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Expressing Price of Perishable Goods Using Trapezoidal Fuzzy Numbers: A Case Study of Price of Fish in Guwahati

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Abstract. In the classical theory of fuzzy sets, it has not been discussed how exactly to construct a fuzzy number in any particular case. In this article, we shall discuss how a trapezoidal fuzzy number appears in describing data represented in terms of intervals. In doing so we shall use the operation of superimposition of sets. Prices of perishable goods such as fish can be used to construct such fuzzy numbers. As a case study, we have collected data on the minimum and maximum daily prices of a particular variety of fish in a market of Guwahati, and we have shown that the data can be described by a trapezoidal fuzzy number.

Keywords: Construction of fuzzy numbers, superimposition of sets, empirical distribution function, the Glivenko-Cantelli theorem of order statistics, the randomness-fuzziness consistency principle.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

In the theory of fuzzy sets, the matter of how exactly to construct fuzzy numbers mathematically has never really been discussed. Researchers simply presume that in any situation to be studied with reference to fuzziness the triangular fuzzy number for example, can be used, but no one has ever tried to see whether that presumption is mathematically valid or not.

In the classical theory of fuzzy sets, there were three noteworthy attempts made to link probability with fuzziness. Had those attempts been successful, a procedure of constructing the membership function with the help of probability would have been there in the literature of the classical theory of fuzziness. The originator of the theory of fuzzy sets, Zadeh [1] had tried to link probability with the concept of fuzziness. However, Zadeh himself mentioned that his principle concerned was heuristic, and not a precise law. Klir and Geer [2] made an attempt to define probability by normalizing the membership function so that the area under the normalized function is equal to unity. Had that been true, one would have got a law of fuzziness from a law of probability just using a transformation.

Dubois *et. al.* [3] defined a probability-possibility consistency principle and claimed that their transformation from probability to fuzziness is the most specific transformation which satisfies the condition of consistency defined by the authors themselves. But the problem of construction of the membership function of a normal fuzzy number mathematically remained.

Using the operation of set superimposition and the Glivenko-Cantelli Theorem of Order Statistics, Baruah ([4, 5, 6, 7]) discussed how to link randomness and fuzziness to describe the membership function of a normal fuzzy number with the help of two independent probability laws. The Randomness-Fuzziness Consistency Principle discussed there leads to the construction of the membership function. It was shown that data of the interval type can be analyzed using this principle.

For interval type of data such as data of daily temperature in a locality, would always have a minimum and a maximum value every day. Earthquake waveform data are also of this type with a minimum and a maximum for every waveform. Similarly, daily stock prices of companies would always have a minimum and a maximum. In such cases in which data are invariable of the interval type, the classical theory of fuzzy sets looked from the viewpoint of superimposition of sets can describe the situation correctly ([8], [9], [10]).

In this article, we have shown that daily data of the price of perishable goods can be described as trapezoidal fuzzy numbers. The price of perishable goods such as raw fish sold in street markets can be seen to be the maximum when the selling starts in the morning every day, and becomes the minimum at the time of closure of sales for the day. Thus the prices of every variety of raw fish in the street markets lie in intervals. We have considered the daily price of a particular variety of fish in a market of Guwahati, and have shown how the price can be described with the help of a trapezoidal fuzzy number.

2. Methodology

We visited a particular locality of the Guwahati fish Market and collected the daily prices of the fish for a period of 15 days from 24th September, 2021 to 8th October 2021. The fish we have selected for the purpose was Common Carp (*Cyprinus carpio*). We noticed that the prices of the fish were maximum when the fish was fresh and minimum when the market was about to be closed for the day. In other words, the prices of raw fish depended on the freshness of the fish on any particular day. Indeed when the day progresses, the price goes on reducing slowly, and as the fish made available in the open market would no longer remain fresh the next day, the vendors would try to sell the fish at a low price if necessary before sales for the day is over.

