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Status-Nirmala Index and its Exponential of a Graph

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Abstract. In this paper, we introduce the status Nirmala index and status Nirmala exponential of a graph. We compute the status Nirmala index and its corresponding exponential of some standard graphs, wheel graphs and friendship graphs. Also we establish some properties of newly defined status Nirmal index.

Keywords: Status Nirmala index, status Nirmala exponential, graph

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*09, 05*C*92

1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1], for other undefined notations and terminologies.

The first and second status indices of a graph G were introduced by Ramane et al. in [2], and they are defined as

$$S_1(G)=\sum_{uv\in E(G)}[\sigma(u)+\sigma(v)], \qquad S_2(G)=\sum_{uv\in E(G)}\sigma_G(u)\sigma_G(v).$$

Recently, some status indices were studied, for example, in [3, 4, 5, 6, 7, 8, 9].

The Nirmala index was introduced by Kulli in [10] and defined it as

$$N(G)=\sum_{uv\in E(G)}\sqrt{d_G(u)+d_G(v)}.$$

Recently, some Nirmala indices were studied, for example, in [11, 12, 13, 14, 15, 16, 17, 18, 19].

Motivated by the definitions of the status and Nirmala indices, we introduce the status-Nirmala index of a graph and defined it as,

$$SN(G)=\sum_{uv\in E(G)}\sqrt{\sigma(u)+\sigma(v)}.$$

We also propose the status Nirmala exponential of a graph and it is defined as

$$SN(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\sigma(u)+\sigma(v)}}.$$

In this paper, we determine the status Nirmala index and it's exponential of certain graphs.

2. Results

2.1. Complete graph K_n on n vertices

Theorem 1. The status Nirmala index of a complete graph K_n is

$$SN(K_n) == \frac{n(n-1)^{3/2}}{\sqrt{2}}.$$

Proof: If K_n is a complete graph with n vertices, then $d_{K_n}(u) = n-1$ and □(u) = n – 1 for any vertex u in K_n. Thus

$$SN(K_n) = \sum_{uv\in E(K_n)} \sqrt{\sigma(u)+\sigma(v)} = \frac{n(n-1)}{2}\sqrt{(n-1)+(n-1)}$$

$$= \frac{n(n-1)^{3/2}}{\sqrt{2}}.$$

2.2. Cycle C_n on n vertices

Theorem 2. Let C_n be a cycle on n vertices. Then

$$SN(C_n) = \frac{n^2}{\sqrt{2}}, \qquad \text{if } n \text{ is even,}$$

$$= \frac{n(n-1)}{\sqrt{2}}, \qquad \text{if } n \text{ is odd.}$$

Proof: If C_n is a cycle with n vertices, then $d_{C_n}(u) = 2$ for every vertex u in C_n.

Case 1. Suppose n is even. Then $\sigma(u) = \frac{n^2}{4}$ for any vertex u in C_n. Therefore

$$SN(C_n) = \sum_{uv\in E(G)} \sqrt{\sigma(u)+\sigma(v)}$$

$$= \sum_{uv\in E(C_n)} \sqrt{\left(\frac{n^2}{4}\right)+\left(\frac{n^2}{4}\right)} = \frac{n^2}{\sqrt{2}}.$$

Case 2. Suppose n is odd. Then $\sigma(u) = \frac{n^2-1}{4}$ for any vertex u in C_n. Thus

$$SN(C_n) = \sum_{uv\in E(C_n)} \sqrt{\sigma(u)+\sigma(v)}$$

$$= \sum_{uv\in E(C_n)} \sqrt{\left(\frac{(n-1)^2}{4}\right)+\left(\frac{(n-1)^2}{4}\right)} = \frac{n(n-1)}{\sqrt{2}}.$$

3. Results for wheel graphs

A wheel W_n is the join of C_n and K_1. Clearly W_n has n+1vertices and $2n$ edges. A graph W_4 is depicted in Figure 1.

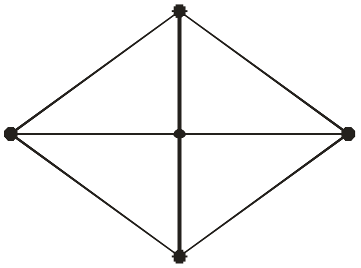

Figure 1: Wheel graph W_4

Let W_n be a wheel with n+1vertices and $2n$ edges. In W_n, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d_{W_n}(u) = d_{W_n}(v) = 3\}$, $\qquad |E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d_{W_n}(u) = 3,\ d_{W_n}(v) = n\}$, $\qquad |E_2| = n$.

Thus there are two types of status edges as given in Table 1.

