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Relatively Prime Split Geodetic Number of a Graph

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Abstract. In this paper we introduce relatively prime split geodetic set of a graph G. A set $S \subseteq V(G)$ is said to be relatively prime split geodetic set in G if S is a relatively prime geodetic set and $< V(G) - S >$ is disconnected. The relatively prime split geodetic set is denoted by $g_{rps}(G)$- set. The minimum cardinality of relatively prime split geodetic set is the relatively prime split geodetic number and it is denoted by $g_{rps}(G)$.

Keywords: Geodetic set, geodetic number, prime split geodetic set, relatively prime, line graph.

AMS Mathematics Subject Classification (2010): 05C12

1. Introduction

By a graph $G = (V, E)$ we mean a finite, connected, undirected graph with neither loops nor multiple edges. The order $|V|$ and size $|E|$ of G are denoted by p and q respectively. For graph theoretic terminology we refer to West [7].

In a connected graph G, the distance between two vertices x and y is denoted by $d(x, y)$ and is defined as the length of a shortest $x - y$ path in G. If $e = \{u, v\}$ is an edge of a graph G with $\deg(u) = 1$ and $\deg(v) > 1$, then we call e a pendant edge, u a pendent vertex and v a support vertex. A set of vertices is said to be independent if no two vertices in it are adjacent. A vertex v of G is said to be an extreme vertex if the subgraph induced by its neighborhood is complete. For any set S of points of G, the induced sub graph $< S >$ is the maximal subgraph of G with point set S. Thus two points of S are adjacent in $< S >$ if and only if they are adjacent in G. An acyclic connected graph is called a tree. An $x - y$ path of length $d(x, y)$ is called geodesic. A vertex v is said to lie on a geodesic P if v is an internal vertex of P. The closed interval $I[x, y]$, consists of x, y and all vertices lying on some $x - y$ geodesic of G and for a non empty set $S \subseteq V(G), I[S] = \bigcup_{x,y \in S} I[x, y]$.

A set $S \subseteq V(G)$ in a connected graph is a geodetic set of G if $I[S] = V(G)$. The geodetic number of G denoted by $g(G)$, is the minimum cardinality of a geodetic set of G. The geodetic number of a disconnected graph is the sum of the geodetic number of its components. A geodetic set of cardinality $g(G)$ is called $g(G)$ – set. Various concepts inspired by geodetic set are introduced in [1, 4]. The concept relatively prime domination was introduced by C. Jayasekaran et. al [5]. The relatively prime geodetic number of a graphs was introduced by C. Jayasekaran et. al [6]. In [7, 8], r- Relatively Prime sets were studied. In this paper we define relatively prime split geodetic number of graphs.

Definition 1.1. [3] The line graph $L(G)$ of a graph G is the graph whose vertices are the edges of G and two vertices in $L(G)$ are adjacent if the corresponding edges of G are adjacent.

Definition 1.2. [5] A set $S \subseteq V(G)$ is said to be relatively prime dominating set of a graph G if it is a dominating set of G with at least two elements and for every pair of vertices u and v in S such that $(degu, deg\ v) = 1$. The minimum cardinatility of a relatively prime dominating set of G is called relatively prime domination number of G and it is denoted by $\gamma_{rpd}(G)$.

Definition 1.3. [8] A geodetic set S of a graph $G = (V, E)$ is a split geodetic set if the induced subgraph $< V(G) - S >$ is disconnected. The split geodetic number $g_s(G)$ of G is the minimum cardinality of a split geodetic set.

Definition 1.4. [2] The jewel graph J_n is a graph with vertex set $V(J_n) = \{u, x, v, y, v_i \,/\, 1 \le i \le n\}$ and $E(J_n) = \{ux, vx, uy, vy, xy, uv_i, vv_i / 1 \le i \le n\}$.

Definition 1.5. [9] The double fan DF_n consists of two fan graph that have a common path. In other words $DF_n = P_n + \overline{K}_2$.

Definition 1.6. [2] The n-pan graph is the graph obtained by joining a cycle C_n to a singleton graph K_1 with a bridge. It is denoted by P_{n_n}.

2. Some basic result

In this section we cite some results to be used in the sequel.

Theorem 2.1. [5] Each end vertices of a graph G belongs to relatively prime geodetic set of G.

Theorem 2.2. [5] Each relatively prime geodetic set of a graph contains its extreme vertices.

Theorem 2.3. [5] For a star graph $K_{1,n}$, $g_{rp}(K_{1,n}) = \begin{cases} 3 \text{ for } n = 2 \\ 0 \text{ for } n \ge 2 \end{cases}$.

Theorem 2.4. [5] For a bistar graph $B_{m,n}$, $g_{rp}(B_{m,n}) = \begin{cases} 3 \text{ for } m = n = 1 \\ 0 \text{ otherwise} \end{cases}$.

