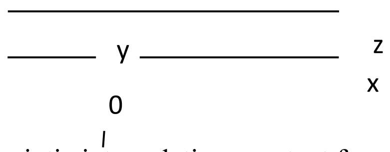

Figure 1: C^3 Rational quintic interpolation constant from with any choice of v_i, w_i.

Now using continuity of third derivatives, we obtain

$$3(2-v_i)h_{i-1},D_{i+1}+9(h_i+h_{i+1})D_i \quad (7)$$
$$+3(t2-_i)h_iD_{i-1}=F_i \qquad i=1,2,.....n$$

where

$$F_i=\frac{h_i}{h_{i-1}}[-60\Delta f_i+(30+4v_i-2w_i)d_{i-1}+30d_i]$$
$$+\frac{h_{i-1}}{h_i}[60\Delta f_{i+1}-24d_i-(36-4v_i+2w_i)d_{i+1}]$$
$$+\frac{h_i}{h_{i-1}^2}[-1+6(-3+v_i)f_{i-1}-6(v_i-w_i)f_{i+1}]$$
$$+\frac{h_{i-1}}{h_1^2}[f_{i+1}\{-20v_i+10w_i+30\}]+(12-8v_i+4w_i)f_i]$$

This is the system of linear equation and satisfies diagonally dominant property and equation have unique solution for unkonwn D_i, i=1,2,.....n, $w_i \geq 0$ and $vi \geq 0$.

Arithmetic Mean Approximation Scheme for derivatives :

The given 2D data $\{(x_i,f_i),i=0,1....n\}$ defined over the interval [a, b] where $x_i<x_{i+1},i=0,1......n-1$ the arithmetic choice of first derivatives at knots x_i, are given by the following formula

$$d_\sigma = \Delta_\sigma + (\Delta_0 - \Delta_1)\frac{h_0}{h_0 + h_i}$$

$$d_n = \Delta_{n-1} + (\Delta_{n-1} - \Delta_{n-2})h_{n-1}(h_{n-1} + (h_{n-2}))^{-1}$$

$$d_n = \frac{\Delta_1 + \Delta_{i-1}}{2},\ i = 1,2,\dots\dots n-1$$

Hence, $\Delta_i = \frac{f_{i+1} - f_i}{h_i}, h_i = x_{i+1} - x_i \quad i = 0,1,\dots\dots n-1.$ (8)

Similarly the second order derivatives D_i at knots.

Geometric Mean (GMM)

Consider

$$d_0 = \begin{cases} 0 & \Delta_\sigma \ or\ \Delta_{2,0} = 0 \\ \Delta_0\left(1 + \frac{h_0}{h_1}\right) \quad \Delta_{0,1}^{\left(-\frac{h_0}{h_i}\right)} & otherwise \end{cases} \tag{9}$$

$$d_n = \begin{cases} 0 & \Delta_{n-1} = 0 \ or\ \Delta_{n,n-2} = 0 \\ \Delta_{n-1}^{\left(1 + \frac{h_{n-1}}{h_{n-2}}\right)} \Delta_{n,n-2}^{(h_{n-1}/h_{n-2})} & otherwise\ x_0 = \end{cases} \tag{10}$$

Delbergo and Gregory [17] give main details about the method that can be used to estimate the first derivative and same as second derivatives by A.M.M. we can find d_0 and d_n.

Similarly the second order derivative D_i at knots x_i are given by the following formula:

$$D_0 = M_0 + (M_0 - M_1)h_0(h_0 + h_1)^{-1}$$

$$D_n = M_{n-1} + (M_{n-1} - M_{n-2})h_{n-1}(h_{n-1} + h_{n-2})$$

$$D_1 = M_i + M_{i-1}, i = 1,2\dots\dots n-1$$

Here $M_1 = \frac{d_{i+1} - d_i}{2}$

$$h_i = x_{i+1} - x_i, \quad i = 0,1,\dots\dots n-1 \tag{11}$$

Remark 1: When $v_i = w_i = 0$, the RQP reduce to QP without any parameter.