Suppose the prices for n consecutive days lie in the intervals $[\alpha_1, \beta_1]$, $[\alpha_2, \beta_2]$, … , $[\alpha_n, \beta_n]$. We shall now superimpose the sets $[\alpha_1, \beta_1]$, $[\alpha_2, \beta_2]$, … , $[\alpha_n, \beta_n]$, every one set over the others. To explain the situation, let us take the first two sets first. Let S define the set operation of superimposition. Indeed, it is known that when we overwrite, the overwritten portions look doubly darker because in the overwritten portion there is double representation. It may be noted that the operation of union of sets does not consider double representation of the elements, and that is why to describe double or multiple representation of elements the operation of union is not sufficient which is why we have to go for the operation of set superimposition. To cite another practical case of superimposition, we may consider the following example. When an opaque object is placed near a small source of light, on a screen placed nearby, the shadow can be seen to have an umbra and a penumbra. The level of darkness in the umbra portion would be doubly darker than that in the penumbra portion. This happens due to superimposition. This is what we are now going to show with reference to two real intervals.

Consider the real intervals $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$. When we superimpose these two intervals, one of the following four results would be possible.

$$[\alpha_1, \beta_1](S)[\alpha_2, \beta_2]$$
$$= [\alpha_1, \alpha_2] \cup [\alpha_2, \beta_1]^{(2)} \cup [\beta_1, \beta_2], if \alpha_1 < \alpha_2 < \beta_1 < \beta_2,$$
$$= [\alpha_1, \alpha_2] \cup [\alpha_2, \beta_2]^{(2)} \cup [\beta_2, \beta_1], if \alpha_1 < \alpha_2 < \beta_2 < \beta_1,$$
$$= [\alpha_2, \alpha_1] \cup [\alpha_1, \beta_1]^{(2)} \cup [\beta_1, \beta_2], if \alpha_2 < \alpha_1 < \beta_1 < \beta_2,$$
$$= [\alpha_2, \alpha_1] \cup [\alpha_1, \beta_2]^{(2)} \cup [\beta_2, \beta_1], if \alpha_2 < \alpha_1 < \beta_2 < \beta_1,$$

where for example$[\alpha_2, \beta_1]^{(2)}$ represent the interval $[\alpha_2, \beta_1]$ with every element being present twice. Indeed, a closer look would reveal that we can write it as

$$[\alpha_1, \beta_1]\ (S)\ [\alpha_2, \beta_2] = [\alpha_{(1)}, \alpha_{(2)}] \cup [\alpha_{(2)}, \beta_{(1)}]^{(2)} \cup [\beta_{(1)}, \beta_{(2)}]$$

where $\alpha_{(1)} = \min(\alpha_1, \alpha_2)$, $\alpha_{(2)} = \max(\alpha_1, \alpha_2)$, $\beta_{(1)} = \min(\beta_1, \beta_2)$, $\beta_{(2)} = \max(\beta_1, \beta_2)$. Here it has been assumed without loss of any generality that $[\alpha_1, \beta_1] \cap [\alpha_2, \beta_2] \neq \emptyset$, which is same as assuming that $\max(\alpha_i) \leq \min(\beta_i)$, $i = 1,2$.

Using the same logic, if we define $[\alpha_1, \beta_1]^{(p)}$ as the interval $[\alpha_1, \beta_1]$ in which all elements are partially present with level of presence equal to p where p is a positive proper fraction, then we shall have

$$[\alpha_1, \beta_1]^{(\frac{1}{2})}(S)(\alpha_2, \beta_2]^{(\frac{1}{2})} = [\alpha_{(1)}, \alpha_{(2)}]^{(\frac{1}{2})} \cup [\alpha_{(2)}, \beta_{(1)}]^{(1)} \cup [\beta_{(1)}, \beta_{(2)}]^{(\frac{1}{2})}$$

As is well known, a set with this kind of partial presence of elements is defined as a fuzzy set. Accordingly, in our case $[\alpha_{(1)}, \alpha_{(2)}]^{(\frac{1}{2})}$represents the real interval $[\alpha_{(1)}, \alpha_{(2)}]$in which all elements are partially present with the level of presence being $1/2$. In other words, $[\alpha_{(1)}, \alpha_{(2)}]^{(\frac{1}{2})}$is a fuzzy number with a membership value equal to $1/2$ for every element. We are indeed going to look into normal fuzziness from the standpoint of superimposition of equally fuzzy sets.