Theorem 2.5. [5] For a connected graph G of order n if $g_{rp}(G)$ exists, then $g(G) \le g_{rp}(G) \le n$.

Theorem 2.6. [5] For a wheel graph W_n $(n \geq 4)$, $g_{rp}(W_n) = \begin{cases} 4 \text{ if } n = 4 \\ 3 \text{ if } n = 5 \\ 0 \text{ otherwise} \end{cases}$.

3. Relatively prime split geodetic number of a graph

Definition 3.1. A set $S \subseteq V(G)$ is said to be relatively prime split geodetic set in G if S is a relatively prime geodetic set and $< V(G) - S >$ is disconnected. The relatively prime split geodetic set is denoted by $g_{rps}(G)$-set. The minimum cardinality of relatively prime split geodetic set is the relatively prime split geodetic number and it is denoted by $g_{rps}(G)$.

Example 3.2. Consider the graph in figure 1. The set $S = \{v_1, v_7\}$ is a mini mum geodetic set and $S' = \{v_1, v_5, v_7\}$ is a minimum relatively prime geodetic set. But $< V(G) - S' > = P_4$ is connected and hence S' cannot be a relatively prime split geodetic number. Now consider $S'' = \{v_1, v_4, v_5, v_7\}$. Then S'' is a minimum relatively prime geodetic set and $< V(G) - S'' >$ is disconnected. Here S'' is a relatively prime split geodetic set of G. Moreover it has the minimum cardinality with this property and hence $g_{rps}(G) = 4$.

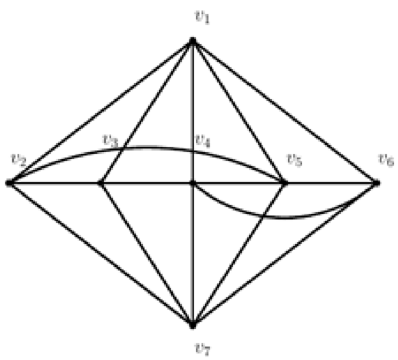

Figure 1: G

Theorem 3.3. Let G be a connected graph of order n. Then
(i) Each relatively prime split geodetic set of G contains its extreme vertices.
(ii) Each end vertex of G belongs to relatively prime spilt geodetic set of G.

Proof: Let G be a connected graph of order n. By definition, each relatively prime split geodetic set is a relatively prime geodetic set.

(i) Hence by Theorem 2.1, each relatively prime split geodetic set of G contains its extreme vertices.

(ii) Further by Theorem 2.2, each end vertex of G belongs to relatively prime split geodetic set of G.

Theorem 3.4. For a connected graph G of order n, if $g_{rps}(G)$ exists, then $g(G) \leq g_{rp}(G) \leq g_{rps}(G)$.

Proof: Let G be a connected graph of order n, such that $g_{rps}(G)$ exists. Since every relatively prime split geodetic set is a relatively prime geodetic set, $g_{rp}(G) \leq g_{rps}(G)$. By Theorem 2.5, $g(G) \leq g_{rp}(G)$. Thus, $g(G) \leq g_{rp}(G) \leq g_{rps}(G)$.

Remark 3.5. For the cycle graph C_n of odd order n, $(n \geq 5), g(C_n) = g_{rp}(C_n) = g_{rps}(C_n) = 3$. Hence all the inequalities in Theorem 3.4 become sharp. Now consider the graph G given in Figure 1. Here $S = \{v_1, v_7\}$ is a geodetic set of G and of minimum order and so $g(G) = 2$. $S' = \{v_1, v_6, v_7\}$ is a minimum relatively prime geodetic set of G and so $g_{rp}(G) = 3$. The set $S'' = \{v_1, v_4, v_6, v_7\}$ is a minimum relatively prime split geodetic set of G and so $g_{rps}(G) = 4$. Thus $g(G) < g_{rp}(G) < g_{rps}(G)$ and hence all the inequalities in Theorem 3.4 become strict.

Theorem 3.6. For cycle C_n of even order $n \geq 6, g_{rps}(C_n) = 3$.
Proof: Let $v_1\, v_2 \ldots v_n\, v_1$ be the cycle C_n of order n. Clearly $S = \{v_i, v_{i+\frac{n}{2}}\}$ where the suffices modulo n, is a minimum geodetic set of C_n and hence $g(C_n) = 2$. By definition, any relatively prime split geodetic set of C_n, must contain at least 3 vertices of C_n. Let $S' = \{v_i, v_{i+1}, v_{i+\frac{n}{2}}\}$ where the suffices modulo n. Then S' is a geodetic set. Now $d(v_i, v_{i+1}) = 1, d\left(v_i, v_{i+\frac{n}{2}}\right) = \frac{n}{2}$ and $d\left(v_{i+1}, v_{i+\frac{n}{2}}\right) = \frac{n}{2} - 1$. Clearly $(1, \frac{n}{2}) = (1, \frac{n}{2} - 1) = (\frac{n}{2}, \frac{n}{2} - 1) = 1$. Also $< V(G) - S' > = P_{\frac{n}{2}-2} \cup P_{\frac{n}{2}-1}$ which is disconnected. Therefore, S' is a minimum relatively prime split geodetic set of C_n and hence, $g_{rps}(C_n) = 3$.