Table 2: A data from work by Sarfaraz et al [6]

I	0	1	2	3	4
x_i	0	2	3	9	11
H	.5	1.5	7	9	13
h_i	2	1	6	2	
D_i	.5	5.5	1/3	2	
$d_i(C^1)$	-2.833	3.40	4.76	1.58	2.4167
$d_i(C^2)$	3.2524	2.7809	-5.301	.4426	
$d_i(C^3)$	3.2524	1.389	27.75	44.43	.4426

Default: C^1 quintic spline

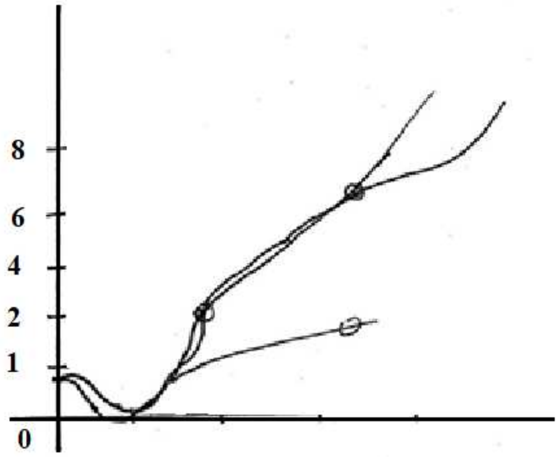

Figure 2: C^2 quintic rational spline**,** C^3 quintic rational spline $v_1 = w_1 = 3$

3. Positive preserving using C^3 rational quintic spline

In this action, the positively preserving by using the proposed C^3 rational quintic spline defined by will be discussed in brief. We will follow same method of Karim and Kong [12, 13] and Abhass et.al. [20, 21].

Given the strictly positive set of data $(x_i, f_i), \quad i = 1,2,....n$

$x_0 < x_1 < ....< x_n$ such that $f_i > 0, i = 0,1,......n$

Now the rational quintic spline will preserve of the data iff $P_i(0) > 0$ and $Q_i(0) > 0$

Since for all *i* $v_i > 0$, $e = 0,1,.....n-1$. Thus $Q(x) > 0$ iff $P_1(0) \geq 0, i = 0,1,.....n-1$.

The quintic polynomial $P_i(Q), i = 0,1,.......n$ can be written as

$$P_i(\theta) = A_i\theta^5 + B_i\theta^4 + C_i\theta^3 + D_i\theta^2 + E_i\theta + F_i \tag{12}$$

Let $\theta = \dfrac{S}{s+1}.s \geq 0$ in x and solve.

We get $$P_i(\theta) = AS^5 + BS^4 + CS^3 + GS^2 + HS + F \tag{13}$$

where

$$A = D_{1+i}h_1^2 + t(-w_1 + 3)d_{1+i}h_i f_{1+i} + (6 + 2v_i - 4w_i) f_{i+1}$$

$$B = D_i h_i^2 + 2(-3 + v_i) d_i h_i f_i + (6 + 2w_i - 4v_i) f_i$$

$$C = d_{i+1}h_i + (-w_i + 3)$$

$$D = d_i h_i + (-3 + v_i)$$

$$E = f_{i+1}$$

$$F = f_i$$

$$If\ (-w_i + 3) > 0 \Rightarrow 3 \geq w_i$$

$v_i - 3 > 0 \Rightarrow v \geq 3$

Theorem 4. For strictly data defined in (18) C^3 rational quintic spline interpolation defined over the interval $[x_0, x_n]$ is positive if in each subinterval $[x_i, x_{i+1}], i = 0,1,.....n-1$.

