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Nano Fuzzy b-open Sets in Nano fuzzy Topological Spaces

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Abstract. The purpose of this paper is to define and study a new class of Nano fuzzy open sets called Nano fuzzy b-open sets in Nano fuzzy topological spaces. We have tried to analyze the basic properties of Nano fuzzy b-open sets. We have also used these Nano fuzzy b-open sets to introduce a new type of Nano fuzzy continuous functions, which are called Nano fuzzy b-open continuous functions and their properties are investigated.

Keywords: Nano fuzzy regular open sets, Nano fuzzy regular closed sets, Nano fuzzy b-open sets, Nano fuzzy b-closed sets, Nano fuzzy b-continuity.

AMS Mathematics Subject Classification (2010): 54A40, 03E72

I. Introduction

Nano topology [6] is useful in the field of engineering as well as medical science. It gives an interdisciplinary forum that is focused on applications of Nano science and Nano structures. With the help of such Nano structures we can be able to design, develop, analyze and control different devices related to technologies in engineering and life science systems.

In 1996, b-open sets were introduced and studied by Andrijevic [2] in a topological space, which was a new class of generalized open sets. The classes of β-open sets [7], semi open sets [3,9] and pre-open sets [3,9] contain this new class of sets called b-open sets. Thivagar [6] has introduced Nano topology. Later, when Nano topology and different Nano forms of weakly open sets (Nano semi-open sets, Nano pre-open sets and Nano β-open sets) were introduced by Thivagar [6], it has given new dimensions to the development of the theory of Nano topology. Mashhour et al. [1] defined continuous and pre-continuous mappings. In 2016, Parimala et al. [8] extended this work by defining Nano b-open sets and Nano b-continuity in Nano topological spaces. In 2015, Tapi and Navalakhe [12] introduced bicontinuity in biclosure spaces. This work can be extended in Nano topological spaces further, in Nano fuzzy topological spaces.

After the introduction of fuzzy theory by Zadeh [5], fuzzification of topological structures became a powerful tool for many practical problems. This has given the motivation to further research. The base of Nano topology is the theory of rough sets and it has been observed by Yao [13] and Dubois [4] that there is connectivity between rough sets and fuzzy sets. This was the motivation behind the theory of Nano fuzzy topology [11]. Further, Nano fuzzy β-open sets [10], Nano fuzzy semi-open sets [10] and Nano fuzzy pre-open sets [10] were defined.

In this paper, we have introduced a new class of Nano fuzzy open sets in Nano fuzzy topological spaces called Nano fuzzy b-open sets and tried to find out the relation of this new class of sets with existing classes of Nano fuzzy open sets.

2. Preliminaries

In this section, we have included some definitions and results which are prerequisites for defining Nano fuzzy b-open sets.

Definition 2.1. [3,8,9] Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

1. Nano semi open if $A \subseteq NCl(NInt(A))$.
2. Nano pre-open if $A \subseteq NInt(NCl(A))$.
3. Nano α-open if $A \subseteq NInt(NCl(NInt(A)))$.

Definition 2.2. [8] If $(U, \tau_R(X))$ is a Nano topological space, then any subset A of U is called Nano regular open set if $A = NInt(NCl(A))$.

Definition 2.3. [8] Let $(U, \tau_R(X))$ is a Nano topological space, then any subset A of U is called Nano b-open set if $A \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$.

The complement of Nano b-open set is called Nano b-closed set.

Definition 2.4. [8] In a Nano topological space $(U, \tau_R(X))$, the Nan b-closure of a set A is denoted by $Ncl_b(A)$ and defined as the intersection of Nano b-closed sets including A. Similarly, the Nano b-interior is denoted by $NInt_b(A)$ and defined as the union of all Nano b-open sets which are included in A.

