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Study of Open Sets in Bi-generalized Topological Spaces

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Abstract. In this paper, we study all kinds of open sets introduced in bi-generalized topological spaces, namely, $\mu_{(m,n)}$-semi open sets, $\mu_{(m,n)}$-pre open sets, $\mu_{(m,n)}$-regular open sets, $\mu_{(m,n)}$-α-open sets, $\mu_{(m,n)}$-β-open sets, $\bar{\mu}_{(m,n)}$-open sets, (*m*, *n*)-open sets and quasi generalized open sets and investigate some of their properties. We choose $\mu_{(m,n)}$-semi open set as the bases open set for our investigation and compare the relationships between the $\mu_{(m,n)}$-semi open sets and other open sets in this bi-generalized topological spaces.

Keywords: Generalized topological spaces, Bi-generalized topological spaces, Open sets.

AMS Mathematics Subject Classification (2010): 54A05, 54A10, 54E55

1. Introduction

Kelly [17] initiated the concept of bi-topological spaces (briefly, *Bi-TS*) in 1963 and thereafter many mathematicians generalized the topological ideas into bi-topological settings. Some open and closed sets in *Bi-TS* were defined by several authors [1,14,25,26,28,30]. Cs*á*sz*á*r [7] introduced the concept of generalized neighborhood systems and generalized topological spaces (briefly, *GTS*). Research in *GTS* is still a hot area of research in which researchers introduced several types of continuity, compactness, homogeneity, and sets are extended from ordinary topological spaces to include *GTS*. As a generalization of *Bi-TS*, Boonpok [4] introduced the concept of bi-generalized topological spaces (briefly, *Bi-GTS*) and studied (*m*, *n*)-closed sets and (*m*, *n*)-open sets in *Bi-GTS*. Also, several authors [3,8,11,12,16,23,27] further extended the concept of various types of closed sets in *Bi-GTS*.

In the literature, different types of open sets in *Bi-GTS* were defined by several authors [5,15,21]. Murugalingam and Gnanam [22] introduced the boundary set on *Bi-GTS*. Further, Sompong [29] defined the dense set in *Bi-GTS*. Zakari [32] defined the almost homeomorphism on *Bi-GTS*. Also, the authors [9,18] introduced the various types of continuous functions in *Bi-GTS*. Gnanam [13] introduced a new kind of connectedness in *Bi-GTS*. In this *Bi-GTS*, separation axioms were defined by several authors [10,19,24,31]. Recently, Ghour [2] introduced certain covering properties and minimal sets in *Bi-GTS*.

In this paper, we studied all kind of open sets introduced in *Bi-GTS* namely, $\mu_{(m,n)}$-semi open sets, $\mu_{(m,n)}$-pre open sets, $\mu_{(m,n)}$-regular open sets, $\mu_{(m,n)}$-α-open sets, $\mu_{(m,n)}$-β-open sets, $\bar{\mu}_{(m,n)}$-open sets, (*m*, *n*)-open sets and quasi generalized open sets and investigated some of their properties. Also we investigated the relationships between the $\mu_{(m,n)}$--semi open sets and other open sets in *Bi-GTS*.

2. Preliminaries

Definition 2.1. [7] Let *X* be a non-empty set and let we denote $\mathcal{P}(X)$ be the power set of *X*. A subset μ of $\mathcal{P}(X)$ is said to be a generalized topology (briefly, *GT*) on *X*, if it satisfying the following axioms:
(1) $\emptyset \in \mu$.
(2) An arbitrary union of elements of μ belongs to μ.

If μ is a *GT* on *X*, then (X, μ) is called a generalized topological space (briefly, *GTS*). The elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets.

Definition 2.2. [6] Let (X, μ) be a *GTS* and $A \subseteq X$. Then, the μ-interior of *A*, denoted by $int_\mu(A)$, is the union of all μ-open sets contained in *A*. The μ-closure of *A*, denoted by $cl_\mu(A)$, is the intersection of all μ-closed sets containing *A*.

Theorem 2.1. [6] Let (X, μ) be a *GTS* and $A \subseteq X$. Then,
(1) $cl_\mu(A) = X - int_\mu(X - A)$.
(2) $int_\mu(A) = X - cl_\mu(X - A)$.