The involving parameter v_i and w_i satisfying the following:

$v_i, w_i \geq 0$

$A + B + C + D + E + F > 0$ iff $3 \geq w$, $v_i \geq 3$ (14)

Remark. The sufficient condition for positive preserving by using C^2 rational quintic interpolant is different from sufficient condition for positive preserving by using C^1 rational quintic interpolant. To achieve C^3 condition in the derivative parameter D_i, *i=1,2,.......n-1* must be calculated from C^3 condition given in (7) mean while to achieve C^2 condition. The second derivative d_i, *i=0,1,.......4* are estimated by using standard approximation methods such as A.M.M.

Arithmetic mean method. Comprising to Abbas [20].

Delbourage [19], Gregery [8], Karim [12], Hussain et.al. (2011) the C^2 continuous and upto first derivatives obtain but in this paper C^3 contemnors shape preserving and second derivative obtained also produce completely positive preserving interpolating curves with C^3 continuously but in some papers the method is computationally expensive and typical Linear equations of second derivatives investigate.

Table 3: A positive dot a form work by Hussain et.al. [2011]

i =	0	1	2	3	4
x =	0	1.0	1.70	1.80	1.90
f_i =	.25	1.0	11.10	2.5	

Table 4: Numerical results

i =	0	1	2	3
$d_i(C')$	-7.296	8.796	123.429	154.570
Δ_i	0.75	14.429	139	
v_i	.5	.5	.5	
w_i	.5	.5	.5	
$D_i(C^3)$	13.94	3.14	0.25	

Table 5: Lead contamination drain of Chenab river. Malik and Hussain et al. ([12,13])

n	1	2	3	4	5
x	20	42	45	47	52
y	.05	.06	1.5	1.65	1.4

Numerical Results:

d_i	-0.4215	.2402	.2775	.0125	-0.1393
$D_i(C^2)$	.0456	.0213	-0.0600	.0814	.0426
$D_i(C^3)$	.0456	7.06	-5.7688	-1.1877	.0426
w_i	.005	.005	.005	.005	.005
v_i	3	3	3	3	3

Numerical result (Change v_i, w_i)

v_i	3	3	3	3	3
w_i	3	3	3	3	3
$D_i(C^3)$	0.456	8.8367	0.97177	1.61727	

Numerical Result

v_i	4	4	4	4	4
w_i	1	1	1	1	1
$D_i(C^3)$	0.456	3.6358	2.0527	-7.3573	
v_i	3.3	3.3	3.3	3.3	3.3
w_i	2	2	2	2	2
$D_i(C^3)$	0.1424	9.7005	-2.9666	1.758	0.039

Table 6: Creative in Human blood

x	2	30	32	35	37	39
y	1.51	0.80	1.05	0.6	0.51	0.58
$D_i(C^3)$	0.1424	10.4522	04.0702	0.12901	0.369	
v_i	3.3	3.3	3.3	3.3	3.3	
w_i	2	2	2	2	2	

Numerical results (Change v_i, w_i)

$D_i(C^3)$	0.1424	0.5994	0.3130	0.7616	0.0369
v_i	3	3	3	3	3
w_i	3	3	3	3	3

Table 7: Data lying above line $y=x+3$

x	0	4	0.21	30	32
y	22.8	8	33.9	38.9	43.6
$v_i=$	$w_i=$	3			
$D_i(C^3)$	*0.054*	- 9.9258	36.7216		

Table 8: Data lying above line y = 0.5x - 1.

x	1	5.5	6
y	1	2	6

$v_i = w_i = 3$

$D_i(C^3)$ 87.250

$w_i = 2.5v_i = 3.5$

$D_i(C^3)$ -61.639

4. Error analysis

In this section, the error Analysis for the function to be interpolated to $f(t) \in C^3[t_0, t_n]$ using over C^3 rational quintic spline which will discussed in detail. Note that the construction rational quintic spline which wll be discussed in details. Note that the constructed rational quintic spline which will be discussed in detail. Note that the constructed rational quintic spline with two parameters is a local interpolant and without loss of generality we take interval.

