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Product Connectivity E-Banhatti Indices of Certain Nanotubes

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Abstract. The connectivity indices are applied to measure the chemical characteristics of chemical compounds in Chemical Sciences. Recently, a novel degree concept has been defined in Graph Theory: Banhatti degree of a vertex in a graph. In this paper, the product connectivity E-Banhatti and the reciprocal product connectivity E-Banhatti indices are defined by using Banhatti degree concept. We also compute these newly defined E-Banhatti indices of wheel graphs and certain nanotubes.

Keywords: product connectivity E-Banhatti index, reciprocal product connectivity E-Banhatti index, graph, nanotube.

AMS Mathematics Subject Classification (2010): 05C05, 05C07, 05C09, 05C92

1. Introduction

Let G be a simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. If $e=uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. The degree $d_G(e)$ of an edge e in G is defined as $d_G(e) = d_G(u) + d_G(v) – 2$ with $e=uv$. For term and concept not given here, we refer the book [1].

Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. Several graph indices have found some applications in Chemistry, especially in QSPR/QSAR research [2].

In [3], Kulli defined the Banhatti degree of a vertex u of a graph G as

$$B(u)=\frac{d_G(e)}{n-d_G(u)},$$

where n is the number of vertices of G and the vertex u and edge e are incident in G.

The first and second E-Banhatti indices were introduced by Kulli in [3] and they are defined as

$$EB_1(G)=\sum_{uv\in E(G)}[B(u)+B(v)],$$

$$EB_2(G)=\sum_{uv\in E(G)}B(u)B(v).$$

Recently, some E-Banhatti indices were studied for example, in [4, 5, 6, 7].
We introduce the product connectivity E-Banhatti index, defined as

$$PEB(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{B(u)B(v)}}.$$

We also propose the reciprocal product connectivity E-Banhatti index of a graph as

$$RPEB(G)=\sum_{uv\in E(G)}\sqrt{B(u)B(v)}.$$

We now introduce the connectivity E-Banhatti polynomials of a graph as follows:

The product connectivity E-Banhatti polynomial of a graph is defined as

$$PEB(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{B(u)B(v)}}}.$$

The reciprocal product connectivity E-Banhatti polynomial of a graph is defined as

$$RPEB(G,x)=\sum_{uv\in E(G)}x^{\sqrt{B(u)B(v)}}.$$

Recently, some connectivity indices were studied for example, in [8, 9, 10, 11, 12].
In this paper, the product connectivity E-Banhatti index, the reciprocal product connectivity E-Banhatti index and their corresponding polynomials of wheel graphs and certain nanotubes are computed.

2. Results for wheel graphs

A wheel graph W_n is the join of C_n and K_1. Then W_n has n+1vertices and $2n$ edges. A graph W_n is shown in Figure 1.

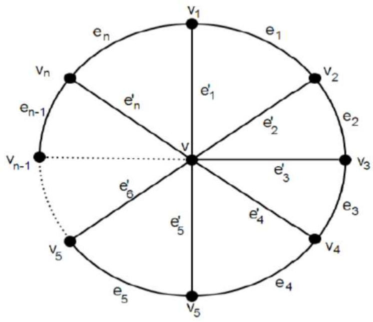

Figure 1: Wheel graph W_n

In W_n , there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}$, $\quad |E_1| = n.$

$E_2 = \{uv \in E(W_n) \mid d(u) =3, d(v) = n\}$, $\quad |E_2| = n.$

Therefore, in W_n, there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge as follow:

$BE_1 = \{uv \in E(W_n) \mid B(u) = B(v) = \frac{4}{(n-2)}\}$, $\quad |BE_1| = n.$

$BE_2 = \{uv \in E(W_n) \mid B(u) = \frac{n+1}{n-2}, B(v) = n+1\}$, $\quad |BE_2| = n.$

We calculate the product connectivity E-Banhatti index and its polynomial form of W_n as follows:

Theorem 1. Let W_n be a wheel graph. Then

(i) $$PEB(W_n)=\frac{1}{4}n(n-2)+\frac{n\sqrt{n-2}}{n+1}.$$

(ii) $$PEB(W_n,x)=nx^{\frac{n-2}{4}}+nx^{\frac{\sqrt{n-2}}{n+1}}.$$

Proof: Applying definition and Banhatti edge partitions of W_n, we conclude

(i) $$PEB(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{B(u)B(v)}}$$

$$=n\left[\left(\frac{4}{n-2}\right)\times\left(\frac{4}{n-2}\right)\right]^{-\frac{1}{2}}+n\left[\left(\frac{n+1}{n-2}\right)\times(n+1)\right]^{-\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$PEB(W_n,x)=\sum_{uv\in E(W_n)}x^{\frac{1}{\sqrt{B(u)B(v)}}}$$

$$=nx^{\left(\frac{4}{n-2}\times\frac{4}{n-2}\right)^{-\frac{1}{2}}}+nx^{\left(\frac{n+1}{n-2}\times(n+1)\right)^{-\frac{1}{2}}}.$$

By simplifying the above equation, we obtain the desired result.