Suppose $[\alpha_i, \beta_i]$are the intervals of the price of the fish on day $i, i = 1, 2, \ldots, 15$. Let $\alpha_{(1)}, \alpha_{(2)}, \ldots, \alpha_{(15)}$ be values of $\alpha_1, \alpha_2, \ldots, \alpha_{15}$ arranged in increasing order of magnitude, and $\beta_{(1)}, \beta_{(2)}, \ldots, \beta_{(15)}$ be values of $\beta_1, \beta_2, \ldots, \beta_{15}$ in increasing order of magnitude. Now, we shall superimpose the 15 equally fuzzy intervals. From the data, we have 15 equally fuzzy intervals of the type: $[\alpha_1, \beta_1]^{(1/15)}, [\alpha_2, \beta_2]^{(1/15)}, \ldots, [\alpha_{15}, \beta_{15}]^{(1/15)}$ with a constant level of partial presence 1/15 for every interval. We shall thus get, subject to the condition that $[\alpha_1, \beta_1] \cap [\alpha_2, \beta_2] \cap \ldots \cap [\alpha_{15}, \beta_{15}]$ is not void,

$$[\alpha_1, \beta_1]^{\left(\frac{1}{15}\right)}(S)[\alpha_2, \beta_2]^{\left(\frac{1}{15}\right)}(S)\ldots(S)[\alpha_{15}, \beta_{15}]^{\left(\frac{1}{15}\right)}$$
$$= [\alpha_{(1)}, \alpha_{(2)}]^{(1/15)} \cup [\alpha_{(2)}, \alpha_{(3)}]^{(2/15)} \cup \ldots \cup [\alpha_{(15)}, \beta_{(1)}]^{(1)}$$
$$\cup [\beta_{(1)}, \beta_{(2)}]^{(14/15)} \cup [\beta_{(2)}, \beta_{(3)}]^{(13/15)} \cup \ldots \cup [\beta_{(14)}, \beta_{(15)}]^{(1/15)}$$

where for example, $[\beta_{(14)}, \beta_{(15)}]^{(1/15)}$ represents the interval $[\beta_{(14)}, \beta_{(15)}]$ with the level of presence of every element being 1/15 for every element in the interval.

We are interested to see what happens when n intervals each with the level of presence $1/n$ are superimposed, and what happens when n becomes infinite. It may be observed that if n such intervals are superimposed, the level of presence of the elements would be k/n in the interval $\left[\alpha_{(k)}, \alpha_{(k+1)}\right]$ and would be $(1-\frac{k}{n})$ in the interval $\left[\beta_{(k)}, \beta_{(k+1)}\right]$ for $k = 1, 2, \dots, (n-1)$. This means that subject to the condition that $[\alpha_1, \beta_1] \cap [\alpha_2, \beta_2] \cap \dots \cap [\alpha_n, \beta_n]$ is not void,

$$[\alpha_1, \beta_1]^{(1/n)}(S)[\alpha_2, \beta_2]^{(1/n)}(S)\dots(S)[\alpha_n, \beta_n]^{(1\backslash n)}$$
$$= \left[\alpha_{(1)}, \alpha_{(2)}\right]^{(1/n)} \cup \left[\alpha_{(2)}, \alpha_{(3)}\right]^{(2/n)} \cup \dots \cup \left[\alpha_{(n)}, \beta_{(1)}\right]^{(1)}$$
$$\cup \left[\beta_{(1)}, \beta_{(2)}\right]^{((n-1)/n)} \cup \left[\beta_{(2)}, \beta_{(3)}\right]^{((n-2)/n)} \cup \dots \cup \left[\beta_{(n-1)}, \beta_{(n)}\right]^{(1/n)}$$