Proposition 2.1. [20] Let (X, μ) be a *GTS* and $A, B \subseteq X$. Then, the following properties holds:
(1) $cl_\mu(X - A) = X - int_\mu(A)$ and $int_\mu(X - A) = X - cl_\mu(A)$.
(2) If $(X - A) \in \mu$, then $cl_\mu(A) = A$ and if $A \in \mu$, then $int_\mu(A) = A$.
(3) If $A \subseteq B$, then $cl_\mu(A) \subseteq cl_\mu(B)$ and $int_\mu(A) \subseteq int_\mu(B)$.
(4) If $A \subseteq cl_\mu(A)$ and $int_\mu(A) \subseteq A$.
(5) $cl_\mu(cl_\mu(A)) = cl_\mu(A)$ and $int_\mu(int_\mu(A)) = int_\mu(A)$.

Definition 2.3. [6] A subset *A* of a *GTS* (X, μ) is called
(1) μ-regular open if $A = int_\mu(cl_\mu(A))$.
(2) μ-pre open if $A \subseteq int_\mu(cl_\mu(A))$.
(3) μ-semi open if $A \subseteq cl_\mu(int_\mu(A))$.
(4) μ-α-open if $A \subseteq int_\mu(cl_\mu(int_\mu(A)))$.
(5) μ-β-open if $A \subseteq cl_\mu(int_\mu(cl_\mu(A)))$.

Definition 2.4. [4] Let *X* be a non-empty set and μ_1, μ_2 be generalized topologies on *X*. The triple (X, μ_1, μ_2) is said to be Bi-generalized topological space (briefly, *Bi-GTS*). The elements of μ_m are called μ_m-open sets, where $m \in \{1,2\}$.

Definition 2.5. [4] Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, μ_m-interior of A with respect to μ_m, denoted by $int_{\mu_m}(A)$, is the union of all μ_m-open sets contained in A. The μ_m-closure of A with respect to μ_m, denoted by $cl_{\mu_m}(A)$, is the intersection of all μ_m-closed sets containing A.

3. Open sets in bi-generalized topological spaces

3.1. $\mu_{(m,n)}$-semi open sets

Definition 3.1. [4] Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a $\mu_{(m,n)}$-semi open set if $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$, where m, $n \in \{1,2\}$ and $m \neq n$. The collection of all $\mu_{(m,n)}$-semi open sets is denoted by $\sigma_{(m,n)}(\mu)$.

Example 3.1. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\mu_2 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$. Then, $\sigma_{(1,2)}(\mu) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.

Lemma 3.1. If A and B are $\mu_{(m,n)}$-semi open sets, then $A \cup B$ is a $\mu_{(m,n)}$-semi open set.

Proof: Suppose that A and B are $\mu_{(m,n)}$-semi open sets. Then, $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$ and $B \subseteq cl_{\mu_n}(int_{\mu_m}(B))$. Since $cl_{\mu_n}(int_{\mu_m}(A)) \subseteq cl_{\mu_n}(int_{\mu_m}(A \cup B))$ and $cl_{\mu_n}(int_{\mu_m}(B)) \subseteq cl_{\mu_n}(int_{\mu_m}(A \cup B))$, we get $A \subseteq cl_{\mu_n}(int_{\mu_m}(A \cup B))$ and $B \subseteq cl_{\mu_n}(int_{\mu_m}(A \cup B))$. Therefore, $A \cup B \subseteq cl_{\mu_n}(int_{\mu_m}(A \cup B))$. Thus, $A \cup B$ is a $\mu_{(m,n)}$-semi open set.

Remark 3.1. If A and B are $\mu_{(m,n)}$-semi open sets, then in general, $A \cap B$ need not be a $\mu_{(m,n)}$-semi open set. This can be seen in the following example:

Example 3.2. Let X = {a, b, c, d}, $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. If A = {a, c}, B = {b, c}, then, $A \cap B$ = {c} $\notin \sigma_{(1,2)}(\mu)$.

Proposition 3.1. Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. If A is $\mu_{(m,n)}$-semi open set, then $cl_{\mu_n}(A) = cl_{\mu_n}(int_{\mu_m}(A))$.

Proof: Suppose that A is a $\mu_{(m,n)}$-semi open set. Then, $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. This implies that $cl_{\mu_n}(A) \subseteq cl_{\mu_n}(cl_{\mu_n}(int_{\mu_m}(A))) = cl_{\mu_n}(int_{\mu_m}(A))$ and so $cl_{\mu_n}(A) \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Since $int_{\mu_m}(A)) \subseteq A$, we get $cl_{\mu_n}(int_{\mu_m}(A)) \subseteq cl_{\mu_n}(A)$. Thus, $cl_{\mu_n}(A) = cl_{\mu_n}(int_{\mu_m}(A))$.