Note 3.7. For $n = 4, g_{rps}(C_n) = 0$.

Theorem 3.8. For path graph P_n of order $n, g_{rps}(G) = 3$ for $n \geq 6$ and $n \neq 7$.
Proof: Let $v_1, v_2, \ldots, v_n$ be a path P_n. Let S be a minimum relatively prime geodetic set of P_n. By Theorem 2.1, the end vertices v_1 and v_n must be in any relatively prime geodetic set and hence $v_1, v_n \in S$.

Case 1. n is even
Subcase 1.1. $n = 4$
To get relatively prime split geodetic set, we must add one more vertex to S. Then S is either $\{v_1, v_2, v_4\}$ or $\{v_1, v_3, v_4\}$ and $< V(G) - S > = K_1$ which is connected. Thus $g_{rps}(P_n) = 0$.
Subcase 1.2. $n \geq 6$
Clearly $S = \{v_1, v_3, v_n\}$ is a geodetic set and $d(v_1, v_3) = 2, d(v_1, v_n) = n - 1$ and $d(v_3, v_n) = n - 3$. Since n is even, both $n - 1$ and $n - 3$ are odd and hence $(2, n - 1) = (2, n - 3) = (n - 1, n - 3) = 1$. Also $< V(G) - S > = K_1 \cup P_{n-4}$ which is disconnected. Therefore S is a minimum relatively prime split geodetic set of P_n and hence $g_{rps}(P_n) = 3$.

Case 2. n is odd
Subcase 2.1. $n = 3$
Clearly $S = \{v_1, v_2, v_3\}$ is the only relatively prime geodetic set and $V(P_n) - S = \phi$. Thus $g_{rps}(P_n) = 0$.
Subcase 2.2. $n = 5$

To get relatively prime split geodetic set, we must add one more vertex to S. Then $S = \{v_1, v_3, v_5\}$ and $< V(G) - S > = K_1 \cup K_1$ which is disconnected. Hence S is a split geodetic set. For $v_i, v_j \in S$, we have $d(v_i, v_j) = 2$ and therefore any two shortest distance between are not relatively prime. Hence it follows that $g_{rps}(P_n) = 0$.

Subcase 2.3. $n \geq 7$

Clearly $S = \{v_1, v_i, v_n\}$ where $i \equiv 0(mod\ 2)$ is a geodetic set and $d(v_1, v_n) = n - 1, d(v_1, v_i) = i - 1, d(v_i, v_n) = n - i$. Since n is odd, $n - 1$ is even and also i is even implies that both $i - 1$ and $n - i$ are odd. This implies that $(n - 1, i - 1) = (n - 1, n - i) = (i - 1, n - i) = 1$. Also $< V(G) - S > = K_{i-2} \cup P_{n-i-1}$, which is disconnected. Therefore S is a minimum relatively prime split geodetic set of P_n and hence $g_{rps}(P_n) = 3$.

Theorem 3.9. For a connected graph G, if $g_{rps}(G)$ exists, then $g_{rps}(G) \geq 3$.

Proof: Let $S \subseteq V(G)$ be a minimum relatively prime split geodetic set of G. Then S is a relatively prime geodetic set and hence by the definition $|S| \geq 3$. Hence $g_{rps}(G) \geq 3$.

Theorem 3.10. For the jewel graph $J_n, g_{rps}(J_n) = 3$.

Proof: Consider the 4 cycle $u_1 u_2\ u_3\ u_4\ u_1$. Join u_2 and u_4, new vertices $v_i, 1 \leq i \leq n$ and join v_i to both u_1 and u_3. The resulting graph G is the jewel graph J_n with vertex set $V(G) = \{u_1, u_2, u_3, u_4, v_i/1 \leq i \leq n\}$ and edge set
$E(G) = \{u_1u_2, u_1u_3, u_1u_4, u_2u_3, u_2u_4, u_3u_4, u_1v_i, u_3v_i/ 1 \leq i \leq n\}$.
Clearly $S = \{u_1, u_3\}$ is a minimum geodetic set. By definition, any relatively prime split geodetic set must contain at least three vertices. Let $S' = \{u_1, u_3, v_i/ 1 \leq i \leq n\}$. Clearly S' is a geodetic set and $d(u_1, u_3) = 2, d(u_1, v_i) = 1, d(u_3, v_i) = 1$ and $(2,1) = (1,1) = 1$. Also $< V(G) - S > = \overline{K}_n$ which is disconnected and hence S' is a minimum relatively prime split geodetic set of J_n. Thus $g_{rps}(J_n) = 3$.