At this point, we would like to define what is known as an empirical distribution function in the statistical literature. An empirical distribution function may be considered as an estimate of the cumulative distribution function defining the randomness concerned (Gibbons and Chakraborti [11], page - 33). Now the principle that we shall apply is as follows. If $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ denote the ordered values ordered in the ascending order of a random sample $X_1, X_2, \dots, X_n$ then empirical distribution function of the random variable would be given by

$$F_n^{(X)} = 0, \text{ if } x < X_{(1)},$$
$$= \frac{k}{n}, \ if X_{(k)} \leq x < X_{(k+1)}, k = 1, 2, \dots, (n-1),$$
$$= 1, if x > X_{(n)},$$

and following the Glivenko-Cantelli Theorem on Order Statistics (Gibbons and Chakraborti [11], page – 35), the empirical distribution function $F_n^{(X)}$ will converge almost surely to the underlying probability distribution function $F(X)$. It can also be expressed as $E(F_n^{(X)}) = F(X)$, where E stands for mathematical expectation.

Similarly, if $Y_{(1)}, Y_{(2)}, \dots, Y_{(n)}$ denote the values ordered in the ascending order of a random sample $Y_1, Y_2, \dots, Y_n$ then the *complementary* empirical distribution function of the random variable would be given by

$$G_n^{(Y)} = 1, \text{ if } y < Y_{(1)},$$
$$= 1 - \frac{k}{n}, \ if Y_{(k)} \leq y < Y_{(k+1)}, k = 1, 2, \dots, (n-1),$$
$$= 0, if \ y > Y_{(n)},$$

so that $(1 - G_n^{(Y)})$ would converge following the Glivenko-Cantelli Theorem to the underlying probability distribution function $\left(1 - G(Y)\right)$ of the random variable concerned.

The minimum values, as well as the maximum values of fish price, are random variables, and therefore the application of the Glivenko – Cantelli Theorem in our case would be valid so as to get an empirical distribution function for the minimum daily prices and an empirical complementary distribution function for the maximum daily prices. For the observed ordered values of the minimum daily prices, the underlying theoretical distribution function can be fitted using the standard statistical method of least squares for curve fitting. Similarly, for the observed ordered values of the maximum daily prices, the theoretical complementary distribution function can be estimated.

3. Construction of the membership function

We have collected the data on price of Common Carp in a market of Guwahati for the specified period of 15 days. The intervals in rupees were [180, 240], [203, 250], [190, 255], [175, 260], [185, 245], [177, 285], [195, 265], [200, 252], [205, 270], [220, 268], [235, 280], [230, 275], [210, 264], [225, 290] and [215, 295]. Then we superimposed the intervals making them equally fuzzy with constant level of partial presence equal to 1/15 in every case.

The minimum daily prices in increasing order of magnitude were: 175, 177, 180, 185, 190, 195, 200, 203, 205, 210, 215, 220, 225, 230 and 235. The maximum prices in increasing order of magnitude were: 240, 245, 250, 252, 255, 260, 264, 265, 268, 270, 275, 280, 285, 290 and 295. Thus for example α_2 = 203 while $\alpha_{(2)}$= 177, and β_2 = 250 while $\beta_{(2)}$= 245.

The observed values of partial presence for the ordered minimum values of the intervals were seen to be almost in a straight line segment with a positive slope, and the observed values of partial presence for the ordered maximum values of the intervals were seen to be in a straight line segment with negative slope. We now proceed to see whether the observed membership values can actually be expressed as a first-degree polynomial function of the ordered minimum prices, and we would also see whether the observed membership values with reference to the observed maximum prices can be expressed as a first-degree polynomial.