$$I = [t_i, t_{i\&i}]$$

By using Perono Kernel theorem [25] the error of interpolation in each interval $It = [t_i, t_{1+i}]$ is dedness as

$$R[f] = f(t) - P_i(t) \tag{15}$$

$$= \frac{1}{2}\int_{t_i}^{t_{1+i}} R_t\left[(t-t)_t^2 d\right]$$

where $f = \left[(t-s)_t^2\right] = \begin{cases} r(j,t) & t_i < j < t \\ 3(j,t) & t < j < t_{1+i} \end{cases}$

with

$$r(r,t) = (t-h)^2 + 3(j,t)$$

$$s(j,t) = v^2\{1 + w_i\theta^2 - 3w_i, \theta + 2w_i$$

$$+ v_i + v_i\theta^2 - 2v_i\theta\} + 2\theta h_j(1-\theta)\{-4 + 3\theta + 2(w_i - \theta)(1-\theta)\} + (1-\theta)^2$$

where $\ell_j = (t_{i+1} - T)$

The absolute error in each interest $I = [x_i, x_{i+1}]$. The absolute error in each interval $I_1 = [x_i, x_{1+i}]$ is given as follows :

$$|f(t_2) - P_i(t)| \le 2 \,\|\, f^3(J) \,\|\int_{j_i}^{t_{1+i}} R_t[(t-J)_+]dt$$

First used to study the properties of the Kernel function *r(j,t)* for estimation of error, *s(J,t)* and evaluate the following details integrates.

$$\int_{t_1}^{t_{i+1}} R_t\left[(t-J)_+^2\right] = \int_{t_1}^{t} |r(T,x)|dJ + \int_{t}^{t_{i+1}} |s(T,x)|dJ \tag{16}$$

To simplify the integral in [16] we begin by funding the roots of $R(t_1 - t) = 0, r(t,t) = 0$ and $s(t,t) = 0$ respectively.

It is easy to see that the roots of *r(t,t)=0* in [0,1] are $\theta = 0$, and $\theta^* = 1 - \frac{w_i}{e_i}$

where $e_i = v_i + w_i$. (17)

Mean while roots of $r(T,t)=0$ $T_K = x$ - $\frac{\theta h_i\left(\theta e_i + (-1)^{K+1} H\right)}{1+\theta e_i}$ K= 1,2

where $H = \sqrt{(e_i - w_i)(1+e_i\theta)} - e_i\theta$

The roots of s(T, t) = 0 are $T_3 = t_{i+1}$.

$$T_4 = t_{i+1} - \frac{2hw_i(1-\theta)-h_i}{4+e_i(1-\theta)}$$

Case I: For $\theta \le \frac{w_i}{e_i} \le 1,\ 0<\theta<\theta^*$.

$$\int_t^{t_4} (t_{i+1}-T)_+^2\, dT$$

$$= \left[-\left(\frac{h_{i+1}-T}{3}\right)^3\right]_t^{t_4}$$

$$= -\frac{1}{3}\left[(t_{i+1}-T_4)^3 - (t_{i+1}-t)^3\right]$$

$$= -\frac{1}{3}\frac{\left(h_i^3\{w_i(1-\theta-1)\}^3\right)}{\left[4+e_i(1-\theta)^3\right]} - (1-\theta)^3 h_i^3$$

$$= -\frac{h_i^3}{3}\frac{\left[8\{w_i(1-\theta)-1\}^3\right.}{\{4+e_i(1-\theta)\}^3} - (1-\theta)^3]K_1 \tag{18}$$

From equation (18), we get

$$-\frac{\theta^3 h_i^3}{3} - \frac{16}{3}h_i^3\left\{\frac{w_i(1-\theta)-1}{4+e_i(1-\theta)}\right\}^3 - \frac{6h_i^3}{Q_i(x)}\frac{[w_i(1-\theta)-1]}{4+e_i(1-\theta)}$$
$$= \ell_i(\theta, w, K_1, K_2) \tag{19}$$

