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Analytical Approach to Fractional Fisher Equations by Laplace-Adomian Decomposition Method

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Abstract. This article implements the Laplace-Adomian decomposition method to obtain approximate analytical solutions in series form for non-linear time-fractional Fisher's equations with initial conditions. The fractional derivatives are given in the sense of Caputo. In addition, the results of this investigation are represented graphically, and they are simple yet highly accurate and compare favourably with the solutions reported in the earliest literature.

Keywords: Caputo fractional derivative; Fisher's equations of fractional order; Laplace transform; Laplace-Adomian decomposition method.

AMS Mathematics Subject Classification (2010): 34A20, 34A08, 35R11, 44A10

1. Introduction

Fractional calculus is a powerful tool for solving problems in many branches of science and technology, including control engineering, physics, signal processing, mathematical biology, viscoelasticity, electromagnetics, and mathematical physics. Fractional partial differential equations (FPDEs) have lately sparked a significant interest in mathematics and its applications. Scientists have used them to model a wide range of chemical, biological, and physical processes [14,15,16]. Nonlinear FPDEs may be solved analytically using a number of different approaches, such as the Adomian decomposition method (ADM) [22], the Homotopy perturbation method (HPM) [23], the Homotopy perturbation transform method (HPTM) [8], the differential transform method (DTM) [12], the Homotopy analysis method (HAM) [7], and the iterative Laplace transform method (ILTM) [1,18,19] and so on.

In 2001, Khuri [10] presented a novel approach to approximating the solution of a class of non-linear differential equations termed the Laplace Adomian decomposition method (LADM). The Volterra integro-differential equations [21], Burger differential equations [13], and Kundu Eckhaus differential equations [5] have been solved using the LADM method. In order to achieve an approximate analytical solution of linear and non-linear fractional diffusion-wave equations, Jafari *et al.* [6] first adopted the LADM technique. In recent years, the LADM has been utilized to solve fractional Telegraph equations [9], fractional Zakharov-Kuznetsov equations [17], and fractional Klien-Gordon equations [2]. The LADM approach is rid of any small or large parameters and has advantages over other approximation approaches such as perturbation. LADM requires neither discretization nor linearization, in contrast to other analytical approaches. Consequently, the outcomes produced by LADM are more efficient and realistic.

As a model for the propagation of a mutant gene, Fisher introduced the classical Fisher's equation, a partial differential equation with constant coefficients as

$$u_t(x,t) = u_{xx}(x,t) + u(x,t)\big(1-u(x,t)\big). \tag{1}$$

In this model, $u(x,t)$ indicates the population density, and $u(u-1)$ denotes the logistic form. This equation appears in chemical kinetics and population dynamics, which encompasses problems like the non-linear evolution of a population in a one-dimensional habitat and the neutron population in a nuclear reaction. In addition, the same equation arises in models of logistic population growth, flame propagation, neurophysiology, autocatalytic chemical reactions, and branching Brownian motion processes.

In this work, the time-fractional model for Fisher's equation may be represented as [1]

$$D_t^{\alpha} u(x,t) = u_{xx}(x,t) + \lambda u(x,t)\big(1-u(x,t)\big), 0 < \alpha \le 1, \tag{2}$$

where $D_t^{\alpha} u(x,t)$ denotes the Caputo fractional derivative of order α and λ is a real parameter.

The key contribution of this paper is to extend the Laplace-Adomian decomposition technique (LADM) in order to construct an approximate analytical solution for the non-linear time-fractional Fisher's equations with initial conditions.

2. Preliminaries

In this part, we present the basic definitions of fractional calculus and the sophisticated properties of the Laplace transform theory.

(a) The fractional derivative of $u(x,t)$ in the Caputo sense is defined as [11, 14]

$$D_t^{\alpha} u(x,t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\eta)^{m-\alpha-1} u^{(m)}(x,\eta)\, d\eta, \tag{3}$$
$$m-1 < \alpha \le m, m \in N,$$

(b) The Laplace transform of a function $f(x)$, $x > 0$ is defined as [20]

$$L[f(x)] = F(s) = \int_0^{\infty} e^{-sx} f(x)\, dx, \tag{4}$$

where s is real or complex number.