Taking the different values of $X_{(k)}$ as the kth observed value of an independent variable X and the level of presence in the interval $[X_{(k)}, X_{(k+1)}]$ as the dependent variable Z, we can fit the left reference function of the fuzzy number. As mentioned above, the left reference function was seen to be a first degree polynomial $Z = a + bX$. The parameters a and b were estimated using the method of least squares. The fitted equation (Fig. 1) was found to be

$$Z = (-2.34622) + 0.014185X.$$

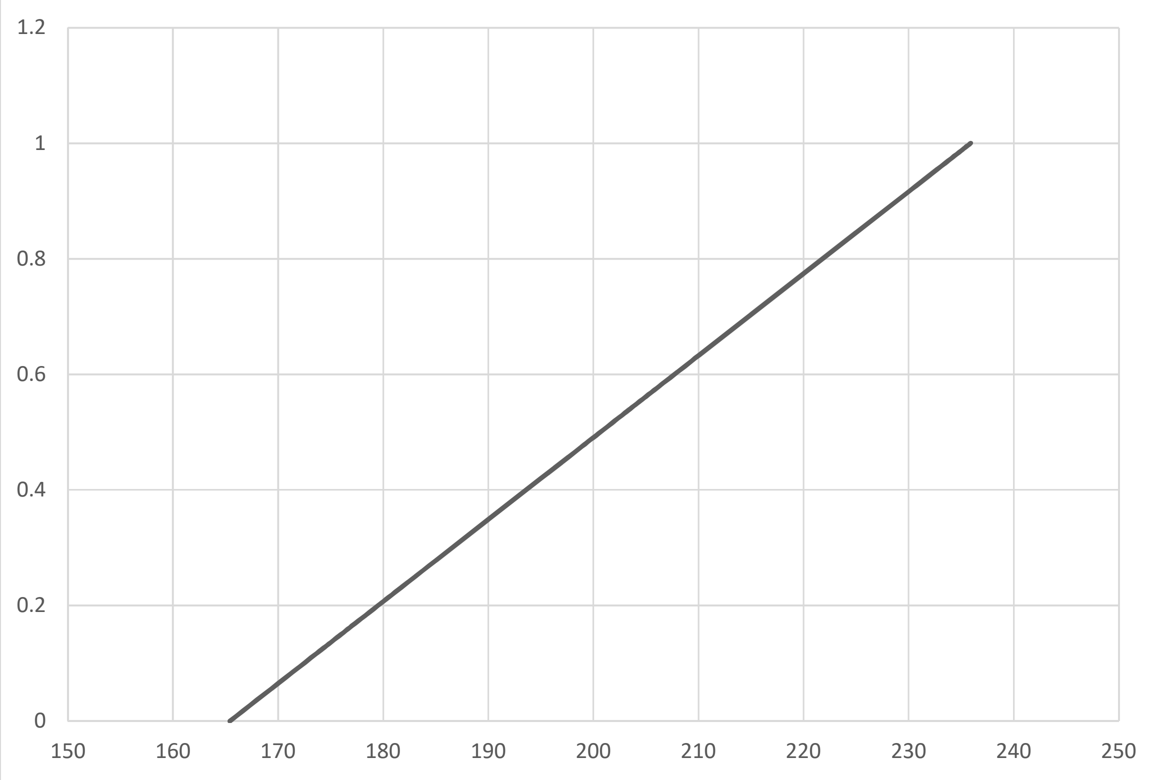

Fig. 1: The left reference function

Statistical acceptability of this fitted equation was tested using the $t-$ test where

$$t = \frac{|r| \sqrt{(n-2)}}{\sqrt{(1 - r^2)}}$$

r being the sample coefficient of correlation and n being the number of pairs of observations (x, z). Here $(n-2)$ is the degree of freedom concerned. The null hypothesis we would test is H_0: ρ = 0 against the two-sided alternative hypothesis H_1: ρ ≠ 0 where ρ is the population correlation coefficient.