We calculate the reciprocal product connectivity E-Banhatti index and its polynomial form of W_n as follows:

Theorem 2. Let W_n be a wheel graph. Then

(i) $$RPEB(W_n)=\frac{4n}{n-2}+\frac{n(n+1)}{\sqrt{n-2}}.$$

(ii) $$RPEB(W_n,x)=nx^{\frac{4}{n-2}}+nx^{\frac{n+1}{\sqrt{n-2}}}.$$

Proof: Applying definition and Banhatti edge partitions of W_n, we conclude

(i) $$RPEB(W_n)=\sum_{uv\in E(W_n)}\sqrt{B(u)B(v)}$$

$$=n\left[\left(\frac{4}{n-2}\right)\times\left(\frac{4}{n-2}\right)\right]^{\frac{1}{2}}+n\left[\left(\frac{n+1}{n-2}\right)\times(n+1)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we obtain the desired result.

(ii) $$RPEB(W_n,x)=\sum_{uv\in E(W_n)}x^{\sqrt{B(u)B(v)}}$$

$$=nx^{\left(\frac{4}{n-2}\times\frac{4}{n-2}\right)^{\frac{1}{2}}}+nx^{\left(\frac{n+1}{n-2}\times(n+1)\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

3. *H*-Naphtalenic Nanotubes

We consider a family of *H*-Naphtalenic nanotubes which is denoted by $NHPX[m, n]$, see Figure 2.

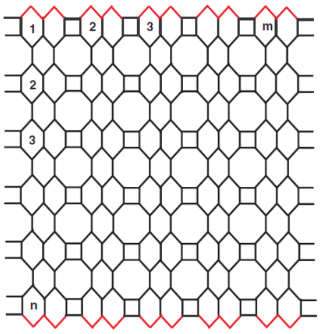

Figure 2: Graph of *H*-Naphtalenic nanotube

The graphs of a nanotube $NHPX[m, n]$ have $10mn$ vertices and $15mn – 2m$ edges are shown in the above graph. Let $G= NHPX[m, n]$.

In G, there are two types of edges as follows:

$E_1 = \{uv \in E(G) \mid d(u) = 2, d(v) = 3\}$, $|E_1| = 8m$.

$E_2 = \{uv \in E(G) \mid d(u) = d(v) = 3\}$, $|E_2| = 15mn – 10m$.

Therefore, in G, we obtain that $\{B(u), B(v): uv \in E(NHPX[m, n])\}$ has two Banhatti edge set partitions.

$$BE_1 = \{uv \in E(G) \mid B(u) = \frac{3}{10mn-2}, B(v) = \frac{3}{10mn-3}\}, \qquad |BE_1| = 8m.$$

$$BE_2 = \{uv \in E(G) \mid B(u) = B(v) = \frac{4}{10mn-3}\}, \qquad |BE_2| = 15mn-10m.$$

We calculate the product connectivity E-Banhatti index and its polynomial form of G as follows:

Theorem 3. Let $G= NHPX[m, n]$. Then

(i) $PEB(G) = \frac{8}{3}m\sqrt{(10mn-2)(10mn-3)} + \frac{1}{4}(15mn-10m)(10mn-3).$

(ii) $PEB(G,x) = 8mx^{\frac{1}{3}\sqrt{(10mn-2)(10mn-3)}} + (15mn-10m)\,x^{\frac{1}{4}(10mn-3)}.$

Proof: Applying definition and Banhatti edge partitions of G, we conclude

(i) $$PEB(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{B(u)B(v)}}$$

$$= 8m\left[\frac{3}{10mn-2}\times\frac{3}{10mn-3}\right]^{-\frac{1}{2}} + (15mn-10m)\left[\frac{4}{10mn-3}\times\frac{4}{10mn-3}\right]^{-\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$PEB(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{B(u)B(v)}}}$$

$$=8mx^{\left[\frac{3}{10mn-2}\times\frac{3}{10mn-3}\right]^{-\frac{1}{2}}}+(15mn-10m)\,x^{\left[\frac{3}{10mn-2}\times\frac{3}{10mn-3}\right]^{-\frac{1}{2}}}.$$

By simplifying the above equation, we get the required solution.

We calculate the reciprocal product connectivity E-Banhatti index and its polynomial form of a wheel graph W_n as follows:

Theorem 4. Let $G= NHPX[m, n]$. Then

(i) $$RPEB(G)=\frac{24m}{\sqrt{(10mn-2)(10mn-3)}}+\frac{4(15mn-10m)}{10mn-3}.$$

(ii) $$RPEB(G,x)=8mx^{\frac{3}{\sqrt{(10mn-2)(10mn-3)}}}+(15mn-10m)\,x^{\frac{4}{10mn-3}}.$$

Proof: Applying definition and Banhatti edge partitions of G, we conclude

(i) $$RPEB(G)=\sum_{uv\in E(G)}\sqrt{B(u)B(v)}$$

$$=8m\left[\frac{3}{10mn-2}\times\frac{3}{10mn-3}\right]^{\frac{1}{2}}+(15mn-10m)\left[\frac{4}{10mn-3}\times\frac{4}{10mn-3}\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$RPEB(G,x)=\sum_{uv\in E(G)} x^{\sqrt{B(u)B(v)}}$$

$$=8mx^{\left(\frac{3}{10mn-2}\times\frac{3}{10mn-3}\right)^{\frac{1}{2}}}+(15mn-10m)\,x^{\left(\frac{4}{10mn-3}\times\frac{4}{10mn-3}\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we obtain the necessary result.

4. Conclusion

In this study, we have introduced some new E-Banhatti indices of a graph. Furthermore, we have computed the product connectivity E-Banhatti index, the reciprocal product connectivity E-Banhatti index and their corresponding polynomial versions of wheel graphs and certain nanotubes.

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Conflict of interest. This is a single-author paper, so there is no scope for a conflict of interest.

Authors' Contributions. This is the sole work of the author.

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