(c) The Laplace transform of the Caputo fractional derivative is defined as [11, 14]

$$L\left[D_t^{\alpha}u\,(x,t)\right]=s^{\alpha}L[u(x,t)]-\sum_{k=o}^{m-1}u^{(k)}(x,0)\,s^{\alpha-k-1},\qquad m-1<\alpha\leq m, m\in N, \tag{5}$$

where $u^{(k)}(x,0)$ is the k-order derivative of $u\,(x,t)$ with respect to t at $t=0$.

3. Basic idea of Laplace-Adomian decomposition method

To illustrate the fundamental concept of the Laplace-Adomian decomposition method [6], we take a general fractional partial differential equation that may be expressed in operator form as

$$D_t^{\alpha}u\left(x,t\right)+Ru\left(x,t\right)+Nu\left(x,t\right)=g\left(x,t\right), m-1<\alpha\leq m,\ \ m\in N, \tag{6}$$

$$u^{(k)}(x,0)=h_k(x),\ k=0,1,2,...,m-1\,, \tag{7}$$

where $D_t^{\alpha}u(x,t)$ is the Caputo fractional derivative of order α, $m-1<\alpha\leq m$, defined by equation (3), R is a linear operator which might include other fractional derivatives of order less than α, N is a non-linear operator which also might include other fractional derivatives of order less than α and $g\left(x,t\right)$ is a known analytic function.

Applying the Laplace transform to equation (6), we have

$$L\left[D_t^{\alpha}u\left(x,t\right)\right]+L\left[Ru\left(x,t\right)+Nu\left(x,t\right)\right]=L\left[g\left(x,t\right)\right]. \tag{8}$$

Using the equation (5), we get

$$L[u(x,t)]=\frac{1}{s^{\alpha}}\sum_{k=0}^{m-1}s^{\alpha-1-k}u^{(k)}(x,0)+\frac{1}{s^{\alpha}}L[g(x,t)] -\frac{1}{s^{\alpha}}L[Ru(x,t)+N\,u(x,t)]. \tag{9}$$

Applying inverse Laplace transform to the equation (9), we obtain

$$u(x,t)=L^{-1}\left[\frac{1}{s^{\alpha}}\left(\sum_{k=0}^{m-1}s^{\alpha-1-k}u^{(k)}(x,0)+L[g(x,t)]\right)\right] -L^{-1}\left[\frac{1}{s^{\alpha}}L[Ru(x,t)+Nu(x,t)]\right]. \tag{10}$$

The ADM solution $u\left(x,t\right)$ is represented by the following infinite series

$$u(x,t)=\sum_{n=0}^{\infty}u_n\,(x,t), \tag{11}$$

and the non-linear term is decomposed as follows

$$Nu(x,t)=\sum_{n=0}^{\infty}A_n\quad, \tag{12}$$

where A_n are the Adomian polynomials given by

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{n} \lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, ... \tag{13}$$

Substituting equations (11) and (13) into equation (10), we get

$$\begin{aligned}\sum_{n=0}^{\infty} u_n(x,t) = L^{-1}\left[\frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-1-k} u^{(k)}(x,0) + \frac{1}{s^\alpha} L[g(x,t)]\right] \\ - L^{-1}\left[\frac{1}{s^\alpha} L\left(R\left(\sum_{n=0}^{\infty} u_n(x,t)\right) + \sum_{n=0}^{\infty} A_n\right)\right].\end{aligned} \tag{14}$$

Using the Adomian method, we construct the elegant form of formal recurrence relations as

$$\left.\begin{aligned} u_0(x,t) &= L^{-1}\left[\frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-1-k} u^{(k)}(x,0) + \frac{1}{s^\alpha} L[g(x,t)]\right], \\ u_{n+1}(x,t) &= -L^{-1}\left[\frac{1}{s^\alpha} L\left(R\left(u_n(x,t)\right) + A_n\right)\right], \; n = 0, 1, 2, ..., \end{aligned}\right\}. \tag{15}$$