Theorem 3.3. Let (X, μ_1, μ_2) be a *Bi-GTS*. If $A \subseteq B \subseteq cl_{\mu_n}(A)$ and A is $\mu_{(m,n)}$- semi open set, then B is a $\mu_{(m,n)}$-semi open set.

Proof: Suppose that A is a $\mu_{(m,n)}$-semi open set. Then, by Proposition 3.1, we get $cl_{\mu_n}(A) = cl_{\mu_n}(int_{\mu_m}(A))$. So $B \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Since $A \subseteq B$, we get $cl_{\mu_n}(int_{\mu_m}(A)) \subseteq cl_{\mu_n}(int_{\mu_m}(B))$. Therefore, $B \subseteq cl_{\mu_n}(int_{\mu_m}(B))$. Thus, B is $\mu_{(m,n)}$-semi open set.

Proposition 3.2. Every μ_m-open set in (X, μ_m) is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).

Proof: Suppose that A is a μ_m-open set. Then, $A = int_{\mu_m}(A)$. Since $A \subseteq cl_{\mu_n}(A) = cl_{\mu_n}(int_{\mu_m}(A))$, we get $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Therefore, A is $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 3.4. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{c\}, X\}$. Where {a, b}, X are $\mu_{(1,2)}$-semi open sets, but these are not μ_1-open sets in (X, μ_1).

3.2. $\mu_{(m,n)}$-pre open set

Definition 3.2. ([4,15]) Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a $\mu_{(m,n)}$-pre open set if $A \subseteq int_{\mu_m}(cl_{\mu_n}(A))$, where m, $n \in \{1,2\}$ and $m \neq n$. The collection of all $\mu_{(m,n)}$-pre open sets is denoted by $\pi_{(m,n)}(\mu)$.

Example 3.5. Let X = {a, b, c, d}, $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ .Then, $\pi_{(1,2)}(\mu) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.

Lemma 3.2. If A and B are $\mu_{(m,n)}$-pre open sets, then $A \cup B$ is $\mu_{(m,n)}$-pre open set.

Proof: Suppose that A and B are $\mu_{(m,n)}$-pre open sets. Then, $A \subseteq int_{\mu_m}(cl_{\mu_n}(A))$ and $B \subseteq int_{\mu_m}(cl_{\mu_n}(B))$. Since $int_{\mu_m}(cl_{\mu_n}(A)) \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$ and $int_{\mu_m}(cl_{\mu_n}(B)) \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$, we get $A \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$ and $B \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$. Therefore, $A \cup B \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$. Thus, $A \cup B$ is a $\mu_{(m,n)}$-pre open set.

Remark 3.2. If A and B are $\mu_{(m,n)}$-pre open sets, then in general, $A \cap B$ need not be a $\mu_{(m,n)}$-pre open set. This can be seen in the following example:

Example 3.6. Let X = {a, b, c, d}, $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. If A = {a, c}, B = {b, c}, then, $A \cap B$ = {c} $\notin \pi_{(1,2)}(\mu)$.

Proposition 3.3. Every μ_m-open set in (X, μ_m) is a $\mu_{(m,n)}$-pre open set in (X, μ_1, μ_2).

Proof: Suppose that A is a μ_m-open set. Then, $A = int_{\mu_m}(A)$. Since $A \subseteq cl_{\mu_n}(A)$, we get $int_{\mu_m}(A) \subseteq int_{\mu_m}(cl_{\mu_n}(A))$. Therefore, $A \subseteq int_{\mu_m}(cl_{\mu_n}(A))$. Thus, A is a $\mu_{(m,n)}$-pre open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 3.7. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a, b\}, \{a, c\}, X\}$ and $\mu_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$ where {b}, {c}, {b, c} are $\mu_{(1,2)}$-pre open sets, but these are not μ_1-open sets in (X, μ_1).

3.3. $\mu_{(m,n)}$-regular open sets

Definition 3.3. [4] Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a $\mu_{(m,n)}$-regular open set if $A = int_{\mu_m}(cl_{\mu_n}(A))$, where m, $n \in \{1,2\}$ and $m \neq n$. The collection of all $\mu_{(m,n)}$-regular open sets is denoted by $\gamma_{(m,n)}(\mu)$.

Example 3.8. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a, b\}, \{a, c\}, X\}$ and $\mu_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. *Then*, $\gamma_{(1,2)}(\mu) = \{\emptyset, \{a, b\}, \{a, c\}, X\}$.

Lemma 3.3. If A and B are $\mu_{(m,n)}$-regular open sets, then $A \cup B$ is a $\mu_{(m,n)}$-regular open set.