Case II: For $\theta \le \frac{w_i}{e_i} \le 1, \theta^* < \theta < 1, \theta^* < \theta < 1$

$$|f(t)-P_i(t)| \le \frac{1}{2}\| f^{(3)}(T)\| = \int_{t_1}^{t_{i+1}} R_t\left[(t-T)_+^3\right] dT = \frac{1}{2}\| f^3(t)\| \ell_2(T)$$

$$G_2(T) = \left\{\int_{t_1}^{T_2} -r(T,t)\,dT + \int_{T_2}^{t} r(T,t)\,dT\right.$$
$$+ \int_t^{t_{i+1}} s(T,t)\,dT \tag{20}$$

Hence $|f(t)-P_i(t)| \le \| f^3(T)\|\ \ell_2(v_i, w_i, \theta)$

where $\ell_2(v_1, w_1, \theta) = \left[\frac{\theta^3 h_i^3}{3\theta_i(x)}\frac{(\theta_i - H)^3}{(1+\theta)^3} - \frac{\theta^3 h_i^3}{3}\right]$

$$K_1(1-\theta)^3 h_i^3 + \frac{1-\theta}{2} h_i^2 K_2 + (1-\theta)h_i \big] / Q_i(x) \quad (21)$$

Case III: For $\frac{w_i}{e_i} > 1,\ 0 < \theta < 1,$

$$|f(t_1) - p_i(t)| \leq \frac{1}{2} \| f^3 \| \int_{t_i}^{t_{i+1}} R_t [t-T]_+^3$$

$$= \frac{1}{2} \| t^3 \| \int_{t_1}^{t} r(T,t)dJ + \int_{t}^{t_{i+1}} s(T,T)dT = \frac{1}{2} \| f^3 \| e_3(v_i, w_i, \theta)$$

where $e_3(v_i, w_i\ \theta) = \frac{\theta^3 h_i^3}{3} - \frac{k_1(1-\theta)^3 h_i^3}{2} + k_2(1-\theta)^2 h_i^2 (1-\theta)^2 \theta h_i$

and $k_1 = \left[1 + w_i(\theta^2 - 3\theta + 2) + v_i(1-\theta)^2\right]$

$$k_2 = [-1 + \{2(w_i - \theta) + 3\}(1-\theta)] \quad (22)$$

Theorem 5. The rational quintic spline intrpolant defined by (1) in each sub interval.
$I_1 = [t_i, t_{i+1}]$, when $t \in c^3(t_i, t_{i+1})$.
The error bound is given by

$$| f(t) + s_i(t) | \leq \| f^{(3)}(T) \| h_i^3 e_i)$$

$$\int_{t_i}^{t_{i+1}} \left[(t-T)_+^2\right] dT$$

$$= \| f^{(3)}(T) \| \left\{ \int_{t_1}^{t} r(T,t)d + \int_{t}^{t_{i+1}} s(T,t)d, T \right.$$

$\theta = \| f^{(3)}(T) \| h_i^3 e_i$ with

$e_i = \max \ell_i(v_i, w_i, \theta)$

$$\ell_i(v_i, w_i, \theta) = \begin{cases} \ell_1(v_i, w_i, \theta) & 0 < \theta < \theta^* \\ \ell_2(v_i, w_1, \theta) & \theta^* \leq \theta \leq 1 \\ \ell_3(v_i, w_i, \theta) & 0 \leq \theta \leq 1 \end{cases}$$

5. Range restricted C^3 rational quintic function

In this section, investigate interpolation schemes to preserve the shape of 2D Data lying above line using C^3 rational quintic function [1] Let $\{a = x_0, x_1, \ldots\ldots x_n = b\}$ be the partition of the interval [a, b] and let $f_1 = f(x_i)$ i=0, 1,......n and lying above straight line *y=mx+c*. Here m is the slope of the given line and y intercept C during (1) $f_i > v_i x_i + C$. The rational quintic function (1) interpolating the given data set
$(x_i, f_i), i = 0,1 \ldots\ldots n$
will be above line *y=mx+c* if $s(x) > mx + C$ (A)