2.1. Properties of fuzzy approximation space [12]

Let R be an arbitrary relation from X to Y. The lower and upper approximation operators of a fuzzy set $\underline{\mathrm{R}}$ and $\overline{\mathrm{R}}$ satisfies the following properties: for all $\alpha, \beta \in F(X)$,

(FL1) $\underline{R}(\alpha) = (\overline{R}(\alpha^C))^C$

(FU1) $\overline{R}(\alpha) = (\underline{R}(\alpha^C))^C$

(FL2) $\underline{R}(\alpha \wedge \beta) = \underline{R}(\alpha) \wedge \underline{R}(\beta)$

(FU2) $\overline{R}(\alpha \vee \beta) = \overline{R}(\alpha) \vee \overline{R}(\beta)$

(FL3) $\alpha \leq \beta \Rightarrow \underline{R}(\alpha) \leq \underline{R}(\beta)$

(FU3) $\alpha \leq \beta \Rightarrow \overline{R}(\alpha) \leq \overline{R}(\beta)$

(FL4) $\underline{R}(\alpha \vee \beta) = \underline{R}(\alpha) \vee \underline{R}(\beta)$

(FU4) $\overline{R}(\alpha \wedge \beta) = \overline{R}(\alpha) \wedge \overline{R}(\beta)$

Definition 2.6. [11] Let X be a non-empty finite set, R be an equivalence relation on X, $\lambda \leq X$ be a fuzzy subset and $\tau_R(\lambda) = \{ 1_\lambda, 0_\lambda, \underline{R}(\lambda), \overline{R}(\lambda), Bd(\lambda)\}$. Then, by property (2.5) $\tau_R(\lambda)$ satisfies the following axioms

(i) 0_λ , $1_\lambda \in \tau_{(R)}(\lambda)$ where $0_\lambda: \lambda \to I$ denotes the null fuzzy sets and $1_\lambda: \lambda \to I$ denotes the
whole fuzzy set.

(ii) Arbitrary union of members of $\tau_{(R)}(\lambda\ )$ is a member of $\tau_{(R)}(\lambda)$.

(iii) Finite intersection of members of $\tau_{(R)}(\lambda)$ is a member of $\tau_{(R)}(\lambda)$.

That is, $\tau_{(R)}(\lambda)$ is a topology on X called the Nano fuzzy topology on X with respect to λ. We call $(X, \tau_{(R)}(\lambda))$ as the Nano fuzzy topological space (NFTS). The elements of the Nano fuzzy topological space that is $\tau_{(R)}(\lambda)$, are called Nano fuzzy open sets and elements of $[\tau_{(R)}(\lambda)]^C$ are called Nano fuzzy closed sets.

Definition 2.7. [11] Let $(X, \tau_{(R)}(\lambda))$ be a Nano fuzzy topological space with respect to λ where $\lambda \leq X$ and if $\mu \leq X$ (fuzzy subsets of X) then the Nano fuzzy interior of μ is defined as union of all Nano fuzzy open subsets of μ and it is denoted by $NfInt(\mu)$. That is, it is the largest Nano fuzzy open subset contained in μ.

Similarly, the Nano fuzzy closure of μ is defined as the intersection of all Nano fuzzy closed sets containing μ. It is denoted by $NfCl(\mu)$ and it is the smallest Nano fuzzy closed set containing μ.

Definition 2.8. [10] Let $(X, \tau_{(R)}(\lambda))$ be a Nano fuzzy topological space and $\mu \leq X$. Then μ is said to be

1. Nano fuzzy semi open if $\mu \leq NfCl(NfInt(\mu))$
2. Nano fuzzy pre-open if $\mu \leq NfInt(NfCl(\mu))$
3. Nano fuzzy α-open if $\mu \leq NfInt(NfCl(NfInt(\mu)))$

NFSO (X, λ), NFPO (X, λ) and $\tau_R^\alpha(\lambda)$ respectively denote the families of all Nano fuzzy semi-open, Nano fuzzy pre-open and Nano fuzzy α-open subsets of X.

