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Computation of E-Banhatti Nirmala Indices of Tetrameric 1,3-Adamantane

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
email: vrkulli@gmail.com

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Abstract. In this paper, we introduce the E-Banhatti Nirmala index and the modified E-Banhatti Nirmala index and their corresponding exponentials of a graph. Furthermore, we determine these newly defined E-Banhatti Nirmala indices and their corresponding exponentials for some standard graphs and tetrameric 1,3-Adamantane.

Keywords: E-Banhatti Nirmala index, modified E-Banhatti Nirmala index, tetrameric 1,3-adamantane.

AMS Mathematics Subject Classification): 05*C*05, 05*C*07, 05*C*09, 05*C*92

1. Introduction

Throughout this paper, we consider simple graphs which are finite, connected, undirected graphs without loops and multiple edges. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. For term and concept not given here, we refer the book [1].

A molecular graph is a simple graph, representing the carbon atom skeleton of an organic molecule of the hydrocarbon. Therefore the vertices of a molecular graph represent the carbon atoms and its edges the carbon-carbon bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. Several graph indices [2] have found some applications in Chemistry, especially in QSPR/QSAR research [3].

In [4], Kulli defined the Banhatti degree of a vertex u of a graph G as

$$B(u)=\frac{d_G(e)}{n-d_G(u)},$$

where n is the number of vertices of G and the vertex u and edge e are incident in G.

The first and second E-Banhatti indices were introduced by Kulli in [4] and they are defined as

$$EB_1(G)=\sum_{uv\in E(G)}[B(u)+B(v)],$$

$$EB_2(G)=\sum_{uv\in E(G)}B(u)B(v).$$

We propose the E-Banhatti Nirmala index of a graph G and defined it as

$$EBN(G) = \sum_{uv \in E(G)} \sqrt{B(u)+B(v)}.$$

We introduce the modified E-Banhatti Nirmala index of a graph G and defined it as

$${}^{m}EBN(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{B(u)+B(v)}}.$$

Considering the E-Banhatti Nirmala and modified E-Banhatti Nirmala indices, we define their corresponding exponentials of a graph G as

$$EBN(G,x) = \sum_{uv \in E(G)} x^{\sqrt{B(u)+B(v)}},$$

$${}^{m}EBN(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{B(u)+B(v)}}}.$$

In Chemical Graph Theory, several graph indices were introduced and studied such as the Wiener index [6, 7], the Zagreb indices [8, 9], the Revan indices [10, 11], the Gourava indices [12, 13], the reverse indices [14, 15] and the Banhatti indices [16, 17].

In this paper, we compute the E-Banhatti Nirmala index and the modified E-Banhatti Nirmala index for some standard graphs and tetrameric 1,3-adamanane.

2. Results for some standard graphs

Proposition 1. If G is an r-regular graph with n vertices and $r \geq 2$, then

$$EBN(G) = nr\sqrt{\frac{r-1}{n-r}}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$. Then G has $\frac{nr}{2}$ edges. For any edge $uv=e$ in G, $d_G(e)= d_G(u)+ d_G(u) -2=2r-2$.

From definition we have

$$EBN(G) = \sum_{uv \in E(G)} \sqrt{B(u)+B(v)}$$

$$= \frac{nr}{2}\sqrt{\left(\frac{2r-2}{n-r}\right)+\left(\frac{2r-2}{n-r}\right)} = nr\sqrt{\frac{r-1}{n-r}}.$$

Corollary 1.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$EBN(C_n) = 2n\sqrt{\frac{1}{n-2}}.$$

Corollary 1.2. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$EBN(K_n) = n(n-1)\sqrt{n-2}.$$

Proposition 2. Let P_n be a path with $n\geq 3$ vertices. Then

$$EBN(P_n) = 2\left[\left(\frac{1}{n-1}\right)+\left(\frac{2}{n-2}\right)\right]^{\frac{1}{2}} + (n-3)\left[\left(\frac{2}{n-2}\right)+\left(\frac{2}{n-2}\right)\right]^{\frac{1}{2}}$$

$$= n\sqrt{\frac{3n-4}{(n-1)(n-2)}} + 2(n-3)\sqrt{\frac{1}{n-2}}.$$

Proposition 3. Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m\leq n$ *and* $n \geq 2$. Then

$$EBN(K_{m,n}) = \sqrt{mn}\sqrt{(m+n)(m+n-2)}.$$

Proof: Let $K_{m,n}$ be a complete bipartite m n graph with $m + n$ vertices and mn *edges* such that $|V_1|= m$, $|V_2|= n$, $V(K_{r,s}) = V_1\cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Then $d_G(e)= d_G(u)+ d_G(v) -2= m + n -2$.