It was found that the sample correlation coefficient r is 0.990143561, and the calculated value of t is 36.1377 for $n = 15$. The tabulated value of t for 13 degrees of freedom at 5% probability level of significance is 2.160. In fact, the tabulated value of t for 13 degrees of freedom at 1% probability level of significance is only 3.012. Accordingly, the calculated value of t is very much larger than the tabulated value of t , and therefore we reject the null hypothesis that the population correlation coefficient ρ = 0. Thus we conclude that the fitted equation is statistically highly acceptable.

In the same way, taking the different values of $Y_{(k)}$ as the kth observed value of an independent variable Y and the level of presence in the interval $[Y_{(k)}, Y_{(k+1)}]$ as the dependent variable W, we can fit the right reference function of the fuzzy number. As mentioned above, the right reference function was seen to first-degree polynomial $W = c + dY$. The parameters c and d were estimated using the method of least squares. The fitted equation (Fig. 2) was found to be

$$W = 5.319942 + (-0.017977)Y.$$

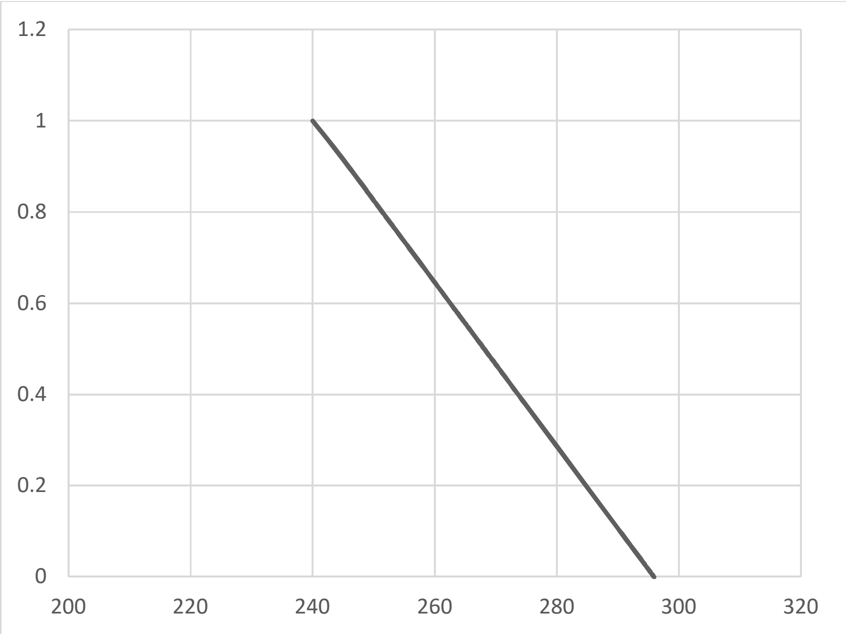

Fig. 2: The right reference function

Statistical acceptability of this equation too was tested as described in the earlier case. It was found that the sample correlation coefficient $r = -0.994019278$, and the calculated value of t is 32.81897 for $n = 15$. Once again, the calculated value of t is very much larger than the tabulated value of $t = 3.012$, and therefore we once again reject the null hypothesis that the population correlation coefficient ρ = 0. Thus we conclude that the fitted equation is statistically highly acceptable.

After combining the estimated left reference function, the constant function 1 in $235.90 \leq x \leq 240.30$, and the right reference function, we can construct the estimated membership curve (Fig. 3), where the left reference function increases from 0 to 1 and the right reference function decreases from 1 to 0.