Theorem 3.11. If G is either complete graph K_n or star graph $K_{1,n}$ or bistar graph $B_{m,n}$, then $g_{rps}(G) = 0$.

Proof: (i) We have $d(u, v) = 1$ for any two vertices u and v in $K_n, (n \geq 3)$. Let $S = V(K_n)$. Clearly S is the minimum relatively prime geodetic set of K_n and hence $g_{rp}\ (K_n) = n$. Since $V(K_n) - S = \phi$, there is no relatively prime split geodetic set of K_n and hence $g_{rps}(K_n) = 0$.

(ii) Let v be the central vertex and $u_i, 1 \leq i \leq n$ be the vertices of $K_{1,n}, (n \geq 2)$. Let S be a minimum relatively prime geodetic set of $K_{1,n}$. By Theorem 2.3, $|S| = 3$ for $n = 2$. In this case $K_{1,2} = P_3$ and $V(K_{1,2}) - S = \phi$. This implies that $K_{1,n}$ has no relatively prime split geodetic set and hence $g_{rps}(K_{1,n}) = 0$.

(iii) Let u_0 and v_0 be the vertices of P_2. Let $u_1, u_2, \ldots, u_m$ be the vertices attached with u_0 and let $v_1, v_2, \ldots, v_n$ be the vertices attached with v_0. The resultant graph is a bistar graph $B_{m,n}, (m, n \geq 1)$ with $V(B_{m,n}) = \{u_0, v_0, u_i, v_i, 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(B_{m,n}) = \{u_0\ v_0, u_0\ u_i, v_0\ v_j, 1 \leq i \leq m, 1 \leq j \leq n\}$. Let S be a minimum relatively prime geodetic set of $B_{m,n}$. By Theorem 2.4, $|S| = 3$ for $m = n = 1$. In this case $B_{1,1} = P_4$ and $< V(B_{m,n}) - S > = K_1$ which is connected and hence $B_{m,n}$ has no relatively prime split geodetic set. Thus $g_{rps}(B_{m,n}) = 0$.

Theorem 3.12. For a wheel $W_n = K_1 + C_{n-1}$ $(n \geq 4)$,

$$g_{rps}(W_n) = \begin{cases} 3 \text{ if } n = 5 \\ 0 \text{ otherwise} \end{cases}.$$

Proof: Let $v_1\, v_2 \ldots v_{n-1}\, v_1$ be the outer cycle C_{n-1} and v be the central vertex of W_n. Then $d(v_i, v_j) = 2$ for $1 \leq i \neq j \leq n-1$ and $\{i,j\} \neq \{1, n-1\}$. We consider the following cases.

Case 1. $n = 4$

Clearly, $W_4 = K_4$. By Theorem 3.11, $g_{rps}(W_4) = 0$.

Case 2. $n = 5$

Clearly $S = \{v_1, v_3\}$ is a minimum geodetic set. By definition, any relatively prime split geodetic set must contain at least three vertices. Let $S' = \{v_1, v_3, v\}$. Then S' is a geodetic set. Now $d(v_1, v_3) = 2, d(v_1, v) = 1, d(v_3, v) = 1$ and $(2,1) = (1,1) = 1$. Also $< V(W_n) - S' > = \overline{K}_2$ which is diconnected and hence S' is a minimum relatively prime split geodetic set. Therefore $g_{rps}(W_n) = 3$.

Case 3. $n \geq 6$

Any minimum geodetic set of W_n is $S_i = \{v_i, v_{i+2}, v_{i+4}, \ldots, v_{i+\left(\left\lceil\frac{n}{2}\right\rceil-1\right)2}\}$ and the subgraph $< V(W_n) - S_i > = K_{1,\left\lfloor\frac{n+1}{2}\right\rfloor}$ which is connected. Let $S_i' = \{v_i, v_{i+2}, v_{i+4}, \ldots, v_{i+\left(\left\lceil\frac{n}{2}\right\rceil-1\right)2}, v\}$. Then S_i' is a geodetic set and $< V(W_n) - S_i' > = \overline{K}_{\left\lfloor\frac{n+1}{2}\right\rfloor}$ which is disconnected. Now $d(v_i, v_{i+2}) = d(v_{i+2}, v_{i+4}) = \ldots = 2$ and hence any two of these shortest distances are not relatively prime. This implies that $g_{rps}(W_n) = 0$.

The result follows from cases 1, 2 and 3.

Theorem 3.13. For a double fan graph DF_n, $g_{rpns}(DF_n) = 3$ if $n \geq 3$.