3. Nano fuzzy b-open sets

Throughout this paper $(X, \tau_{(R)}(\lambda))$ represents Nano fuzzy topological space with respect to λ where $\lambda \leq X$ (a fuzzy subset of X) and R is an equivalence relation on X where X/R denotes the family of equivalence classes of X by R.

Definition 3.1. Let $(X, \tau_R(\mu))$ is a Nano fuzzy topological space with respect to μ where $\mu \leq X$ (a fuzzy subset of X) and R be an equivalence relation on X. Then μ is said to be Nano fuzzy b-open set if $\mu \leq NfCl(NfInt(\mu)) \vee NfInt(NfCl(\mu))$.
The complement of Nano fuzzy b-open sets is called Nano fuzzy b-closed set.

Theorem 3.2. Every Nano fuzzy open set is Nano fuzzy b-open.
Proof: Let μ be a Nano fuzzy open set in $(X, \tau_R(\mu))$. Then

$$\mu \leq NfCl(\mu) \tag{3.1}$$

And

$$NfInt(\mu) = \mu \tag{3.2}$$

We can observe from equation (3.1) that $NfInt(\mu) \leq NfInt(NfCl(\mu))$. From equations (3.1) and (3.2), we can conclude that $NfInt(\mu) \leq NfCl\big(NfInt(\mu)\big)$ which implies that $NfInt(\mu) \leq NfCl(NfInt(\mu)) \vee NfInt(NfCl(\mu))$ and using equation (3.2) we can write that $\mu \leq NfCl(NfInt(\mu)) \vee NfInt(NfCl(\mu))$. So μ is Nano fuzzy b-open.

Theorem 3.3. Every Nano fuzzy semi-open set is Nano fuzzy b-open.
Proof: Let μ be a Nano fuzzy semi-open set in $(X, \tau_R(\mu))$. Then $\mu \leq NfCl(NfInt((\mu))$. Hence $\mu \leq NfCl(NfInt(\mu)) \vee NfInt(NfCl(\mu))$ and μ is Nano fuzzy b-open in $(X, \tau_R(\mu))$.

The converse of the above theorem need not be true as shown by the following example.

Example 3.4. Let $X = \{a, b, c, d\}$ and $X/R = \big\{\{a, b\}, \{c\}, \{d\}\big\}$ be any equivalence relation on it. Let $\mu = \{a_{0.2}, b_{0.3}, c_{0.7}, d_1\}$ then $\underline{R}(\mu) = \{a_{0.2}, b_{0.2}, c_{0.7}, d_1\}$ and $\overline{R}(\mu) = \{a_{0.3}, b_{0.2}, c_{0.7}, d_1\}$ and boundary is $Bd(\mu) = \{a_{0.1}, b_{0.1}, c_0, d_0\}$. Then Nano fuzzy topology is $\tau_R(\mu) = \{0_\lambda, 1_\lambda, \underline{R}(\mu), \overline{R}(\mu), Bd(\mu)\}$. For $\gamma = \{a_{0.4}, b_{0.2}, c_{0.1}, d_{0.6}\}$. We have observed that γ is Nano fuzzy b-open set but it is not Nano fuzzy semi-open set.

Theorem 3.5. Every Nano fuzzy pre-open set is Nano fuzzy b-open.
Proof: Let μ be a Nano fuzzy semi-open set in $(X, \tau_R(\mu))$. Then $\mu \leq NfInt(NfCl(\mu))$. Hence $\mu \leq NfCl(NfInt(\mu)) \vee NfInt(NfCl(\mu))$ and μ is Nano fuzzy b-open in $(X, \tau_R(\mu))$.