The membership function of the estimated trapezoidal fuzzy number is

$$\begin{aligned} \mu_{A(x)} &= (-2.34622) + 0.014185x, 165.40 \leq x \leq 235.90, \\ &= 1, \text{ if } 235.90 \leq x \leq 240.30, \\ &= 5.319942 + (-0.017977)x, 240.30 \leq x \leq 295.93, \\ &= 0, \text{ otherwise.} \end{aligned}$$

Expressed in the classical style, the membership function can be seen to be *approximately*

$$\begin{aligned} \mu_{A(x)} &= \frac{x - 165.40}{235.90 - 165.40}, 165.40 \leq x \leq 235.90, \\ &= 1, \text{ if } 235.90 \leq x \leq 240.30, \\ &= \frac{295.93 - x}{295.93 - 240.30}, 240.30 \leq x \leq 295.93, \\ &= 0, \text{ otherwise.} \end{aligned}$$

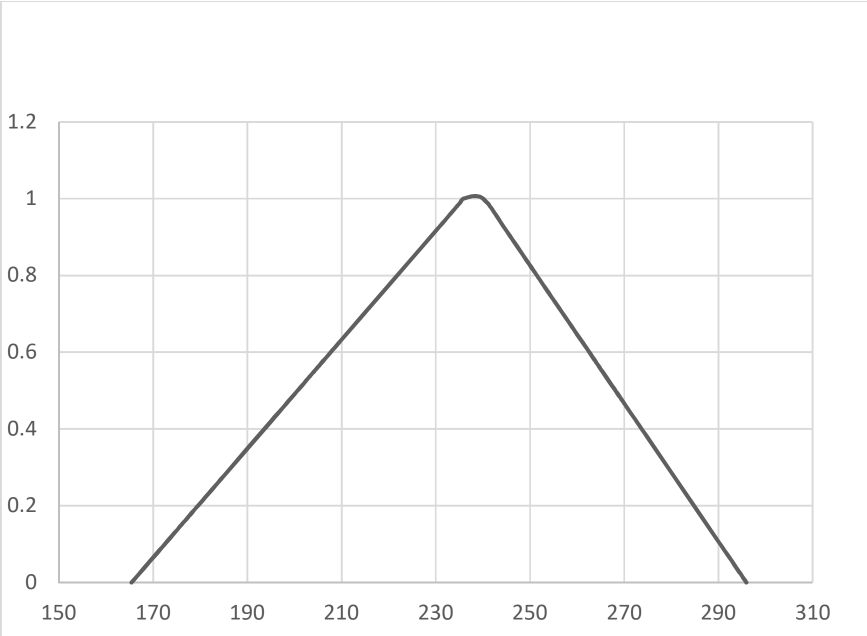

Fig. 3: The trapezoidal fuzzy number

Thus the probability distribution function in $165.40 \leq x \leq 235.90$ is say

$$F(x) = \frac{x - 165.40}{235.90 - 165.40}$$

and therefore the probability density function is $f(x) = \frac{dF(x)}{dx} = \frac{1}{70.5}$, signifying a uniform probability law in the interval concerned. Similarly, the probability distribution function in $240.30 \leq x \leq 295.93$ is

$$G(x) = 1 - \frac{295.93 - x}{295.93 - 240.30}$$

and therefore the probability density function is $g(x) = \frac{dG(x)}{dx} = \frac{1}{55.63}$, signifying a uniform probability law in the interval concerned.

We have thus seen that the membership function of the minimum prices is a probability distribution function of a random variable in the interval $165.40 \leq x \leq 235.90$, while the membership function of the maximum prices is a complementary probability distribution function of a random variable in the interval $240.30 \leq x \leq 295.93$. Within the interval $235.90 \leq x \leq 240.30$, the membership function is equal to 1. The fuzzy number expressing the daily price of Common Carp in a market in Guwahati is thus a trapezoidal fuzzy number.

4. Conclusions and discussions

In this article, from the daily prices of a particular variety of fish in a market of Guwahati, we have shown how a trapezoidal fuzzy number can express the minimum and the maximum daily prices. For perishable goods, such as raw fish and raw meat, the fuzzy number can be expected to be of this type.

For daily data on rainfall, for example, statistical analysis is sufficient, and therefore for rainfall data, the theory of probability can explain the situation. This is so because the daily data on rainfall are in terms of observations expressed as real numbers that involve a random error term following some probability law such as the normal probability law.