Therefore, the approximate analytical solution of equations (6) and (7) in truncated series form is given by

$$u(x,t) \cong \lim_{N\to\infty} \sum_{m=0}^{N} u_m(x,t). \tag{16}$$

In general, the solutions in the aforementioned series form converge rapidly. The classical approach to convergence of this type of series has been presented by Cherruault and Adomian [3] and Cherruault *et al.* [4].

4. Implementation of the Laplace-Adomian decomposition method

In this part, the above-mentioned reliable method is implemented to solve the non-linear time-fractional Fisher's equations with initial conditions.

Example 1. Consider the following non-linear Fisher’s equation concerning the time fractional derivative, given by [1, 12]

$$D_t^\alpha u = \frac{\partial^2 u}{\partial t^2} + 6u(1-u), \quad 0 < \alpha \le 1, \tag{17}$$

with the initial condition

$$u(x,0) = \frac{1}{\left(1+e^x\right)^2}, \tag{18}$$

where $D_t^\alpha u(x,t)$ is the Caputo fractional derivative of order α given by equation (3).

Taking the Laplace transform of the above equation (17) and making use of the result given by equation (18), we have

$$L[u(x,t)] = \frac{1}{s}\frac{1}{\left(1+e^x\right)^2} + \frac{1}{s^\alpha}\left[L\left(\frac{\partial^2 u}{\partial x^2} + 6u(1-u)\right)\right]. \tag{19}$$

Applying inverse Laplace transform to the equation (19), we obtain

$$u(x,t)=\frac{1}{\left(1+e^{x}\right)^{2}}+L^{-1}\left[\frac{1}{s^{\alpha}}L\left(\frac{\partial^{2}u}{\partial x^{2}}+6u(1-u)\right)\right]. \tag{20}$$

Substituting the results from equations (11) and (12) in the equation (20) and making use of the results given by the equation (15), we determine the components of the LADM solution as follows

$$u_{0}(x,t)=\frac{1}{\left(1+e^{x}\right)^{2}}, \tag{21}$$

$$u_{n+1}(x,t)=\ L^{-1}\left[\frac{1}{s^{\alpha}}L\left(\frac{\partial^{2}u_{0}}{\partial x^{2}}\right)\right]+L^{-1}\left[\frac{1}{s^{\alpha}}L\left(A_{n}\right)\right], n=0,1,2,..., \tag{22}$$

where A_n are the Adomian polynomials for the non-linear term $Nu=6u(1-u)$.
Now, for $n=0,1,2,...,$ and using equations (13) and (22), we have

$$A_{0}=\frac{6e^{x}(2+e^{x})}{\left(1+e^{x}\right)^{4}}, \tag{23}$$

$$u_{1}(x,t)=10\frac{e^{x}}{\left(1+e^{x}\right)^{3}}\frac{t^{\alpha}}{\Gamma(\alpha+1)}, \tag{24}$$

$$A_{1}=\frac{60\left(e^{3x}+2e^{2x}-e^{x}\right)}{\left(1+e^{x}\right)^{5}}\frac{t^{\alpha}}{\Gamma(\alpha+1)}, \tag{25}$$

$$u_{2}(x,t)=\frac{50\,e^{x}\left(-1+e^{2x}\right)}{\left(1+e^{x}\right)^{4}}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}, \tag{26}$$

$$A_{2}=\frac{50e^{x}}{\left(1+e^{x}\right)^{6}}\left[6(-1+e^{2x})(-1+2e^{x}+e^{2x})-12e^{x}\frac{\Gamma(2\alpha+1)}{(\alpha+1)^{2}}\right]\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}, \tag{27}$$