Example 3.6. Let $X = \{a, b, c, d\}$ and $X/R = \big\{\{a, b\}, \{c\}, \{d\}\big\}$ be any equivalence relation on it. Let $\mu = \{a_{0.2}, b_{0.3}, c_{0.7}, d_1\}$ then $\underline{R}(\mu) = \{a_{0.2}, b_{0.2}, c_{0.7}, d_1\}$ and $\overline{R}(\mu) = \{a_{0.3}, b_{0.2}, c_{0.7}, d_1\}$ and boundary is $Bd(\mu) = \{a_{0.1}, b_{0.1}, c_0, d_0\}$. Then Nano fuzzy topology is $\tau_R(\mu) = \{0_\lambda, 1_\lambda, \underline{R}(\mu), \overline{R}(\mu), Bd(\mu)\}$. For $\gamma = \{a_{0.4}, b_{0.2}, c_{0.1}, d_{0.6}\}$. We have been observed that $\gamma \leq NfCl(NfInt(\gamma)) \vee NfInt(NfCl(\gamma))$ but $\gamma \nleq NfInt\big(NfCl(\gamma)\big)$. So, γ is Nano fuzzy b-open set but it is not Nano fuzzy pre-open set.

Theorem 3.7. Every Nano fuzzy α-open set is Nano fuzzy b-open.
Proof: Let μ be a Nano fuzzy α-open set in $(X, \tau_R(\mu))$. Then
$\mu \leq NfInt(NfCl(NfInt\big((\mu)\big))$.
Hence $\mu \leq NfInt(NfCl(NfInt\big((\mu)\big)) \leq NfCl(NfInt(\mu)) \vee NfInt(NfCl(\mu))$ and μ is Nano fuzzy b-open in $(X, \tau_R(\mu))$.
The converse of this theorem need not be true which can be seen with the help of the following example.

Example 3.8. Let $X = \{a, b, c, d\}$ and $X/R = \{\{a, b\}, \{c\}, \{d\}\}$ be any equivalence relation on it. Let $\mu = \{a_{0.2}, b_{0.3}, c_{0.7}, d_1\}$ then $\underline{R}(\mu) = \{a_{0.2}, b_{0.2}, c_{0.7}, d_1\}$ and $\overline{R}(\mu) = \{a_{0.3}, b_{0.2}, c_{0.7}, d_1\}$ and boundary is
$Bd(\mu) = \{a_{0.1}, b_{0.1}, c_0, d_0\}$. Then Nano fuzzy topology is
$\tau_R(\mu) = \{0_\lambda, 1_\lambda, \underline{R}(\mu), \overline{R}(\mu), Bd(\mu)\}$. For $\gamma = \{a_{0.4}, b_{0.2}, c_{0.1}, d_{0.6}\}$. It has been observed that $\gamma \nleq NfInt(NfCl\left(NfInt((\gamma))\right)$ but
$\gamma \leq NfCl(NfInt(\gamma)) \vee NfInt(NfCl(\gamma))$ and hence γ is Nano fuzzy b-open set but it is not Nano fuzzy α-open set.

Theorem 3.9. The arbitrary union of two Nano fuzzy b-open sets is a Nano fuzzy b-open set in $(X, \tau_R(\mu))$.
Proof: Let α and β be two Nano fuzzy b-open sets. Then
$\alpha \leq NfCl(NfInt(\alpha)) \vee NfInt(NfCl(\alpha))$ and
$\beta \leq NfCl(NfInt(\beta)) \vee NfInt(NfCl(\beta))$. So
$\alpha \vee \beta \leq NfCl(NfInt(\alpha)) \vee NfInt(NfCl(\alpha)) \vee NfCl(NfInt(\beta)) \vee NfInt(NfCl(\beta))$
$\leq NfCl(NfInt(\alpha \vee \beta)) \vee NfInt(NfCl(\alpha \vee \beta))$. Therefore, $\alpha \vee \beta$ is Nano fuzzy b-open.

Definition 3.10. The Nano fuzzy b-closure of a fuzzy set μ, denoted by $NfCl_b(\mu)$, is the intersection of Nano fuzzy b-closed sets containing μ. The Nano fuzzy b-interior of a set μ, is denoted by $NfInt_b(\mu)$, and it is the union of Nano fuzzy b-open sets contained in μ.