Proof: Suppose that A and B are $\mu_{(m,n)}$-regular open sets. Then, $A = int_{\mu_m}(cl_{\mu_n}(A))$ and $B = int_{\mu_m}(cl_{\mu_n}(B))$. Since $int_{\mu_m}(cl_{\mu_n}(A)) \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$ and $int_{\mu_m}(cl_{\mu_n}(B)) \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$, we get $A \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$ and $B \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$. Therefore, $A \cup B \subseteq int_{\mu_m}(cl_{\mu_n}(A \cup B))$. And it is clear that $int_{\mu_m}(cl_{\mu_n}(A \cup B)) \subseteq A \cup B$. Therefore, $A \cup B = int_{\mu_m}(cl_{\mu_n}(A \cup B))$. Thus, $A \cup B$ is a $\mu_{(m,n)}$-regular open set.

Remark 3.3. If A and B are $\mu_{(m,n)}$-regular open sets, then in general, $A \cap B$ need not be a $\mu_{(m,n)}$-regular open set. This can be seen in the following example:

Example 3.9. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a, b\}, \{a, c\}, X\}$ and $\mu_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. If $A = \{a, b\}$, $B = \{a, c\}$, *then*, $A \cap B = \{a\} \notin \gamma_{(1,2)}(\mu)$.

Proposition 3.4. let A be a μ_n-closed set in (X, μ_n). Then, A is a $\mu_{(m,n)}$-regular open set in (X, μ_1, μ_2) if and only if A is a μ_m-open set in (X, μ_m).

Proof: Suppose that A is a $\mu_{(m,n)}$-regular open set. Then, $A = int_{\mu_m}(cl_{\mu_n}(A))$. Since A is μ_n-closed set, we get $cl_{\mu_n}(A) = A$. Therefore, $A = int_{\mu_m}(A)$. Hence A is a μ_m-open set in (X, μ_m).

Conversely, suppose that A is a μ_m-open set. Then, $A = int_{\mu_m}(A)$. Since A is μ_n-closed set, we get $A = cl_{\mu_n}(A)$. Therefore, $A = int_{\mu_m}(cl_{\mu_n}(A))$. Hence A is a $\mu_{(m,n)}$-regular open set.

3.4. $\mu_{(m,n)}$-α-open sets

Definition 3.4. [4] Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a $\mu_{(m,n)}$-α-open set if $A \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A)))$, where $m, n \in \{1,2\}$ and $m \neq n$. The collection of all $\mu_{(m,n)}$-α-open sets is denoted by $\alpha_{(m,n)}(\mu)$.

Example 3.10. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a, b\}, \{a, c\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$. Then, $\alpha_{(1,2)}(\mu) = \{\emptyset, \{a, b\}, \{a, c\}, X\}$.

Lemma 3.4. If A and B are $\mu_{(m,n)}$-α-open sets, then $A \cup B$ is a $\mu_{(m,n)}$-α-open set.

Proof: Suppose that A and B are $\mu_{(m,n)}$-α-open sets. Then, $A \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A)))$ and $B \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(B)))$. Since $int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A))) \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A \cup B)))$ and $int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(B))) \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A \cup B)))$, we get $A \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A \cup B)))$ and $B \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A \cup B)))$. Therefore, $A \cup B \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A \cup B)))$. Thus, $A \cup B$ is a $\mu_{(m,n)}$-α-open set.

Remark 3.4. If A and B are $\mu_{(m,n)}$-α-open sets, then in general, $A \cap B$ need not be a $\mu_{(m,n)}$-α-open set. This can be seen in the following example:

Example 3.11. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a, b\}, \{a, c\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$. If A = {a, b}, B = {a, c}, then, $A \cap B$ = {a} $\notin \alpha_{(1,2)}(\mu)$.

Proposition 3.5. Every μ_m-open set in (X, μ_m) is a $\mu_{(m,n)}$-α-open set in (X, μ_1, μ_2).
Proof: Suppose that A is a μ_m-open set. Then, $A = int_{\mu_m}(A)$. Since $A \subseteq cl_{\mu_n}(A) = cl_{\mu_n}(int_{\mu_m}(A))$, we get $int_{\mu_m}(A) \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A)))$. Therefore, $A \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A)))$. Hence A is a $\mu_{(m,n)}$-α-open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 3.12. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, X\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{a, c\}, X\}$. Then, $\alpha_{(1,2)}(\mu) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Therefore, {a, c} is $\mu_{(1,2)}$-α-open set, but this is not a μ_1-open set in (X, μ_1).