$$u_{3}(x,t)=\frac{50e^{x}}{\left(1+e^{x}\right)^{6}}\left[5-6e^{x}-15e^{2x}+20e^{3x}-12e^{x}\frac{\Gamma(2\alpha+1)}{\left(\Gamma(\alpha+1)\right)^{2}}\right]\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \tag{28}$$

and so on. The remaining components may be obtained similarly.
Thus, the series-form approximate analytical solution can be obtained as

$$u\left(x,t\right)=u_{0}(x,t)+u_{1}(x,t)+u_{2}(x,t)+u_{3}(x,t)+...$$

$$= \frac{1}{\left(1+e^{x}\right)^{2}} + 10\,\frac{e^{x}}{\left(1+e^{x}\right)^{3}}\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 50\,\frac{e^{x}(-1+2e^{x})}{\left(1+e^{x}\right)^{4}}\frac{t^{2x}}{\Gamma(2\alpha+1)}$$

$$+ 50e^{x}\left(5-6e^{x}-15e^{2x}+20e^{3x}-12e^{x}\frac{\Gamma(2\alpha+1)}{\left(\Gamma(\alpha+1)\right)^{2}}\right)\frac{t^{3\alpha}}{\left(1+e^{x}\right)^{6}\Gamma(3\alpha+1)}+\ldots \tag{29}$$

Special Cases

(i) The result in (29) was derived by Zhang and Liu [23] using the method of HPM.

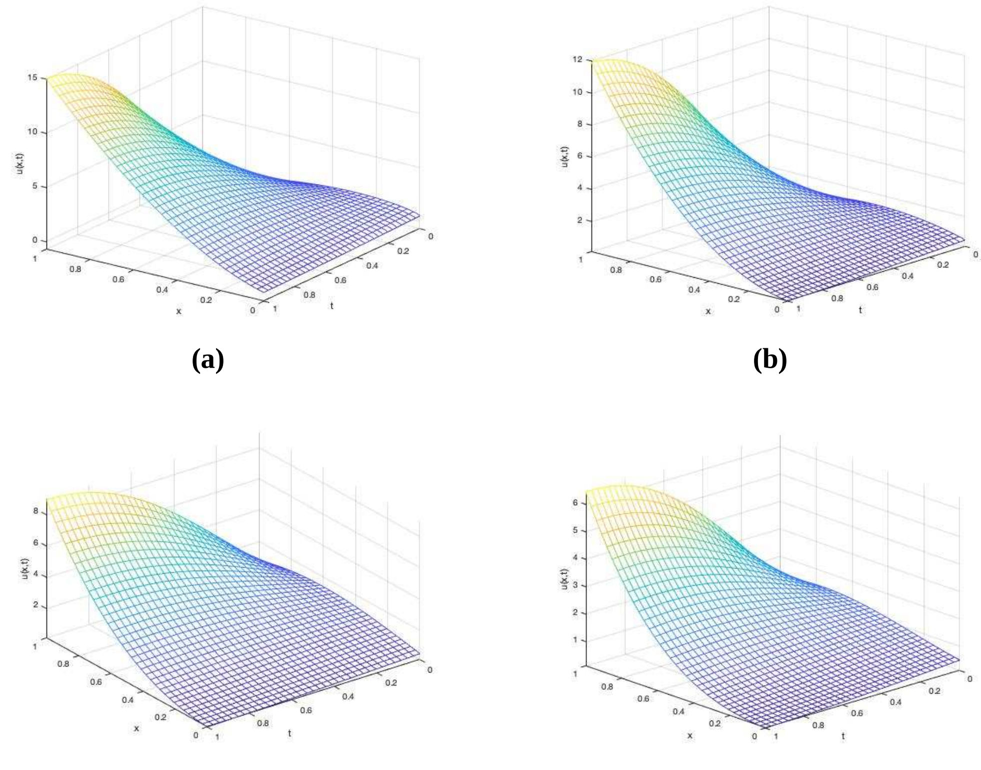
(ii) The result in (29) was obtained by Bairwa [1] using ILTM Method.