Theorem 3.11.
(1) $\mu \leq NfCl_b(\mu)$ and $\mu = NfCl_b(\mu)$ iff μ is a Nano fuzzy b-closed set.
(2) $NfInt_b(\mu) \leq \mu$ and $\mu = NfInt_b(\mu)$ iff μ is a Nano fuzzy b-open set.
Proof: Obvious.

Preposition 3.12. The intersection of Nano fuzzy α-open set and a Nano fuzzy b-open set is a Nano fuzzy b-open set.

4. Nano fuzzy b-continuity

Definition 4.1. Let $(X, \tau_R(\alpha))$ and $(Y, \tau_{R'}(\beta))$ be Nano fuzzy topological spaces. Then a mapping $f: (X, \tau_R(\alpha)) \to (Y, \tau_{R'}(\beta))$ is Nano fuzzy continuous on X if the inverse image of every Nano fuzzy b-open set in Y is a Nano fuzzy b-open in X.

Theorem 4.2. A function $f: (X, \tau_R(\alpha)) \to (Y, \tau_{R'}(\beta))$ is Nano fuzzy b-continuous if and only if the inverse image of every Nano fuzzy b-closed set in Y is Nano fuzzy closed in X.
Proof: Let f be Nano fuzzy b-continuous and γ be Nano fuzzy b-closed in Y. That is, $1_\beta - \gamma$ is Nano fuzzy b-open in Y where 1_β is the whole fuzzy set. Since, f is Nano fuzzy continuous, $f^{-1}(\gamma)$ is Nano fuzzy closed in X. Thus the inverse image of every Nano fuzzy b-closed set in Y is Nano fuzzy closed in X if f is Nano fuzzy continuous in X. Conversely, let the inverse image of every Nano fuzzy b-closed set in Y is Nano fuzzy closed in X. Let, α Be Nano fuzzy b-open in Y. Then $1_\beta - \alpha$ is Nano fuzzy b-closed in Y. Then $f^{-1}(1_\beta - \alpha)$ is Nano fuzzy b-closed in X. That is, $1_\beta - f^{-1}(\alpha)$ is Nano fuzzy closed in X.

Therefore, $f^{-1}(\alpha)$ is Nano fuzzy open in X. Thus, the inverse image of every Nano fuzzy b-open set in Y Nanis Nano fuzzy open in X. That is, f is Nano fuzzy b-continuous on X.

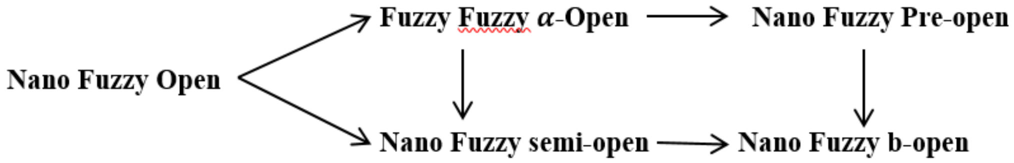
Theorem 4.3. A function $f: (X, \tau_R(\alpha)) \to (Y, \tau_{R'}(\beta))$ is Nano fuzzy b-continuous if and only if $f(NfCl_b(\mu)) \leq NfCl_b\big(f(\mu)\big)$ for every fuzzy subset $\mu\ of\ X$.
Proof: Let f be Nano fuzzy b-continuous and $\mu \leq X$. Then $f(\mu) \leq Y$. Obviously, $NfCl_b\big(f(\mu)\big)$ is Nano fuzzy b-closed in Y. Since, f is Nano fuzzy b-continuous, $f^{-1}\Big(NfCl_b\big(f(\mu)\big)\Big)$ is Nano fuzzy closed in X. Since $f(\mu) \leq NfCl_b\big(f(\mu)\big)$, $\mu \leq f^{-1}\Big(NfCl_b\big(f(\mu)\big)\Big)$. Thus, $f^{-1}\Big(NfCl_b\big(f(\mu)\big)\Big)$ is Nano fuzzy b-closed set containing μ. Therefore, $NfCl_b(\mu)\ \leq f^{-1}\Big(NfCl_b\big(f(\mu)\big)\Big)$. That is, $f(NfCl_b(\mu)) \leq NfCl_b\big(f(\mu)\big)$.
Conversely, let $f(NfCl_b(\mu)) \leq NfCl_b\big(f(\mu)\big)$ for every fuzzy subset $\mu\ of\ X$. If γ is Nano fuzzy b-closed in Y, since $f^{-1}(\gamma) \leq X$, $f(NfCl_b(f^{-1}(\gamma))) \leq$
$NfCl_b\Big(f\big(f^{-1}(\gamma)\big)\Big) \leq NfCl_b(\gamma)$. That is, $NfCl_b(f^{-1}(\gamma) \leq\ f^{-1}(NfCl_b(\gamma))$.
Therefore, $NfCl_b(f^{-1}(\gamma)) = f^{-1}(\gamma)$. Therefore, $f^{-1}(\gamma)$ is Nano fuzzy closed in X for every Nano fuzzy b-closed set γ in Y. That is, f is Nano fuzzy b-continuous.