3.5. $\mu_{(m,n)}$-β-open sets

Definition 3.5. [4] Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a $\mu_{(m,n)}$-β-open set if $A \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A)))$, where m, $n \in \{1,2\}$ and $m \neq n$. The collection of all $\mu_{(m,n)}$-β-open sets is denoted by $\beta_{(m,n)}(\mu)$.

Example 3.13. Let X = {a, b, c, d}, $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. Then, $\beta_{(1,2)}(\mu) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.

Lemma 3.5. If A and B are $\mu_{(m,n)}$-β-open sets, then $A \cup B$ is a $\mu_{(m,n)}$-β-open set.
Proof: Suppose that A and B are $\mu_{(m,n)}$-β-open sets. Then, $A \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A)))$ and $B \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(B)))$. Since $cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A))) \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A \cup B)))$ and $cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(B))) \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A \cup B)))$, we get $A \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A \cup B)))$ and $B \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A \cup B)))$. Therefore, $A \cup B \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A \cup B)))$. Thus, $A \cup B$ is a $\mu_{(m,n)}$-β-open set.

Remark 3.5. If A and B are $\mu_{(m,n)}$-β-open sets, then in general, $A \cap B$ need not be a $\mu_{(m,n)}$-β-open set. This can be seen in the following example:

Example 3.14. Let X = {a, b, c, d}, $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. If A = {a, c}, B = {b, c}, then, $A \cap B$ = {c} $\notin \beta_{(1,2)}(\mu)$.

Proposition 3.6. Every μ_m-open set in (X, μ_m) is a $\mu_{(m,n)}$-β-open set in (X, μ_1, μ_2).
Proof: Suppose that A is a μ_m-open set. Then, $A = int_{\mu_m}(A)$. Since $A \subseteq cl_{\mu_n}(A)$, we get $A = int_{\mu_m}(A)) \subseteq int_{\mu_m}(cl_{\mu_n}(A))$, So $cl_{\mu_n}(A) \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A)))$. Therefore, $A \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A)))$. Hence A is a $\mu_{(m,n)}$-β-open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 3.15. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{c\}, X\}$. Then, $\beta_{(1,2)}(\mu) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Therefore, {b}, {a, b}, {b, c}, X are $\mu_{(1,2)}$-β-open sets, but these are not μ_1-open sets in (X, μ_1).

3.6. $\bar{\mu}_{(m,n)}$-open sets

Definition 3.6. [5] Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a $\bar{\mu}_{(m,n)}$-open set if there exists a μ_m-open set U of X such that $U \subseteq A \subseteq cl_{s_{\mu_n}}(U)$, where $cl_{s_{\mu_n}}(U)$ is the intersection of all μ_n-semi closed sets containing U. That is, the smallest μ_n-semi closed set containing U, where $m, n \in \{1,2\}$ and $m \neq n$.

Example 3.16. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{c\}, X\}$. Then, $\emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}, X$ are $\bar{\mu}_{(1,2)}$-open sets.

Lemma 3.6. If A and B are $\bar{\mu}_{(m,n)}$-open sets, then $A \cup B$ is $\bar{\mu}_{(m,n)}$-open set.

Proof: Suppose that A and B are $\bar{\mu}_{(m,n)}$-open sets. Then, there exists a μ_m-open set U of X such that $U \subseteq A \subseteq cl_{s_{\mu_n}}(U)$ and $U \subseteq B \subseteq cl_{s_{\mu_n}}(U)$. This implies that $U \subseteq A \cup B \subseteq cl_{s_{\mu_n}}(U)$. Thus, $A \cup B$ is a $\bar{\mu}_{(m,n)}$-open set.

Remark 3.6. If A and B are $\bar{\mu}_{(m,n)}$-open sets, then in general, $A \cap B$ need not be a $\bar{\mu}_{(m,n)}$-open set. This can be seen in the following example:

Example 3.17. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a, b\}, \{b, c\}, X\}$ and $\mu_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. If A = {a, b}, B = {b, c}, then, $A \cap B$ = {b} $\notin \bar{\mu}_{(1,2)}$-open set.

Theorem 3.18. Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a $\bar{\mu}_{(m,n)}$-open set if and only if $A \subseteq cl_{s_{\mu_n}}(int_{\mu_m}(A))$.

Proof: Let A be a $\bar{\mu}_{(m,n)}$-open set. Then, there exists a μ_m-open set U such that $U \subseteq A \subseteq cl_{s_{\mu_n}}(U)$. Since U is μ_m-open set, we get $U = int_{\mu_m}(U) \subseteq int_{\mu_m}(A)$. This implies that $A \subseteq cl_{s_{\mu_n}}(U) \subseteq cl_{s_{\mu_n}}(int_{\mu_m}(A))$. Thus, $A \subseteq cl_{s_{\mu_n}}(int_{\mu_m}(A))$.