We write
$s(x)=a_i(1-\theta)+b_i\theta \quad in[x_i,x_{i+1}]$
Here $a_i=m_ix_i+C, b_i=mx_{i+1}+C$ and $h_i=x_{i+1}-x_i$
So $\dfrac{P_i(\theta)}{q_i(\theta)}>a_i(1-\theta)+b_i\theta$
After simplification

$$P_i(\theta)>[a_i(1-\theta)+b_i\theta]q_i(\theta)(1-\theta+\theta) \tag{23}$$
$$\Rightarrow p_i(\theta)>a_iq_i(\theta)(1-\theta)^2+a_iq_i(\theta)\theta(1-\theta)\ +b_i\theta q_i(\theta)(1-\theta)+b_iq_i(\theta)\theta^2$$
$$\Rightarrow P_i(\theta)>a_iq_i(1-\theta)^2+(a_i+b_i)q_i(\theta)\ \theta(1-\theta)+b_iq_i(\theta)\theta^2$$

Now $B_0p_1(\theta)+B_1p_2(\theta)+B_2p_3(\theta)+B_3P_4(\theta)\ +B_4p_5(\theta)+B_5P_6(\theta)$
$>\left[a_i(1-\theta)^2+(a_i+b_i)\theta(1-\theta)+b_i\theta^2\right]q_i(\theta)$
where

$$B_0=f_i-a_i$$
$$B_1=f_{i+1}-b_i$$
$$B_2=d_ih_i+4a_i-a_iv_i-b_i+(-3+(v_i))f_i$$
$$B_3=d_{i+1}h_i+4b_i+a_i+w_ib_i$$
$$B_4=[D_ih_i^2-3d_ih_i+v_i(d_ih_i-4f_1+10a_i)-2b_i)-8a_i+10b_i-2a_iw_i+6f_i+2w_if_i]$$
$$B_5=[D_{i+1}h_i^2+w_i(-2d_{i+1}h_i-4f_{i+1}-8b_i)+6d_{i+1}h_i+2(3-v_i)f_{i+1}-12(a_i+b_i)-20b_i]$$
$$B_2>0 \text{ if } v_i>\frac{\{d_ih_i+4a_i-b_i-3f_i\}}{(f_i-a_i)}$$

where $h_i=\dfrac{\hat{h}_i}{k_i}$

$$B_3>0 \text{ if } w_i>\frac{d_{i+1}h_i-4b_i+a_{i+3}f_{i+1}}{(f_{i+1}-b_i)}$$
$$B_4>0 \text{ if } v_i>\frac{[D_ih_i^2-3d_ih_i-8a_i+6f_i+10b_i-2a_iw_i+2w_if_i)}{[2b_i-10a_i+4f_i-d_ih_i]}$$

and $k_i>h_i(m_i-d_i)/3(f_i-a_i)$

$$B_5>0 if\ w_i>\frac{[D_{i+1}h_i^2+6d_{i+1}h_i\ 2(3+v_i)f_{i+1}-12(a_i+b_i)-20b_i]}{d_{i+1}h_i+4f_{i+1}+8b_i}$$

and $\dfrac{h_i(d_{i+1}-m_i)}{3(f_{i+1}-b)}>k_i$

Theorem 6. The C^3 rational quintic function (1) preserve the shape of data lying above straight line *y=mx+c* if the shape parameters v_i, w_i satisfies the following.