Proof: Let $v_1\, v_2 \ldots v_n$ be a path. Add two vertices u_1 and u_2 which are adjacent to each $v_i, 1 \leq i \leq n$. The resultant graph is the double fan DF_n. Clearly $V(DF_n) = \left\{v_1, v_2, \ldots, v_n, u_1, u_2 \frac{}{1} \leq i \leq n\right\}$, $E(DF_n) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_1\, v_i, u_2 v_i \,/\, 1 \leq i \leq n\}$ and $|V(DF_n)| = n + 2, |E(DF_n)| = 3n - 1$. In $DF_n, S = \{u_1, u_2\}$ is a minimum geodetic set. To get relatively prime split geodetic set we add one more vertices to S. Let $S' = \{u_1, u_2, v_i\}$ where $2 \leq i \leq n-1$. Clearly S' is a geodetic set and $< V(G) - S' > = P_{n-i} \cup P_{i-1}$, which is disconnected. Now $d(u_1, u_2) = 2, d(u_1, v_1) = 1, d(u_2, v_1) = 1$ and $(1,2) = (1,1) = 1$. Hence S' is a minimum relatively prime split geodetic set of DF_n. Thus $g_{rpns}(DF_n) = 3$.

Theorem 3.14. For a 1 - pan graph P_{n_1} of even order $n \geq 4, g_{rps}(P_{n_1}) = 3$.

Proof: Let $v_1\, v_2 \ldots v_n\, v_1$ be a cycle C_n and let K_1 be the vertex v. Join u with v_1, we get a 1-pan graph P_{n_1}. Clearly $V(P_{n_1}) = \{u, v_1\}$ and $E(P_{n_1}) = \{u\, v_1, v_j\, v_{j+1} / 1 \leq j \leq n\}$ where the suffices modulo n. In $P_{n_1}, S = \{u, v_{1+\frac{n}{2}}\}$ is a minimum geodetic set. To get relatively prime split geodetic set we add one more vertices to S. Let $S' = \left\{u, v_1, v_{1+\frac{n}{2}}\right\}$. Clearly S' is a geodetic set and $< V(G) - S' > = P_{\frac{n-2}{2}} \cup P_{\frac{n-2}{2}}$ which is disconnected.

Now $d(u, v_1) = 1, d\left(u, v_{1+\frac{n}{2}}\right) = \frac{n}{2} + 1, d\left(v_1, v_{1+\frac{n}{2}}\right) = \frac{n}{2}$ and $(1, \frac{n}{2} + 1) = (1, \frac{n}{2}) = (\frac{n}{2} + 1, \frac{n}{2}) = 1$. Hence S' is a minimum relatively prime split geodetic set of P_{n_1}. Thus $g_{rps}(P_{n_1}) = 3$.

Theorem 3.15. For a Dumbbell graph Db_n, $g_{rps}(Db_n) = \begin{cases} 3 \text{ if } n \text{ is even} \\ 0 \text{ if } n \text{ is odd} \end{cases}$.

Proof: The dumbbell graph Db_n is obtained by joining two disjoint cycles $u_1 u_2 \ldots u_n u_1$ and $v_1 v_2 \ldots v_n v_1$ with an edge $u_1 v_1$. Then the vertex set $V(Db_n) = \{u_i, v_i / 1 \le i \le n\}$ and edge set $E(Db_n) = \{u_1 v_1, u_1 u_n, v_1 v_n, u_i u_{i+1}, v_i v_{i+1} / 1 \le i \le n - 1\}$. Clearly Db_n has $2n$ vertices and $2n + 1$ edges. Now we consider the following cases.

Case 1. n is even

Clearly $S = \{u_{\frac{n}{2}+1}, v_{\frac{n}{2}+1}\}$ where the suffices modulo n, is a minimum geodetic set. By definition any relatively prime split geodetic set must contains at least three vertices. Let $S' = \{u_{\frac{n}{2}+1}, u_{\frac{n}{2}-1}, v_{\frac{n}{2}+1}\}$ where the sufficies modulo n. Clearly S' is a geodetic set and $< V(G) - S' >$ is disconnected. Now $d(u_{\frac{n}{2}+1}, u_{\frac{n}{2}-1}) = 2, d(u_{\frac{n}{2}+1}, v_{\frac{n}{2}+1}) = \left\lceil \frac{n+1}{2} \right\rceil, d(u_{\frac{n}{2}-1}, v_{\frac{n}{2}+1}) = \left\lfloor \frac{n-1}{2} \right\rfloor$ and $(1, \left\lceil \frac{n+1}{2} \right\rceil) = (1, \left\lfloor \frac{n-1}{2} \right\rfloor) = (\left\lceil \frac{n+1}{2} \right\rceil, \left\lfloor \frac{n-1}{2} \right\rfloor) = 1$. Hence S' is a minimum relatively prime split geodetic set of Db_n. Thus $g_{rps}(Db_n) = 3$.

Case 2. n is odd

If $n = 3$, then $S = \{u_2, u_3, v_2, v_3\}$ is a minimum geodetic set and $< V(G) - S > = K_2$ is connected and hence $g_{rps}(Db_n) = 0$.