(iii) The result in (29) was deduced by Khan *et al*. [7] by the application of HAM.

(iv) For $\alpha=1,$ the result in (29) reduces to the following exact solution

$$u(x,t)=\frac{1}{\left(1+e^{x-5t}\right)^{2}}. \tag{30}$$

This result was earlier achieved by Wazwaz and Gorguis [22] using the ADM approach.



(a) **(b)**

(c) **(d)**

Figure 1: The surface shows the solution of the $u(x,t)$: **(a)** approximate solution for $\alpha=0.55$, **(b)** approximate solution for $\alpha=0.70$, **(c)** approximate solution for $\alpha=0.85$, **(d)** approximate solution for $\alpha=1$.

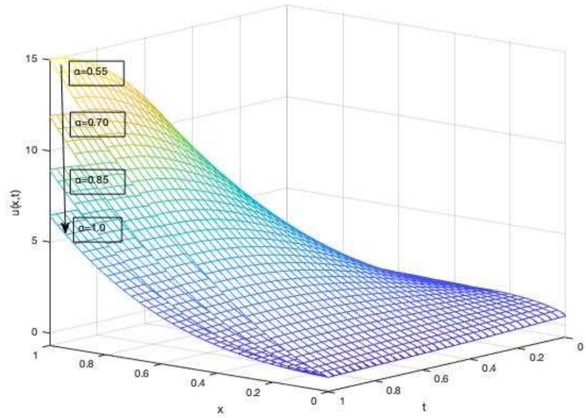


Figure 2: The surface shows the comparison graph of the solution $u(x,t)$ for different values of parameters, $\alpha = 0.55, \alpha = 0.70,\ \alpha = 0.85, \alpha = 1$

Example 2. Consider the following non-linear Fisher's equation concerning the time fractional derivative, given by [1, 12]

$$D_t^{\alpha} u = \frac{\partial^2 u}{\partial x^2} + u(1-u),\ \ 0 < \alpha \le 1\,, \tag{31}$$

with the initial condition

$$u(x,0) = \beta, \tag{32}$$

where $D_t^{\alpha} u(x,t)$ is the Caputo fractional derivative of order α given by (3) and β be a constant parameter.

Taking the Laplace transform of equation (31), and making use of the result given by (32), we obtain

$$L[u(x,t)] = \frac{\beta}{s} + \frac{1}{s^{\alpha}}\left[L\left(\frac{\partial^2 u}{\partial x^2} + u(1-u) \right) \right]. \tag{33}$$

Applying the inverse Laplace transform to the equation (33) yields

$$u(x,t) = \beta + L^{-1}\left[\frac{1}{s^{\alpha}} L\left(\frac{\partial^2 u}{\partial u^2} + u(1-u) \right) \right]. \tag{34}$$

Substituting the results of the equations (11) and (12) in the equation (34) and applying the equation (15), we determine the components of the LADM solution as follows

$$u_0(x,t) = \beta, \tag{35}$$

$$u_{n+1}(x,t) = L^{-1}\left[\frac{1}{s^{\alpha}}L\left(\frac{\partial^2 u_0}{\partial x^2}\right)\right] + L^{-1}\left[\frac{1}{s^{\alpha}}L\left(A_n\right)\right], n = 0,1,2,..., \tag{36}$$

where A_n are the Adomian polynomials for the non-linear term $Nu = u(1-u)$.
Now, for $n = 0,1,2,...,$ and using equations (13) and (36), we have

$$A_0 = \beta\left(1-\beta\right), \tag{37}$$

$$u_1(x,t) = \beta\left(1-\beta\right)\frac{t^{\alpha}}{\Gamma(\alpha+1)}, \tag{38}$$

$$A_1 = \beta\left(1-\beta\right)\left(1-2\beta\right)\frac{t^{\alpha}}{\Gamma(\alpha+1)}, \tag{39}$$

$$u_2(x,t) = \beta\left(1-\beta\right)\left(1-2\beta\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}, \tag{40}$$

