Computation of E-Banhatti Nirmala Indices of Tetrameric 1,3-Adamantane

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Abstract. In this paper, we introduce the E-Banhatti Nirmala index and the modified E-Banhatti Nirmala index and their corresponding exponentials of a graph. Furthermore, we determine these newly defined E-Banhatti Nirmala indices and their corresponding exponentials for some standard graphs and tetrameric 1,3-Adamantane.

Keywords: E-Banhatti Nirmala index, modified E-Banhatti Nirmala index, tetrameric 1,3-adamantane.

AMS Mathematics Subject Classification): 05C05, 05C07, 05C09, 05C92

1. Introduction
Throughout this paper, we consider simple graphs which are finite, connected, undirected graphs without loops and multiple edges. Let $G$ be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$. For term and concept not given here, we refer the book [1].

A molecular graph is a simple graph, representing the carbon atom skeleton of an organic molecule of the hydrocarbon. Therefore the vertices of a molecular graph represent the carbon atoms and its edges the carbon-carbon bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. Several graph indices [2] have found some applications in Chemistry, especially in QSPR/QSAR research [3].

In [4], Kulli defined the Banhatti degree of a vertex $u$ of a graph $G$ as

$$B(u) = \frac{d_G(e)}{n - d_G(u)},$$

where $n$ is the number of vertices of $G$ and the vertex $u$ and edge $e$ are incident in $G$.

The first and second E-Banhatti indices were introduced by Kulli in [4] and they are defined as

$$EB_1(G) = \sum_{u \in V(G)} \left[B(u) + B(v)\right],$$

$$EB_2(G) = \sum_{u \in V(G)} B(u)B(v).$$
We propose the E-Banhatti Nirmala index of a graph $G$ and defined it as

$$EBN(G) = \sum_{uv \in E(G)} \sqrt{B(u) + B(v)}.$$

We introduce the modified E-Banhatti Nirmala index of a graph $G$ and defined it as

$$mEBN(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{B(u) + B(v)}}.$$

Considering the E-Banhatti Nirmala and modified E-Banhatti Nirmala indices, we define their corresponding exponentials of a graph $G$ as

$$EBN(G, x) = \sum_{uv \in E(G)} x^{\sqrt{B(u) + B(v)}},$$

$$mEBN(G, x) = \sum_{uv \in E(G)} \frac{1}{x^{\sqrt{B(u) + B(v)}}}.$$

In Chemical Graph Theory, several graph indices were introduced and studied such as the Wiener index [6, 7], the Zagreb indices [8, 9], the Revan indices [10, 11], the Gourava indices [12, 13], the reverse indices [14, 15] and the Banhatti indices [16, 17].

In this paper, we compute the E-Banhatti Nirmala index and the modified E-Banhatti Nirmala index for some standard graphs and tetrameric 1,3-adamanane.

2. Results for some standard graphs

**Proposition 1.** If $G$ is an $r$-regular graph with $n$ vertices and $r \geq 2$, then

$$EBN(G) = nr \sqrt{\frac{r-1}{n-r}}.$$

**Proof:** Let $G$ be an $r$-regular graph with $n$ vertices and $r \geq 2$. Then $G$ has \( \frac{nr}{2} \) edges. For any edge $uv=e$ in $G$, $d_G(e)=d_G(u)+d_G(u)-2=2r-2$.

From definition we have

$$EBN(G) = \sum_{uv \in E(G)} \sqrt{B(u) + B(v)}$$

$$= \frac{nr}{2} \sqrt{\frac{2r-2}{n-r} + \frac{2r-2}{n-r}} = nr \sqrt{\frac{r-1}{n-r}}.$$

**Corollary 1.1.** Let $C_n$ be a cycle with $n \geq 3$ vertices. Then

$$EBN(C_n) = 2n \sqrt{\frac{1}{n-2}}.$$
Corollary 1.2. Let $K_n$ be a complete graph with $n \geq 3$ vertices. Then
\[ EBN(K_n) = n(n-1)\sqrt{n-2}. \]

Proposition 2. Let $P_n$ be a path with $n \geq 3$ vertices. Then
\[
EBN(P_n) = 2\left[\left(\frac{1}{n-1}\right) + \left(\frac{2}{n-2}\right)\right] + (n-3)\left[\left(\frac{2}{n-2}\right)^{1/2} + \left(\frac{2}{n-2}\right)\right]
\]
\[= n\sqrt{\frac{3n-4}{(n-1)(n-2)}} + 2(n-3)\sqrt{\frac{1}{n-2}}.\]

Proposition 3. Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ and $n \geq 2$. Then
\[ EBN(K_{m,n}) = \sqrt{mn}\sqrt{(m+n)(m+n-2)}. \]

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m+n$ vertices and $mn$ edges such that $|V_1| = m$, $|V_2| = n$, and $V(K_{m,n}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of $V_1$ is incident with $n$ edges and every vertex of $V_2$ is incident with $m$ edges. Then
\[ d_G(e) = d_G(u) + d_G(v) - 2 = m + n - 2. \]
\[ EBN(K_{m,n}) = \sum_{uv \in E(G)} \sqrt{B(u) + B(v)} \]
\[= mn\left[\left(\frac{m+n-2}{m+n-n}\right) + \left(\frac{m+n-2}{m+n-m}\right)\right]^{1/2} \]
\[= \sqrt{mn}\sqrt{(m+n)(m+n-2)}. \]

Corollary 3.1. Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then
\[ EBN(K_{n,n}) = 2n\sqrt{n(n-1)}. \]

Corollary 3.2. Let $K_{1,n}$ be a star with $n \geq 2$. Then
\[ EBN(K_{1,n}) = \sqrt{n}\sqrt{(n+1)(n-1)}. \]