Let $n \ge 5$. Clearly, $S = \left\{u_{\left\lceil \frac{n}{2} \right\rceil}, u_{\left\lceil \frac{n}{2}+1 \right\rceil}, v_{\left\lceil \frac{n}{2} \right\rceil}, v_{\left\lceil \frac{n}{2}+1 \right\rceil}\right\}$ where the sufficies modulo n, is a minimum geodetic set and $< V(G) - S' >$ is connected. To get relatively prime split geodetic set we must add one more vertex to S. Let $S' = \{u_{\left\lceil \frac{n}{2} \right\rceil}, u_{\left\lceil \frac{n}{2}+1 \right\rceil}, u_1, v_{\left\lceil \frac{n}{2} \right\rceil}, v_{\left\lceil \frac{n}{2}+1 \right\rceil}\}$ where the sufficies modulo n, is a minimum geodetic set and $< V(G) - S' >$ is disconnected. Now $d\left(u_{\left\lceil \frac{n}{2} \right\rceil}, u_{\left\lceil \frac{n}{2}+1 \right\rceil}\right) = 1, d\left(u_{\left\lceil \frac{n}{2} \right\rceil}, u_1\right) = \left\lfloor \frac{n}{2} \right\rfloor, d\left(u_{\left\lceil \frac{n}{2}+1 \right\rceil}, u_1\right) = \left\lfloor \frac{n}{2} \right\rfloor, d\left(v_{\left\lceil \frac{n}{2} \right\rceil}, v_{\left\lceil \frac{n}{2}+1 \right\rceil}\right) = 1, d\left(u_1, v_{\left\lceil \frac{n}{2} \right\rceil}\right) = \left\lceil \frac{n}{2} \right\rceil, d(u_1, v_{\left\lceil \frac{n}{2}+1 \right\rceil}) = \left\lceil \frac{n}{2} \right\rceil$ where $u_i, v_j \in S'$ and hence the shortest distance between any two vertices in S' is either $\left\lceil \frac{n}{2} \right\rceil$ or $\left\lfloor \frac{n}{2} \right\rfloor$. It follows that $g_{rps}(Db_n) = 0$.

Theorem 3.16. For the complete bipartite $K_{m,n}$,

$$g_{rps}(K_{m,n}) = \begin{cases} 3 \text{ if } m = 2, n \ge 3 \text{ or } m \ge 3, n = 2 \\ 0, \quad \text{otherwise} \end{cases}.$$

Proof: Let $X = \{u_1, u_2, \ldots, u_m\}$ and $Y = \{v_1, v_2, \ldots, v_n\}$ be a partition of vertex set of $K_{m,n}$. Let S be a minimum relatively prime split geodetic set. We consider the following cases.

Case 1. $m = 1, n \ge 2$ or $n = 1, m \ge 2$

In both cases, the graph is a star graph. By Theorem 3.3(ii) the end vertices $\{v_1, v_2, \ldots, v_n\} \subseteq S$. Since $d(v_1, v_2) = 2$, $d(v_1, v_3) = 2$ and $(d(v_1, v_2), d(v_1, v_3)) = 2$, S cannot be a relatively prime split geodetic set of $K_{1,n}$. Thus $g_{rps}(K_{1,n}) = 0$.

Case 2. $m = n = 2$

Then the graph $K_{2,2}$ is C_4. By Note 3.7, $g_{rps}(G) = 0$.

Case 3. $m = 2$ and $n \geq 3$ or $m \geq 3$ and $n = 2$

Clearly $S = \{u_1, u_2\}$ is a minimum geodetic set and the subgraph $< V(G) - S > = \overline{K}_n$ which is disconnected and hence S is a split geodetic set. By defintion any relatively prime split geodetic set must contains atleast three vertices. Let $S' = \{u_1, u_2, v_k / 1 \leq k \leq n\}$ is a minimum geodetic set and the subgraph $< V(G) - S > = \overline{K}_{n-1}$ which is disconnected and hence S' is a split geodetic set. Since $d(u_1, u_2) = 2, d(u_1, v_k) = 1, d(u_2, v_k) = 1$ and $(1, 1) = (1,2) = 1$. Hence S' be a minimum relatively prime split geodetic set of $K_{m,n}$. Similarly $S^* = \{u_i, v_1, v_2 / 1 \leq i \leq m\}$ is a minimum relatively prime split geodetic set of $K_{m,n}$. Hence $g_{rps}(K_{m,n}) = |S'| = |S^*| = 3$.

Case 4. $m, n \geq 3$

Here $S = \{u_i, u_j, v_k, v_l\}$ where $1 \leq i \neq j \leq m, 1 \leq k \neq l \leq n$ is a minimum geodetic set and the subgraph $< V(G) - S > = C_4$ which is connected. To get $< V(G) - S >$ as disconnected, let $S' = \{u_1, u_2, \ldots, u_m\}$. Then S' is a minimum geodetic set and the subgraph $< V(G) - S' > = \overline{K}_n$ which is disconnected and hence S' is a split geodetic set. Since $d(u_i, u_j) = 2$ where $u_i, u_j \in S'$, S' is not relatively prime. Hence it follows that $g_{rps}(K_{m,n}) = 0$. The result follows from cases 1, 2, 3 and 4.