Conversely, let $A \subseteq cl_{s_{\mu_n}}(int_{\mu_m}(A))$ and take $U = int_{\mu_m}(A)$. Then, $int_{\mu_m}(A) \subseteq A \subseteq cl_{s_{\mu_n}}(int_{\mu_m}(A))$. Hence A is $\bar{\mu}_{(m,n)}$-open set.

Proposition 3.7. Every μ_m-open set in (X, μ_m) is a $\bar{\mu}_{(m,n)}$-open set in (X, μ_1, μ_2).

Proof: Suppose that A is a μ_m-open set. Then, $A = int_{\mu_m}(A)$. Since $A \subseteq cl_{s_{\mu_n}}(A)$, we get $A \subseteq cl_{s_{\mu_n}}(int_{\mu_m}(A))$. Therefore, by Theorem 3.18, we get A is $\bar{\mu}_{(m,n)}$-open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 3.19. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{a\}, X\}$. Then, {a, b}, X are $\bar{\mu}_{(1,2)}$-open sets, but these are not μ_1-open sets in (X, μ_1).

3.7. (*m*, *n*)-open sets

Definition 3.7. [4] Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a (m, n)-open set if $A = int_{\mu_m}(int_{\mu_n}(A))$, where $m, n \in \{1,2\}$ and $m \neq n$.

Example 3.20. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{b\}, \{a, b\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{c\}, X\}$. Then, (1, 2)-open set is {a, b}.

Lemma 3.7. If A and B are (m, n)-open sets, then $A \cup B$ is a (m, n)-open set.
Proof: Suppose that A and B are (m, n)-open sets. Then, $A = int_{\mu_m}(int_{\mu_n}(A))$ and $B = int_{\mu_m}(int_{\mu_n}(B))$. Since $int_{\mu_m}(int_{\mu_n}(A)) \subseteq int_{\mu_m}(int_{\mu_n}(A \cup B))$ and $int_{\mu_m}(int_{\mu_n}(B)) \subseteq int_{\mu_m}(int_{\mu_n}(A \cup B))$, we get $A \subseteq int_{\mu_m}(int_{\mu_n}(A \cup B))$ and $B \subseteq int_{\mu_m}(int_{\mu_n}(A \cup B))$. Then, $A \cup B \subseteq int_{\mu_m}(int_{\mu_n}(A \cup B))$. Since $int_{\mu_n}(A \cup B) \subseteq A \cup B$, we get $int_{\mu_m}(int_{\mu_n}(A \cup B)) \subseteq int_{\mu_m}(A \cup B) \subseteq A \cup B$. Therefore, $A \cup B = int_{\mu_m}(int_{\mu_n}(A \cup B))$. Thus, $A \cup B$ is a (m, n)-open set.

Remark 3.7. If A and B are (m, n)-open sets, then in general, $A \cap B$ need not be a (m, n)-open set. This can be seen in the following example:

Example 3.21. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a, b\}, \{b, c\}, X\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{b, c\}, X\}$. If $A = \{a, b\}$, $B = \{b, c\}$, then, $A \cap B$ = {b} $\notin$ (1, 2)-open set.

Proposition 3.8. let A be a subset of a *Bi-GTS* (X, μ_1, μ_2) and A is μ_n-open set in (X, μ_n). Then, A is a (m, n)-open set in (X, μ_1, μ_2) if and only if A is a μ_m-open set in (X, μ_m).
Proof: Suppose that A is a (m, n)-open set. Then, $A = int_{\mu_m}(int_{\mu_n}(A))$. Since A is μ_n-open set, we get $A = int_{\mu_n}(A)$. This implies that $A = int_{\mu_m}(A)$. Hence A is a μ_m-open set in (X, μ_m).

Conversely, suppose that A is a μ_m-open set. Then, $A = int_{\mu_m}(A)$. Since A is μ_n-open set, we get $A = int_{\mu_n}(A)$. This implies that $A = int_{\mu_m}(int_{\mu_n}(A))$. Hence A is a (m, n)-open set.

3.8. Quasi generalized open sets

Definition 3.8. [21] Let (X, μ_1, μ_2) be a *Bi-GTS* and A be a subset of X. Then, A is said to be a quasi generalized open set (briefly, q_μ-open set) if for every $x \in A$, then there exist a μ_1-open set U such that $x \in U \subseteq A$, or a μ_2-open set V such that $x \in V \subseteq A$.