$\sigma(u), \sigma(v) \setminus uv \in E(W_n)$	$(2n - 3, 2n - 3)$	$(n, 2n - 3)$
Number of edges	n	N

Table 1: Status edge partition of W_n

In the following theorem, we compute the status Nirmala index of a wheel graph W_n.

Theorem 3. The status Nirmala index of a wheel graph W_n is

$$SN(W_n) = n\sqrt{4n-6} + n\sqrt{3n-3}.$$

Proof: By definition and Table 1, we deduce

$$SN(W_n) = \sum_{uv \in E(W_n)} \sqrt{\sigma(u) + \sigma(v)}$$
$$= n\sqrt{(2n-3)+(2n-3)} + n\sqrt{n+(2n-3)}$$
$$= n\sqrt{4n-6} + n\sqrt{3n-3}.$$

In the next theorem, we compute the status Nirmala exponential of a wheel graph W_n.

Theorem 4. The status Nirmala exponential of a wheel graph W_n is

$$SN(W_n, x) = nx^{\sqrt{4n-6}} + nx^{\sqrt{3n-3}}.$$

Proof: From definition and by using Table 1, we have

$$SN(W_n, x) = \sum_{uv \in E(W_n)} x^{\sqrt{\sigma(u)+\sigma(v)}}$$
$$= nx^{\sqrt{(2n-3)+(2n-3)}} + nx^{\sqrt{n+(2n-3)}}$$

$$= nx^{\sqrt{4n-6}} + nx^{\sqrt{3n-3}}.$$

4. Results for friendship graphs

A friendship graph F_n, $n \geq 2$, is a graph that can be constructed by joining n copies of C_3 with a common vertex. A graph F_4 is presented in Figure 2.

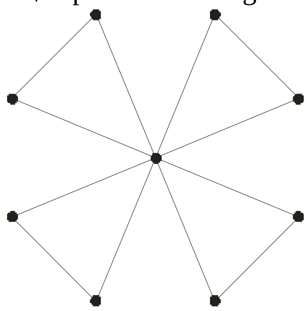

Figure 2: Friendship graph F_4

Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. By calculation, we obtain that there are two types of edges as follows:

$$E_1 = \{uv \in E(F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}, \qquad |E_1| = n.$$

$$E_2 = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}, \qquad |E_2| = 2n.$$

Therefore, in F_n, there are two types of status edges as given in Table 2.

$\sigma(u), \sigma(v) \setminus uv \in E(F_n)$	$(4n-2, 4n-2)$	$(2n, 4n-2)$
Number of edges	n	$2n$

Table 2: Status edge partition of F_n

In Theorem 5, we compute the status Nirmala index of a friendship graph F_n.

Theorem 5. The status Nirmala index of a wheel graph F_n is

$$SN(F_n) = n\sqrt{8n-4} + 2n\sqrt{6n-2}.$$

Proof: By using definition and Table 2, we deduce

$$SN(F_n) = \sum_{uv \in E(F_n)} \sqrt{\sigma(u) + \sigma(v)}$$

$$= n\sqrt{(4n-2)+(4n-2)} + n\sqrt{2n+(4n-2)}$$

$$= n\sqrt{8n-4} + 2n\sqrt{6n-2}.$$

In the following theorem, we compute the status Nirmala exponential of a friendship graph F_n.

Theorem 6. The status Nirmala exponential of a wheel graph F_n is

$$SN(F_n, x) = nx^{\sqrt{8n-4}} + 2nx^{\sqrt{6n-2}}.$$

Proof: From definition and by using Table 2, we have

$$SN(F_n, x) = \sum_{uv \in E(W_n)} x^{\sqrt{\sigma(u)+\sigma(v)}}$$
$$= nx^{\sqrt{(4n-2)+(4n-2)}} + 2nx^{\sqrt{2n+(4n-2)}}$$
$$= nx^{\sqrt{8n-4}} + 2nx^{\sqrt{6n-2}}.$$

5. Property of status nirmala index

In Theorem 7, we establish a property of $SN(G)$.

Theorem 7. Let G be a connected graph with n vertices and m edges. Then

$$SN(G) \le \sqrt{mS_1(G)}.$$

Proof: Using the Cauchy-Schwarz inequality, we obtain

$$\left(\sum_{uv \in E(G)} \sqrt{\sigma(u)+\sigma(v)}\right)^2 \le \sum_{uv \in E(G)} 1 \sum_{uv \in E(G)} (\sigma(u)+\sigma(v)).$$
$$= mS_1(G).$$

Thus $$SN(G) \le \sqrt{mS_1(G)}.$$

6. Conclusion

In this study, a novel invariant is considered which is the status Nirmala index. Also we have defined the status Nirmala exponential of a graph. Furthermore, the status Nirmala index and its corresponding exponential for certain standard graphs, wheel graphs, friendship graphs are determined.

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