$$EBN(K_{m,n}) = \sum_{uv\in E(G)} \sqrt{B(u)+B(v)}$$

$$= mn\left[\left(\frac{m+n-2}{m+n-n}\right)+\left(\frac{m+n-2}{m+n-m}\right)\right]^{\frac{1}{2}}$$

$$= \sqrt{mn}\sqrt{(m+n)(m+n-2)}.$$

Corollary 3.1. Let $K_{n,n}$ be a complete bipartite graph with $n\geq 2$. Then

$$EBN(K_{n,n}) = 2n\sqrt{n(n-1)}.$$

Corollary 3.2. Let $K_{1,n}$ be a star with $n\geq 2$. Then

$$EBN(K_{1,n}) = \sqrt{n}\sqrt{(n+1)(n-1)}.$$

3. Results for Tetrameric 1,3-Adamantane

In Chemistry, diamondoids are variants of the carbon cage known as a damantane (C_{10}, H_{16}), the smallest unit cage structure of the diamond crystal lattice. We focus on the molecular graph structure of the family of tetrameric 1,3-adamantane, denoted by $TA[n]$. Let G be the graph of a tetrameric 1,3-adamantane $TA[n]$. The graph of a tetrameric 1,3-adamantane $TA[4]$ is presented in Figure 1.

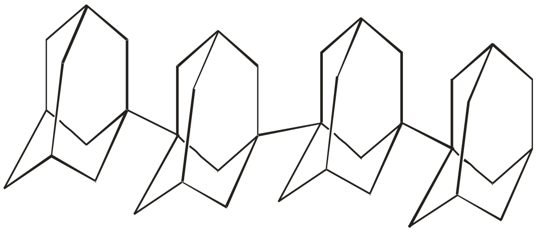

Figure 1

By calculation, G has $10n$ vertices and $13n – 1$edges. Also by calculation, we obtain three edge partitions of G based on the degrees of the end vertices of each edge as follows:

$E_1=\{uv\in E(G) \mid d_G(u)=2, d_G(v)=3\}$, $|E_1| = 6n + 6$.
$E_2=\{uv\in E(G) \mid d_G(u)=2, d_G(v)=4\}$, $|E_2| = 6n – 6$.
$E_3=\{uv\in E(G) \mid d_G(u)=d_G(v) = 4\}$, $|E_3| = n – 1$.

Therefore, in $TA[n]$, there are three types of edges based on the Banhatti degree of end vertices of each edge as follow:

$BE_1 = \{uv \in E(G) \mid B(u) = \frac{3}{10n-2}, B(v) = \frac{3}{10n-3}\}$, $|BE_1| =6n+6$.

$BE_2 = \{uv \in E(G) \mid B(u) = \frac{4}{10n-2}, B(v) = \frac{4}{10n-4}\}$, $|BE_2| = 6n-6$.

$BE_3 = \{uv \in E(G) \mid B(u) = \frac{6}{10n-4}, B(v) = \frac{6}{10n-4}\}$, $|BE_3| = n-1$.

We determine the E-Banhatti Nirmala index of $TA[n]$.

Theorem 1. Let G be the graph of a tetrameric 1,3-adamantane $TA[n]$ with $10n$ vertices and $13n-1$ edges. Then

$$EBN\left(TA[n]\right)=(6n+6)\sqrt{\frac{60n-15}{(10n-2)(10n-3)}}+(6n-6)\sqrt{\frac{20n-6}{(5n-1)(5n-2)}}+(n-1)\sqrt{\frac{6}{5n-2}}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of $TA[n]$, we obtain

$$EBN\left(TA[n]\right)=\sum_{uv\in E(TA[n])}\sqrt{B(u)+B(v)}$$

$$=(6n+6)\sqrt{\left(\frac{3}{10n-2}\right)+\left(\frac{3}{10n-3}\right)}+(6n-6)\sqrt{\left(\frac{4}{10n-2}\right)+\left(\frac{4}{10n-4}\right)}$$

$$+(n-1)\sqrt{\left(\frac{6}{10n-4}\right)+\left(\frac{6}{10n-4}\right)}.$$

After simplification, we get the desired result.

We compute the modified E-Banhatti Nirmala index of $TA[n]$.

Theorem 2. Let G be the graph of a tetrameric 1,3-adamantane $TA[n]$ with $10n$ vertices and $13n-1$ edges. Then

$$^{m}EBN(TA[n])=(6n+6)\sqrt{\frac{(10n-2)(10n-3)}{60n-15}}+(6n-6)\sqrt{\frac{(5n-1)(5n-2)}{20n-6}}+(n-1)\sqrt{\frac{5n-2}{6}}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of $TA[n]$, we have

$$^{m}EBN(TA[n])=\sum_{uv\in E(TA[n])}\left[B(u)+B(v)\right]^{-\frac{1}{2}}$$

$$=(6n+6)\left[\left(\frac{3}{10n-2}\right)+\left(\frac{3}{10n-3}\right)\right]^{-\frac{1}{2}}+(6n-6)\left[\left(\frac{4}{10n-2}\right)+\left(\frac{4}{10n-4}\right)\right]^{-\frac{1}{2}}$$
$$+(n-1)\left[\left(\frac{6}{10n-4}\right)+\left(\frac{6}{10n-4}\right)\right]^{-\frac{1}{2}}$$

gives the desired result after simplification.