Example 3.22. Let X = {a, b, c, d}, $\mu_1 = \{\emptyset, \{a, b\}\}$ and $\mu_2 = \{\emptyset, \{a, c\}\}$. Then, q_μ-open set is {a, b, c}.

Lemma 3.8. If A and B are q_μ-open sets, then $A \cup B$ is a q_μ-open set.
Proof: Suppose that A and B are q_μ-open sets. Then, for every $x \in A$ and $x \in B$, then there exist a μ_1-open set U such that $x \in U \subseteq A$ and $x \in U \subseteq B$, or a μ_2-open set V such that $x \in$

$V \subseteq A$ and $x \in V \subseteq B$. This implies that for every $x \in A \cup B$, then there exist a μ_1-open set U such that $x \in U \subseteq A \cup B$, or a μ_2-open set V such that $x \in V \subseteq A \cup B$. Thus, $A \cup B$ is a q_μ-open set.

Remark 3.8. If A and B are q_μ-open sets, then in general, $A \cap B$ need not be a q_μ-open set. This can be seen in the following example:

Example 3.23. Let X = {a, b, c, d}, $\mu_1 = \{\emptyset, \{d\}, \{a, b\}, \{a, b, d\}\}$ and $\mu_2 = \{\emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$. If A = {a, b, c}, B = {a, c, d}, then, $A \cap B$ = {a, c} $\notin q_\mu$-open set.

4. Comparison of open sets in bi-generalized topological spaces

We choose $\mu_{(m,n)}$-semi open set as the base open set for the comparison of all open sets in the *Bi-GTS*.

Proposition 4.1. Every $\mu_{(m,n)}$-α-open set is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).
Proof: Let A be a $\mu_{(m,n)}$-α-open set. Then, $A \subseteq int_{\mu_m}(cl_{\mu_n}(int_{\mu_m}(A)))$. Take $B = cl_{\mu_n}(int_{\mu_m}(A))$. Let $int_{\mu_m}(B)$ be the union of all open sets contained in B, that is, $int_{\mu_m}(B) = \bigcup_{i \in I} G_i$, where $G_i \subseteq B$. Then, $A \subseteq \bigcup_{i \in I} G_i$, where $G_i \subseteq B$. This implies that $A \subseteq B$. Therefore, $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Hence A is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 4.1. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then, {a, b}, X are $\mu_{(1,2)}$-semi open sets, but these are not $\mu_{(1,2)}$-α-open sets in (X, μ_1, μ_2).

Proposition 4.2. Every $\mu_{(m,n)}$-semi open set is a $\mu_{(m,n)}$-β-open set in (X, μ_1, μ_2).
Proof: Let A be a $\mu_{(m,n)}$-semi open set. Then, $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Since $A \subseteq cl_{\mu_n}(A)$, we get $cl_{\mu_n}(int_{\mu_m}(A)) \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A)))$. This implies that $A \subseteq cl_{\mu_n}(int_{\mu_m}(cl_{\mu_n}(A)))$. Hence A is $\mu_{(m,n)}$-β-open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 4.2. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{c\}, \{a, b\}, X\}$. Then, {b}, {b, c} are $\mu_{(1,2)}$-β-open sets, but these are not $\mu_{(1,2)}$-semi open sets in (X, μ_1, μ_2).

Proposition 4.3. [5] Every $\bar{\mu}_{(m,n)}$-open set is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 4.3. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{c\}, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Then, {a, c} is a $\mu_{(1,2)}$-semi open set, but these is not *a* $\bar{\mu}_{(1,2)}$-open set in (X, μ_1, μ_2).

Proposition 4.4. let A be a μ_n-closed set of a *Bi-GTS* (X, μ_1, μ_2). If A is $\mu_{(m,n)}$-pre open set in (X, μ_1, μ_2), then A is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).
Proof: Let A be a $\mu_{(m,n)}$-pre open set. Then, $A \subseteq int_{\mu_m}(cl_{\mu_n}(A))$. Since A is μ_n-closed set, we get $A = cl_{\mu_n}(A)$. This implies that $A \subseteq int_{\mu_m}(A)$ and so $cl_{\mu_n}(A) \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Therefore, $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Hence A is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 4.4. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{c\}, \{a, b\}, X\}$ and also {a, b}, X are μ_2-closed sets. Then, {a, b}, X are $\mu_{(1,2)}$-semi open sets in (X, μ_1, μ_2), but these are not $\mu_{(1,2)}$-pre open sets in (X, μ_1, μ_2).