$$A_2 = \left(\beta-2\beta^2+8\beta^3-4\beta^4\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \left(\beta^2-2\beta^3+\beta^4\right)\frac{t^{2\alpha}}{\left[\Gamma(\alpha+1)\right]^2}, \tag{41}$$

$$u_3(x,t) = \left(\beta-2\beta^2+8\beta^3-4\beta^4\right)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \left(\beta^2-2\beta^3+\beta^4\right)\frac{\Gamma(2\alpha+1)}{\left[\Gamma(\alpha+1)\right]^2}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \tag{42}$$

$$\begin{aligned} A_3 = {} & \left(1-2\beta\right)\left[\left(\beta-5\beta^2+8\beta^3-4\beta^4\right) - \left(\beta^2-2\beta^3+\beta^4\right)\frac{\Gamma(2\alpha+1)}{\left[\Gamma(\alpha+1)\right]^2}\right]\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} \\ & -2\left(\beta-\beta^2\right)\left(\beta^2-2\beta^3+\beta^4\right)\frac{t^{3\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}, \end{aligned} \tag{43}$$

$$\begin{aligned} u_4(x,t) = {} & \left(1-2\beta\right)\left[\left(\beta-5\beta^2+8\beta^3-4\beta^4\right) - \left(\beta^2-2\beta^3+\beta^4\right)\frac{\Gamma(2\alpha+1)}{\left[\Gamma(\alpha+1)\right]^2}\right]\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} \\ & -2\left(\beta-\beta^2\right)\left(\beta^2-2\beta^3+2\beta^4\right)\frac{\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}, \end{aligned} \tag{44}$$

and so on. The remaining components may be obtained similarly.

Thus, the series-form approximate analytical solution can be obtained as $u\left(x,t\right) = u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)+u_4(x,t)+...$

$$
\begin{aligned}
&= \beta + \beta(1-\beta)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \beta(1-\beta)(1-2\beta)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\
&\quad + \left(\beta - 2\beta^{2} + 8\beta^{3} - 4\beta^{4}\right)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \left(\beta^{2} - 2\beta^{3} + \beta^{4}\right)\frac{\Gamma(2\alpha+1)}{[\Gamma(\alpha+1)]^{2}}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} \\
&\quad + (1-2\beta)\left[\left(\beta - 5\beta^{2} + 8\beta^{3} - 4\beta^{4}\right) - \left(\beta^{2} - 2\beta^{3} + \beta^{4}\right)\frac{\Gamma(2\alpha+1)}{[\Gamma(\alpha+1)]^{2}}\right]\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} \\
&\quad - 2\left(\beta - \beta^{2}\right)\left(\beta^{2} - 2\beta^{3} + 2\beta^{4}\right)\frac{\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + \ldots
\end{aligned}
\tag{45}
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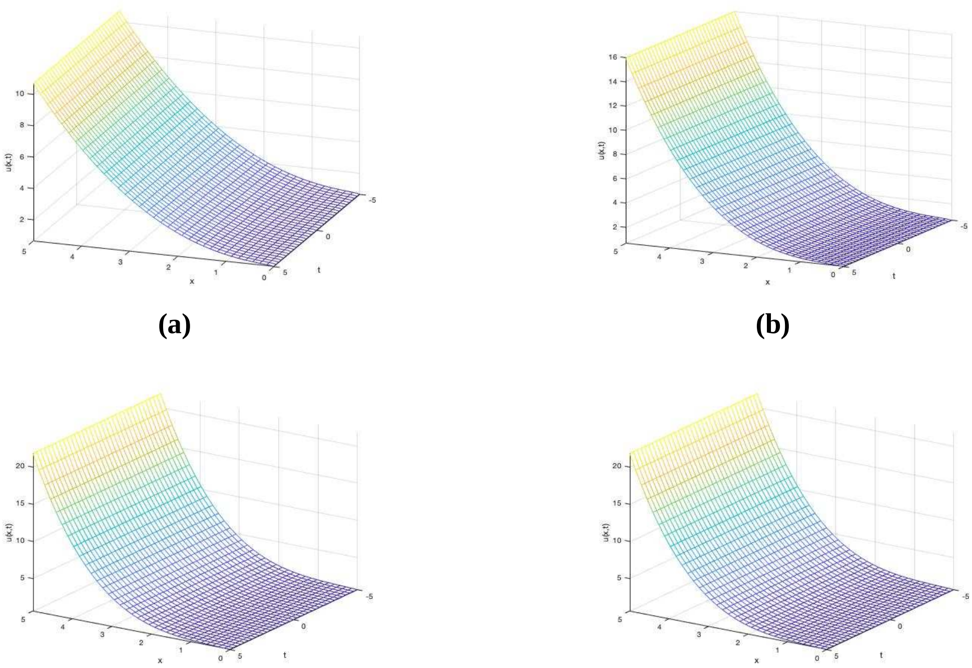