By using definitions and by cardinalities of the Banhatti edge partition of $TA[n]$, we obtain the E-Banhatti Nirmala and modified E-Banhatti Nirmala exponentials of $TA[n]$.

Theorem 3. The E-Banhatti Nirmala exponential of $TA[n]$ is given by

$$EBN(TA[n],x)=(6n+6)x^{\sqrt{\frac{60n-15}{(10n-2)(10n-3)}}}+(6n-6)x^{\sqrt{\frac{20n-6}{(5n-1)(5n-2)}}}+(n-1)x^{\sqrt{\frac{6}{5n-2}}}.$$

Theorem 4. The modified E-Banhatti Nirmala exponential of $TA[n]$ is given by

$$^{m}EBN(TA[n],x)=(6n+6)x^{\sqrt{\frac{(10n-2)(10n-3)}{60n-15}}}+(6n-6)x^{\sqrt{\frac{(5n-1)(5n-2)}{20n-6}}}+(n-1)x^{\sqrt{\frac{5n-2}{6}}}.$$

4. Conclusion

In this study, we have introduced the E-Banhatti Nirmala index and the modified E-Banhatti Nirmala index and their corresponding exponentials of a graph. These newly defined E-Banhatti Nirmala indices and their corresponding exponentials for some standard graphs and tetrameric 1,3-adamantane have been determined. This study is a new direction in Graph Indices.

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Conflict of interest. The authors declare that they have no conflict of interest.

Authors' Contributions. All the authors contributed equally to this work.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, Graph indices, in *Hand Book of Research in Advanced Applications of Graph Theory in Modern Society*, M.Pal, S.Samanta, A.Pal (eds.) IGI Global, USA (2020) 66-91. .
3. I.Gutman and O.E.Polansky, *Mathematical Concepts in Organic Chemistry*, Springer, Berlin (1986).
4. V.R.Kulli, New direction in the theory of graph index in graphs, submitted.
5. V.R.Kulli, Hyper E-Banhatti indices of certain networks, submitted.
6. H.Wiener, Structural determination of parattin boiling points, *Journal of the American Chemical Society*, 69(1) (1947) 17-20.
7. P.W.Fowler, G.Caporossi and P.Hansen, Distance matrices, Wiener indices and related invariants of fullerenes, *The Journal of Physical Chemistry A*, 105(25) (2001) 6232-6242.
8. I.Gutman and N. Trinajstić, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.*, 17 (1972) 535-538.
9. K.C.Das, I.Gutman and B.Horoldagva, Comparing Zagreb indices and coindices of trees, *MATCH Commun. Math. Comput. Chem.*, 67 (2012) 189-198.
10. V.R.Kulli, The sum connectivity Revan index of silicate and hexagonal networks, *Annals of Pure and Applied Mathematics*, 14(3) (2017) 401-406.
11. R.Aguilar-Sanchez, I.F.Herrera-Gonzalez, J.A.Mendez-Bermudez and J.M.Sigarreta, Revan degree indices on random graphs, arXiv:2210.04749v1[math.CO]10 Oct.2022.
12. V.R.Kulli, The Gourava indices and coindices of graphs, *Annals of Pure and Applied Mathematics*, 14(1) (2017) 33-38.
13. B.Basavanagoud and S.Policepatil, Chemical applicability of Gourava and hyper Gourava indices, *Nanosystems: Physics, Chemistry, Mathematics* 12(2) (2021) 142-150.
14. S.Ediz, Maximal graphs of the first reverse Zagreb beta index, *TWMS J. Appl.Eng. Math.* 8 (2018) 306-310.
15. V.R.Kulli, The reverse Zagreb and reverse hyper Zagreb indices and their polynomials of rhombus silicate networks, *Annals of Pure and Applied Mathematics*, 16(1) (2018) 47-51.
16. V.R.Kulli, On *K* Banhatti indices of graphs, *Journal of Computer and Mathematical Sciences*, 7 (2016) 213-218.
17. D.Zhao, M.A.Zahid, R.Irfan, M.Arshad, A.Fahad, Z.Ahmad and L.Li, R.Kulli, Banhatti, Revan and hyper indices of silicon carbide Si2C3-III[*n*,*m*], *Open Chemistry*, 19 (2021) 646-652.