Proposition 4.5. let A be a μ_n-closed set of a *Bi-GTS* (X, μ_1, μ_2). If A is a $\mu_{(m,n)}$-regular open set in (X, μ_1, μ_2), then A is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).
Proof: Let A be a $\mu_{(m,n)}$-regular open set. Then, $A = int_{\mu_m}(cl_{\mu_n}(A))$. Since A is μ_n-closed set, we get $A = cl_{\mu_n}(A)$. This implies that $A = int_{\mu_m}(A)$. Since $A \subseteq cl_{\mu_n}(A)$, we get $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Therefore, $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Hence A is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 4.5. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{c\}, \{a, b\}, X\}$ and also {a, b}, X are μ_2-closed sets. Then, {a, b}, X are $\mu_{(1,2)}$-semi open sets in (X, μ_1, μ_2), but these are not $\mu_{(1,2)}$-regular open sets in (X, μ_1, μ_2).

Proposition 4.6. let A be a μ_n-open set of a *Bi-GTS* (X, μ_1, μ_2). If A is a (m, n)-open set in (X, μ_1, μ_2), then A is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).
Proof: Let A be a (m, n)-open set in (X, μ_1, μ_2). Then, $A = int_{\mu_m}(int_{\mu_n}(A))$. Since A is μ_n-open set, we get $A = int_{\mu_n}(A)$. This implies that $A = int_{\mu_m}(A)$. Since $A \subseteq cl_{\mu_n}(A)$, we get $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Therefore, $A \subseteq cl_{\mu_n}(int_{\mu_m}(A))$. Hence A is a $\mu_{(m,n)}$-semi open set in (X, μ_1, μ_2).

The converse of the above proposition need not be true in general. This can be seen in the following example:

Example 4.6. Let $X = \{a, b, c\}$, $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{c\}, \{a, b\}, \{a, c\}, X\}$ and also {a, b}, X are μ_2-open sets. Then, {a, b}, X are $\mu_{(1,2)}$-semi open sets in (X, μ_1, μ_2), but these are not *(1,2)*-open sets in (X, μ_1, μ_2).

Since the quasi generalized open set is defined by μ_1-open set or μ_2-open set. So this cannot be compared with $\mu_{(m,n)}$-semi open set.

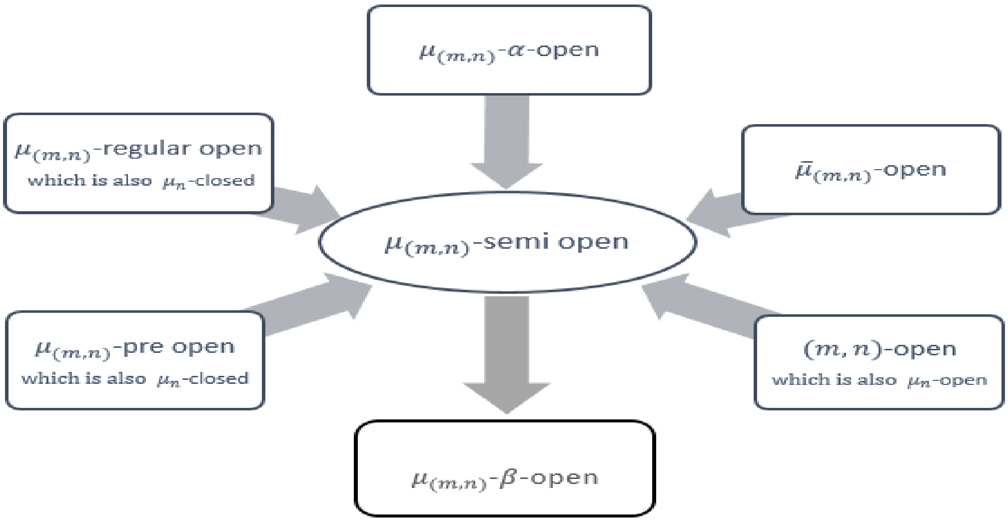

Figure 4.1: Relationships between the $\mu_{(m,n)}$-semi open set and other open sets in *Bi-GTS*.

5. Conclusion

In this paper, we studied all kind of open sets in *Bi-GTS* namely, $\mu_{(m,n)}$-semi open sets, $\mu_{(m,n)}$-pre open sets, $\mu_{(m,n)}$-regular open sets, $\mu_{(m,n)}$-α-open sets, $\mu_{(m,n)}$-β-open sets, $\bar{\mu}_{(m,n)}$-open sets, (m, n)-open sets and quasi generalized open sets and investigated some of the properties of these open sets. Also we compared the relationships between the $\mu_{(m,n)}$-semi open set and other open sets in *Bi-GTS*.

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