Theorem 3.17. For $m, n \geq 2$,

$$g_{rps}(P_m + P_n) = \begin{cases} 4 \text{ if } m = 3 \text{ and } n \geq 3 \text{ or } m \geq 3 \text{ and } n = 3 \\ 0, \text{ otherwise} \end{cases}.$$

Proof: Let P_m be the path $u_1 u_2 \ldots u_m$ and P_n be the path $v_1 v_2 \ldots v_n$. Let G be the graph $P_m + P_n$. Clearly $V(G) = \{u_i, v_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{u_i v_j, u_i u_{i+1}, v_j v_{j+1} / 1 \leq i \leq m, 1 \leq j \leq n\}$. Now we consider the following cases.

Case 1. $m = n = 2$.

Then the graph $P_2 + P_2 = K_4$. By Theorem 3.11, $g_{rps}(P_m + P_n) = 0$.

Case 2. $m = 2, n = 3$ *or* $m = 3, n = 2$

Clearly $S = \{v_1, v_3\}$ is a minimum geodetic set. To get relatively prime split geodetic set, we must add one more vertex. Let $S' = S \cup \{u\}$ where $u \in \{v_1, v_3, u_2\}$. Then S' is a minimum geodetic set and the subgraph $< V(G) - S' > = K_2$ is connected. Hence S' cannot be a minimum relatively prime split geodetic set and $g_{rps}(P_m + P_n) = 0$.

Case 3. $m = 3$ and $n \geq 3$ or $m \geq 3$ and $n = 3$.

Without loss of generality, let $m = 3$ and $n \geq 3$. Clearly $S = \{u_1, u_3\}$ is a minimum geodetic set. To get relatively prime geodetic set, we must add one more vertex. Let $S_i' = \{u_1, u_3, v_i / 2 \leq i \leq n-1\}$. Then S_i' is a minimum geodetic set and the subgraph $< V(G) - S_i' > = P_3$ is connected. To get relatively prime split geodetic set, we must add one more vertex. Let $S_i'' = \{u_1, u_2, u_3, v_i / 2 \leq i \leq n-1\}$. Then S_i'' is a minimum geodetic set and the subgraph $< V(G) - S_i'' > = K_1 \cup P_{i-1} \cup P_{m-i}$ is disconnected. Now $d(u_1, u_2) = 1, d(u_1, u_3) = 2, d(u_1, v_i) = 2, d(u_2, u_3) = 1, d(u_2, v_i) =$

$1, d(u_3, v_i) = 1$ and $(1,1) = (1,2) = 1$. Hence S_i'' is a minimum relatively prime split geodetic set. Then $g_{rps}(P_m + P_n) = 4$.

Case 4. $m \geq 4$ and $n \geq 1, n \neq 3$

Clearly $S = \{v_1, v_3, \dots, v_{n-2}, v_n\}$ for n is odd and $S = \{v_1, v_3, \dots, v_{n-1}, v_n\}$ for n is even is a minimum geodetic set of $P_m + P_n$. Here $d(v_j, v_k) = 2$ where $v_j, v_k \in S$ and hence the shortest distance between any two vertices in S is 2. It follows that $g_{rps}(P_m + P_n) = 0$. The result follows from cases 1, 2, 3 and 4.

Theorem 3.18. For $m, n \geq 1$,

$$g_{rps}(C_m + K_n) = \begin{cases} m+n-2 \text{ if } m = 4 \text{ and } n \geq 1 \\ 0, \quad \text{otherwise} \end{cases}.$$

Proof: Let $v_1\, v_2 \dots v_m\, v_1$ be the vertices of C_m. Let $u_1, u_2, \dots, u_n$ be the vertices of K_n. Clearly $V(C_m + K_n) = \{v_i, u_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(C_m + K_n) = \{v_i\, u_j, v_i\, v_{i+1}, u_i\, u_j / 1 \leq i \leq m, 1 \leq j \leq n\}$. Now we consider the following cases.