(a) **(b)** **(c)** **(d)**

Figure 3: The surface shows the solution of the $u(x,t)$: **(a)** approximate solution for, $\alpha = 0.55$, **(b)** approximate solution for $\alpha = 0.70$, **(c)** approximate solution for $\alpha = 0.85$, **(d)** approximate solution for $\alpha = 1$.

Special Cases

(i) The result in (45) was derived by Zhang and Liu [23] using a method of HPM.
(ii) The result in (45) was deduced by Mirzazadeh [12] by the application of DTM.
(iii) The result in (45) was obtained by Bairwa [1] using ILTM technique.

(iv) For $\alpha = 1,$ the result in (45) reduces to the following exact solution

$$u(x,t) = \frac{\beta e^{x}}{1-\beta+\beta e^{t}} \ . \tag{46}$$

This result was earlier achieved by Wazwaz and Gorguis [22] using the ADM approach.

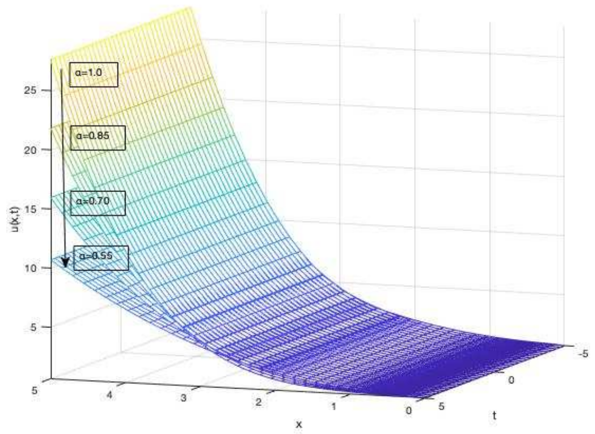

Figure 4: The surface shows the comparison graph of the solution $u(x,t)$ for the different values of the parameters, $\alpha = 0.55, \alpha = 0.70,\ \alpha = 0.85, \alpha = 1.$

5. Concluding remarks

The analytical approximate solutions to non-linear time-fractional Fisher's equations with initial conditions were determined using the Laplace-Adomian decomposition technique. Furthermore, the findings of this study are illustrated graphically using the mathematical software MATLAB. The results of the investigation show that the suggested method works very well in terms of simplicity and efficiency, and it may be used to examine other problems in the field of non-linear differential equations of fractional order.

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Conflict of interest. The authors declare that they have no conflict of interest.

Authors' Contributions. All the authors have equal contribution.

REFERENCES

1. R.K.Bairwa, Analytical Solution of the time-fractional Fisher equation by using Iterative Laplace Transform Method, *J. Raj. Acad. Phys. Sci*,, 17(3-4) (2019) 191-200.
2. R.K.Bairwa and K.Singh, Analytical Solution of time-fractional Klien-Gordon Equation by using Laplace-Adomian Decomposition Method, *Annals of Pure and Applied Mathematics*, 24(1) (2021) 27-35.