Constrains:

$x_i > 0, w_i > 0$

$v_i > \max[0, \ell_1, \ell_2]$

$w_i > \max\{0, \ell_3, h_i\}$

$k_i > \max\{0, \ell_5, \ell_6\}$

i.e. $v_i > \max.\{0, \ell_1, \ell_2, \ell_3, \ell_4\}$

$$e_1 = -\frac{\{d_i h_i + 4a_i + b_i - 3f_i\}}{f_i - a_i}$$

$$e_2 = \frac{\left[D_i h_i^2 - 3d_i h_i - 8a_i + 6f_i + 10b_i - 2a_i w_i + 2w_i f_i\right]}{\left[2b_i - 10a_i + 4f_i - d_i h_i\right]}$$

$$e_3 = \frac{d_{i+1} h_i + 4b_i + a_i + 3f_{i+1}}{\left(f_{i+1} - \ell_i\right)}$$

$$e_4 = \frac{D_{i+1} h_i^2 + 6d_{i+1} + h_i(3 + v_i)h_{i+1} - 12(a_i + b_i) - 20b_i}{2a_{i+1} h_i + 4f_{i+1} + 8h_i}$$

$$e_5 = \frac{h_i(m_1 - d_i)}{3(f_i - a_i)}$$

$$e_6 = \frac{h_i(d_{i+1} - m_i)}{3(f_{i+1} - b_i)}$$

6. Algorithm

Step I. Input the 2D data set $(x_i, f_i) i = 0,1,2,\ldots\ldots n$ lying above the straight line $y = mx + c$.

Step II. Compute the values of $h_i = x_{i+1} - x_i, q_i = mx_i + C$ and $b_i = mh_i + a_i$.

Step III. Compute the values of first and second order derivative d_i and D_i at knots x_i, $i=0,1,2,\ldots.n$ by the arithmetic mean scheme or direct linear equation.

Step IV. Calculate value of the shape parameter using theorem 3.

Step V. Substitute the values of f_i, d_i, D_i and shape parameters in C^3 rational quintic function (1) to obtain the curves lying above the straight line.
y=mx+c.

6. Result and discussion

In the following numerical example the importance of the proposed shape preserving schemes is demonstrated through graphical results. It is conclude that the C^2 continuous quintic Harmitte Polynomial (Farin) falls to preserve the inherit shape of the data and second derivative values is not possible in Malik, Zuner [7] other than A.M. but in the results obtain second derivative equal values by linear equation of second derivate C^3-Continuous shape. Preserving rational quintic function fills remain thing of (Farin 2005 [3, 4], Malik [7]).

7. Conclusion

The new C^3 rational quintic spline interpolation with (quintic/cubic) have obtained and the idea has been extended to shape preserving interpolation for positive data using the constructed rational quintic spline.

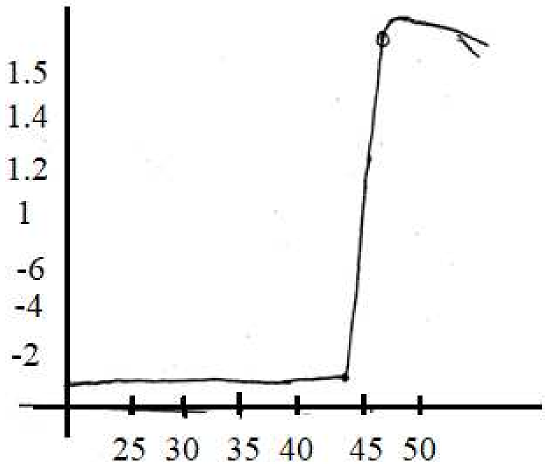

Figure 3:

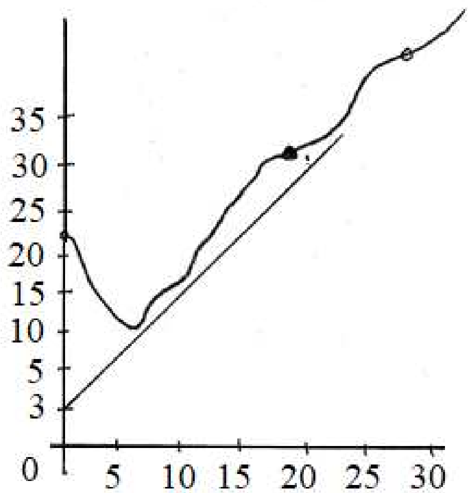

Figure 4: Rational quintic spline above straight line.

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