However, not all data are expressed as real numbers. Unlike rainfall data, data on daily temperature, stock prices, earthquake waveform etc. are of the interval type with a minimum and a maximum each day. In such cases, the minimum values of the intervals would follow some probability law and the maximum values of the intervals would follow another probability law. The probability laws may actually be similar but they have to be expressed with different parameters. Accordingly, if we have to deal with such interval type of data, we have to deal with two probability laws, and the two probability laws can define a normal fuzzy number. We have explained in this article that two probability laws are sufficient to define a law of fuzziness. Indeed, in ([1, 2, 3]) attempts were made to define a probability law on a space on which a law of fuzziness was already defined. As we have seen, on a space on which a normal law of fuzziness is defined, trying to define one single law of probability on that same space is of no meaning as far as the construction of the membership function of the fuzzy number is concerned.

It may be noted that Didier Dubois and Henri Prade did actually conclude that the membership function of a normal fuzzy number should be explained in terms of two functions – the Dubois-Prade left reference function and the Dubois-Prade right reference function. However, they did not pursue the matter any further. Using the operation of superimposition of equally fuzzy intervals, we have seen how for a normal fuzzy number, the left reference function can be explained as the distribution function of a random variable following a probability law and how the right reference function can be explained as a complementary distribution function of another random variable following another probability law. The triangular fuzzy number is said to be the simplest fuzzy number. In our view, the triangular fuzzy number can be explained with the help of two uniform probability laws, and the uniform probability law is the simplest probability law. Therefore, the simplicity of the triangular fuzzy numbers is rooted in the simplicity of the uniform probability law.

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Conflict of interest. The authors declare that they have no conflict of interest.

Authors' Contributions. All the authors contributed equally to this work.

REFERENCES

1. L.A.Zadeh, Fuzzy sets as a basis of theory of possibility, *Fuzzy Sets and Systems*, 1, (19780) 3-28.
2. G.J.Klir and J.F.Geer, Information-preserving probability-possibility transformations: recent developments, In: R. Lowen, M. Roubens (Eds), *Fuzzy Logic. Theory and Decision Library* (Series D: System Theory, Knowledge Engineering and Problem Solving), Vol. 12, Springer, (1993) 417-428.

3. D.Dubois, H.Prade and S.Sandri, On possibility/probability transformations, In: R.Lowen, M. Roubens (Eds), *Fuzzy Logic. Theory and Decision Library* (Series D: System Theory, Knowledge Engineering and Problem Solving), Vol 12, Springer, (1993) 103-112.
4. H.K.Baruah, The randomness-fuzziness consistency principle, *International Journal of Energy, Information and Communications*, 1(1) (2010) 37–48.
5. H.K.Baruah, The theory of fuzzy sets: beliefs and realities, *International Journal of Energy Information and Communications*, 2 (2) (2011) 1 – 22.
6. H.K.Baruah, Construction of the membership function of a fuzzy number, *ICIC Express Letters*, 5 (2) (2011) 545-549.
7. H.K.Baruah, In search of the root of fuzziness: the measure theoretic meaning of partial presence, *Annals of Fuzzy Mathematics and Informatics*, 2 (1), (2011) 57 – 68.
8. D.Das, A.Dutta, S.Mahanta and H.K.Baruah, Construction of normal fuzzy numbers: a case study with earthquake waveform data, *Journal of Process Management - New Technologies*, 1 (1) (2013) 1- 6.
9. A.Dutta, D.Das and H.K.Baruah, Construction of the membership function of normal fuzzy numbers, *Journal of Process Management - New Technologies*, 1(2) (2013) 82-86.
10. D.Das, S.Mahanta, R.Chutia and H.K.Baruah, Construction of normal fuzzy numbers: case studies with stock exchange data, *Annals of Fuzzy Mathematics and Informatics*, 5 (3) (2013) 505–514.
11. J. D. Gibbons and S. Chakraborti, Nonparametric Statistical Inference, Sixth Edition, CRC Press, Taylor & Francis, (2021).