3. Results for Tetrameric 1,3-Adamantane
In Chemistry, diamondoids are variants of the carbon cage known as a damantane ($C_{10}, \ H_{16}$), the smallest unit cage structure of the diamond crystal lattice. We focus on the molecular graph structure of the family of tetrameric 1,3-adamantane, denoted by $TA[n]$. Let $G$ be the graph of a tetrameric 1,3-adamantane $TA[n]$. The graph of a tetrameric 1,3-adamantane $TA[4]$ is presented in Figure 1.
By calculation, $G$ has $10n$ vertices and $13n-1$ edges. Also by calculation, we obtain three edge partitions of $G$ based on the degrees of the end vertices of each edge as follows:

$$
E_1=\{uv\in E(G) \mid d_G(u)=2, \ d_G(v)=3\}, \quad |E_1|=6n+6.
$$
$$
E_2=\{uv\in E(G) \mid d_G(u)=2, \ d_G(v)=4\}, \quad |E_2|=6n-6.
$$
$$
E_3=\{uv\in E(G) \mid d_G(u)=d_G(v)=4\}, \quad |E_3|=n-1.
$$

Therefore, in $TA[n]$, there are three types of edges based on the Banhatti degree of end vertices of each edge as follow:

$$
BE_1=\{uv\in E(G) \mid B(u)=\frac{3}{10n-2}, \ B(v)=\frac{3}{10n-3}\}, \quad |BE_1|=6n+6.
$$
$$
BE_2=\{uv\in E(G) \mid B(u)=\frac{4}{10n-2}, \ B(v)=\frac{4}{10n-4}\}, \quad |BE_2|=6n-6.
$$
$$
BE_3=\{uv\in E(G) \mid B(u)=\frac{6}{10n-4}, \ B(v)=\frac{6}{10n-4}\}, \quad |BE_3|=n-1.
$$

We determine the E-Banhatti Nirmala index of $TA[n]$.

**Theorem 1.** Let $G$ be the graph of a tetrameric 1,3-adamantane $TA[n]$ with $10n$ vertices and $13n-1$ edges. Then

$$
EBN(TA[n])=(6n+6)\sqrt{\frac{60n-15}{(10n-2)(10n-3)}}+(6n-6)\sqrt{\frac{20n-6}{(5n-1)(5n-2)}}+(n-1)\sqrt{\frac{6}{5n-2}}.
$$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $TA[n]$, we obtain

$$
EBN(TA[n])=\sum_{uv\in E(TA[n])}\sqrt{B(u)+B(v)}
$$

$$
=(6n+6)\sqrt{\frac{3}{10n-2}}+(6n-6)\sqrt{\frac{3}{10n-3}}+(6n-6)\sqrt{\frac{4}{10n-2}}+(n-1)\sqrt{\frac{4}{10n-4}}
$$
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\[ + (n-1) \left( \frac{6}{10n-4} \right) + \left( \frac{6}{10n-4} \right) \]

After simplification, we get the desired result.

We compute the modified E-Banhatti Nirmala index of TA[n].

**Theorem 2.** Let \( G \) be the graph of a tetrameric 1,3-adamantane \( TA[n] \) with 10n vertices and 13n−1 edges. Then

\[ ^m EBN \left( TA[n] \right) = (6n+6) \sqrt{\frac{(10n-2)(10n-3)}{60n-15}} + (6n-6) \sqrt{\frac{(5n-1)(5n-2)}{20n-6}} + (n-1) \sqrt{\frac{5n-2}{6}}. \]

**Proof:** From definition and by cardinalities of the Banhatti edge partition of \( TA[n] \), we have

\[ ^m EBN \left( TA[n] \right) = \sum_{u \in E(TA[n])} \left[ B(u) + B(v) \right]^{\frac{1}{2}} \]

\[ = (6n+6) \left[ \left( \frac{3}{10n-2} \right) + \left( \frac{3}{10n-3} \right) \right]^{\frac{1}{2}} + (6n-6) \left[ \left( \frac{4}{10n-2} \right) + \left( \frac{4}{10n-4} \right) \right]^{\frac{1}{2}} \]

\[ + (n-1) \left[ \left( \frac{6}{10n-4} \right) + \left( \frac{6}{10n-4} \right) \right]^{\frac{1}{2}} \]

gives the desired result after simplification.

By using definitions and by cardinalities of the Banhatti edge partition of \( TA[n] \), we obtain the E-Banhatti Nirmala and modified E-Banhatti Nirmala exponentials of \( TA[n] \).

**Theorem 3.** The E-Banhatti Nirmala exponential of \( TA[n] \) is given by

\[ EBN \left( TA[n] , x \right) = (6n+6) x^{\frac{1}{60n-15}} (10n-2)(10n-3) + (6n-6) x^{\frac{1}{20n-6}} (5n-1)(5n-2) + (n-1) x^{\frac{1}{5n-2}}. \]

**Theorem 4.** The modified E-Banhatti Nirmala exponential of \( TA[n] \) is given by

\[ ^m EBN \left( TA[n] , x \right) = (6n+6) x^{\frac{1}{60n-15}} \left( \frac{(10n-2)(10n-3)}{60n-15} \right) + (6n-6) x^{\frac{1}{20n-6}} \left( \frac{(5n-1)(5n-2)}{20n-6} \right) + (n-1) x^{\frac{1}{5n-2}}. \]

**4. Conclusion**

In this study, we have introduced the E-Banhatti Nirmala index and the modified E-Banhatti Nirmala index and their corresponding exponentials of a graph. These newly defined E-Banhatti Nirmala indices and their corresponding exponentials for some standard graphs and tetrameric 1,3-adamantane have been determined. This study is a new direction in Graph Indices.
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REFERENCES