Case 1. m is even and $n \geq 1$

Subcase 1.1. $m = 4$ and $n \geq 1$

For $1 \leq i \leq 4$, $S_i = \{v_i, v_{i+\frac{m}{2}}\}$ is a minimum geodetic set of $C_4 + K_n$ and $< V(G) - S_i > = K_{n+2} - \{e\}$ which is connected where $e = v_{i-1}\, v_{i+1}$. For $< V(G) - S_i >$ to be disconnected, we must include all vertices of K_n. Let $S_i' = \{u_1, u_2, \dots, u_n, v_i, v_{i+\frac{m}{2}}\}$. Then S_i' is a geodetic set and $< V(G) - S_i' > = 2\, K_1$ which is disconnected. Now $d(u_i, u_j) = 1, d(u_i, v_i) = 1, d(u_j, v_{i+\frac{n}{2}}) = 1, d(v_i, v_{i+\frac{n}{2}}) = 2$ and $(1,\ 2) = (1,1) = 1$. Hence S_i' is a minimum relatively prime split geodetic set and so $g_{rps}(C_4 + K_n) = m + n - 2$.

Subcase 1.2. m ≥ 6 and $n \geq 1$

Clearly $S_i = \{v_1, v_3, \dots, v_{m-1}\}$ is a minimum geodetic set of $C_m + K_n$ and $< V(G) - S_i >$ is connected. For $< V(G) - S_i >$ to be disconnected, we must include all vertices of K_n. Then $S_i' = \{u_1, u_2, \dots, u_n, v_1, v_3, \dots, v_{m-1}\}$ is geodetic set and the subgraph $< V(G) - S_i' > = (n-2)K_1$ which is disconnected. Now $d(v_k, v_l) = 2, d(u_i, v_k) = 1, d(u_i, v_l) = 1, d(u_j, v_k) = 1, d(u_j, v_l) = 1, d(u_i, u_j) = 1$ where $v_j, v_k \in S_i'$ and hence the shortest distance between any two vertices in S_i' is 2. It follows that $g_{rps}(C_m + K_n) = 0$.

Case 2. m is odd and $n \geq 1$

Subcase 2.1. $m = 3$ and $n \geq 1$

Clearly $C_m + K_n = K_m + K_n = K_{m+n}$. By Theorem 3.11, $g_{rps}(C_m + K_n) = 0$.

Subcase 2.2. $m \geq 5$ and $n \geq 1$

Clearly $S_i = \{v_1, v_3, \dots v_{m-2}, v_{m-1}\}$ is a minimum geodetic set of $C_m + K_n$ and $< V(G) - S_i >$ is connected. For $< V(G) - S_i >$ to be disconnected, we must include all vertices of K_n . Then $S' = \{u_1, u_2, \dots, u_n, v_1, v_3, \dots v_{m-2}, v_{m-1}\}$ is geodetic set and the subgraph $< V(G) - S_i' > = (n-2)K_1$ which is disconnected. Now $d(v_k, v_l) = 2, d(u_i, v_k) = 1, d(u_i, v_l) = 1, d(u_j, v_k) = 1, d(u_j, v_l) = 1, d(u_i, u_j) = 1$ where $v_j, v_k \in S_i'$ and hence the shortest distance between any two vertices in S_i' is 2. It follows that $g_{rps}(C_m + K_n) = 0$. The result follows from cases 1 and 2.

Theorem 3.19. For cycle C_n of even order $n \geq 6, g_{rps}(C_n) = \frac{n}{\alpha_0(C_n)} + 1$ where $\alpha_0(C_n)$ is the vertex covering number of G.

Proof: Let $v_1\, v_2 \ldots v_n\, v_1$ be the cycle C_n of even order n and let $\alpha_0(C_n)$ be the vertex covering number of G. Clearly $S = \{v_i, v_{i+\frac{n}{2}}\}$ is a minimum geodetic set of C_n. We have by Theorem 3.6, $g_{rps}(C_n) = 3$. Also vertex covering number $\alpha_0(C_n) = \frac{n}{2}$. Hence $g_{rps}(C_n) = \frac{n}{\frac{n}{2}} + 1 = \frac{n}{\alpha_0(C_n)} + 1$.

Theorem 3.20. Let $L(C_n)$ be the line graph of C_n of even order n. Then $g_{rps}(L(C_n)) = 3$ for $n \geq 6$.

Proof: We have $L(C_n) = C_n$. The result follows from Theorem 3.6.

Theorem 3.21. Let $L(P_n)$ be the line graph of P_n. Then $g_{rps}(L(P_n)) = 3$ for $n \geq 9$.

Proof: We have $L(P_n) = P_{n-1}$. By Theorem 3.8, $g_{rps}(L(P_n)) = g_{rps}(P_{n-1}) = 3$ for $n - 1 \geq 8$ and hence $n \geq 9$.

Theorem 3.22. Let $L(K_{1,n})$ be the line graph of $K_{1,n}$. Then $g_{rps}(L(K_{1,n})) = 0$.

Proof: We have $L(K_{1,n}) = K_n$. By Theorem 3.11, $g_{rps}(L(K_{1,n})) = g_{rps}(K_n)$ hence $L(K_{1,n}) = 0$.

4. Conclusion

In this paper, we have found the relatively prime split geodetic number of some standard graphs like cycle graph, path graph, wheel graph, double fan graph, 1-pan graph, jewel graph, and complete bipartite graph.

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