3. Y.Cherruault and G.Adomian, Decomposition methods: a new proof of convergence, *Mathematical and Computer Modelling*, 18(12) (1993) 103-106.
4. Y.Cherruault, G.Adomian, K.Abbaoui and R.Rach, Further remarks on convergence of decomposition method, *International Journal of Bio-Medical Computing*, 38(1) (1995) 89-93.
5. O.González-Gaxiola, The Laplace-Adomian decomposition method applied to the Kundu-Eckhaus equation, *International Journal of Mathematics and Its Applications*, 5(1-A) (2017) 1-12.
6. H.Jafari, C.M.Khalique and M.Nazari, Application of the Laplace decomposition method for solving linear and nonlinear fractional diffusion-wave equations, *Applied Mathematics Letters*, 24(11) (2011) 1799-1805.
7. N.A.Khan, N.U.Khan, A.Ara and M.Jamil, Approximate analytical solutions of fractional reaction-diffusion equations, *Journal of King Saud University-Science*, 24, (2012) 111-118.
8. Y, Khan and Q.Wu, Homotopy perturbation transform method for nonlinear equations using He's polynomials, *Comp. Math. Appli.*, 61(8) (2011) 1963-1967.
9. H.Khan R.Shah, P.Kumam, D.Baleanu and M.Arif, An efficient analytical technique, for the solution of fractional-order telegraph equations, *Mathematics*, 7(5) (2019) 426.
10. S.A.Khuri, A Laplace decomposition algorithm applied to a class of nonlinear differential equations, *Journal of Applied Mathematics*, 1(4) (2001) 141-155.
11. K.S.Miller and B.Ross, An introduction to the fractional calculus and fractional differential equations, *John Wiley & Sons, New York, USA*, (1993).
12. M.Mirzazadeh, A novel approach for solving fractional Fisher equation using differential transform method, *Pramana J. Phys.*, 86 (5) (2016) 957-963.
13. A.Naghipour and J.Manafian, Application of the Laplace Adomian decomposition and implicit methods for solving Burgers' equation, *TWMS Journal of Pure and Applied Mathematics*, 6(1) (2015) 68-77.
14. I.Podlubny, Fractional Differential Equations, vol. 198, *Academic Press, New York, USA*, (1999).
15. I.A.Rus, A.Petrusel and G.Petrusel, Fixed Point Theory: 1950-2000, Romanian Contribution, *House of the Book of Science, Cluj-Napoca*, (2002).
16. J.Sabatier, O.P.Agrawal, and J.A.T.Machado, Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering, *Springer, Dordrecht, The Netherlands*, (2007).
17. R.Shah, H.Khan, D.Baleanu, P.Kumam and M.Arif, A novel method for the analytical solution of fractional Zakharov-Kuznetsov equations, *Advances in difference equations*, 2019(1) (2019) 1-14.
18. S.C.Sharma and R.K.Bairwa, A reliable treatment of Iterative Laplace transform method for fractional Telegraph equations, *Annals of Pure and Applied Mathematics*, 9(1) (2014) 81-89.
19. S.C.Sharma and R.K.Bairwa, Iterative Laplace transform method for solving fractional heat and wave-like equation, *Res. J. Math. Stat. Sci.*, 3(2) (2015) 4-9.
20. I.Sneddon, The Use of Integral Transforms, *Tata McGraw-Hill, New Delhi*, (1979).

21. A.M.Wazwaz, The combined Laplace transform-Adomian decomposition method for handling nonlinear Volterra integro-differential equations, *Applied Mathematics and Computation*, 216(4) (2010) 1304-1309.
22. A.M.Wazwaz, and A.Gorguis, An analytic study of Fisher's equation by using Adomian decomposition method, *Appli. Math. Comput.*, 154 (2004) 609–620.
23. X.Zhang and J.Liu , An analytic study on time-fractional Fisher's equation by using Homotopy perturbation method, *Walailak J. Sci. & Tech.*, 11 (11) (2014) 975-985.