Theorem 4.4. A function $f: (X, \tau_R(\alpha)) \to (Y, \tau_{R'}(\beta))$ is Nano fuzzy b-continuous if and only if $NfCl_b(f^{-1}(\gamma))\ \leq\ f^{-1}(NfCl_b(\gamma))$ for every fuzzy subset $\gamma\ of\ Y$.
Proof: Let f be Nano fuzzy b-continuous and $\gamma \leq Y$, $NfCl_b(\gamma)$ is Nano fuzzy b-closed in Y and hence $f^{-1}(NfCl_b(\gamma))$ is Nano fuzzy closed in X. Therefore,
$NfCl_b(f^{-1}(NfCl_b(\gamma)))= f^{-1}(NfCl_b(\gamma))$.
Since, $\gamma \leq NfCl_b(\gamma),\ f^{-1}(\gamma) \leq\ f^{-1}(NfCl_b(\gamma))$.
Therefore, $NfCl_b(f^{-1}(\gamma)) \leq\ NfCl_b(f^{-1}(NfCl_b(\gamma))) =\ f^{-1}(NfCl_b(\gamma))$. That is, $NfCl_b(f^{-1}(\gamma)) \leq\ f^{-1}(NfCl_b(\gamma))$.

Conversely, let γ be Nano fuzzy b-closed in Y. Then $NfCl_b(\gamma) = \gamma$. By assumption, $NfCl_b\big(f^{-1}(\gamma)\big) \leq\ f^{-1}\big(NfCl_b(\gamma)\big) = f^{-1}(\gamma)$.
Thus, $NfCl_b\big(f^{-1}(\gamma)\big) \leq f^{-1}(\gamma)$ but $f^{-1}(\gamma) \leq NfCl_b\big(f^{-1}(\gamma)\big)$.
Therefore, $NfCl_b\big(f^{-1}(\gamma)\big) = f^{-1}(\gamma)$. That is, $f^{-1}(\gamma)$ is Nano fuzzy closed in X for every Nano fuzzy b-closed in Y. Therefore, f is Nano fuzzy b-continuous on X.

5. Conclusion

In this paper, we have defined a new class of Nano fuzzy open/closed sets called Nano fuzzy b-open sets and Nano fuzzy b-closed sets. Also the theory has given new addition by defining a new class of Nano fuzzy continuous functions called Nano fuzzy b-continuous functions. In the process of developing the theory of Nano fuzzy topological spaces, these classes can play an important role. We hope we will be able to find more categories and types of such sets and functions which will provide us a platform to find their real life applications. These Nano fuzzy sets are inter-related to each other which has been observed in theorems also. This connection is shown in following figure:



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Conflict of interest. The authors declare that they have no conflict of interest.

Authors' Contributions. All the authors contributed equally to this work.

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